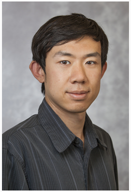
On the Local Minima of the Empirical Risk

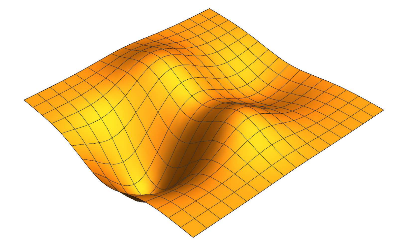
Chi Jin*[1], Lydia T. Liu*[1], Rong Ge[2], Michael I. Jordan[1]

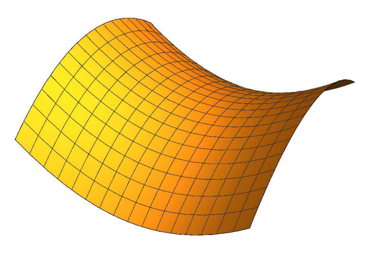
[1]EECS, University of California, Berkeley. [2]Duke University.



Overview

Nonconvex Optimization.





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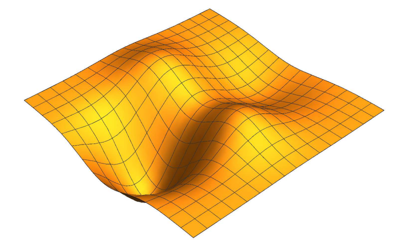

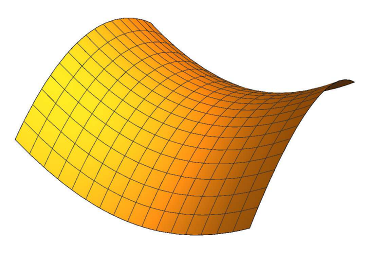

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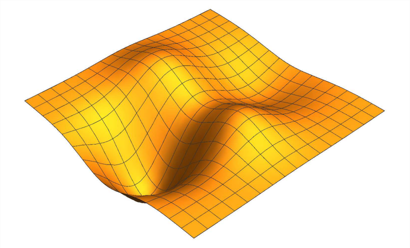

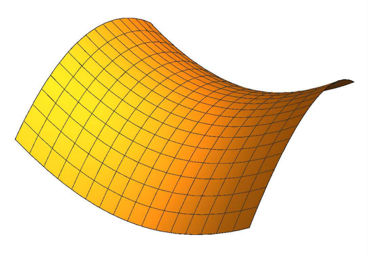

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- **How to deal with spurious local min?**

Local Minima

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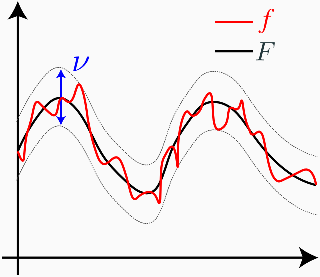


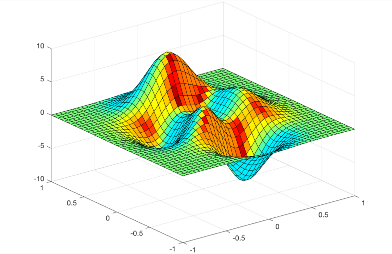
Avoiding "shallow" local minima

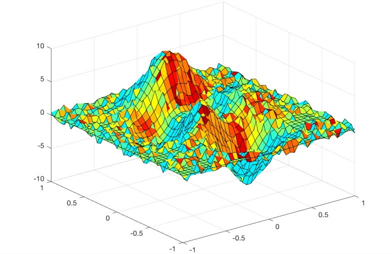
Goal: finds **approximate local minima** of **smooth** nonconvex function F, given only access to an errorneous version f where $\sup_{\mathbf{x}} |F(\mathbf{x}) - f(\mathbf{x})| \le \nu$

Statistical Learning.

Minimize population risk R while only have access to emprical risk $\hat{R}_n$.

$$R(\theta) = \mathbb{E}_{\mathbf{z}\sim\mathcal{D}}[L(\theta;\mathbf{z})], \qquad \hat{R}_n(\theta) = \frac{1}{n}\sum_{i=1}^{n} L(\theta;\mathbf{z}_i).$$





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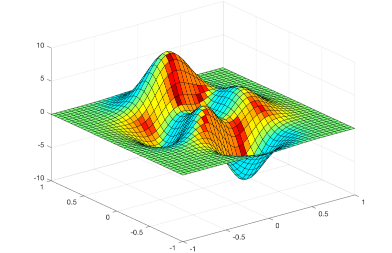
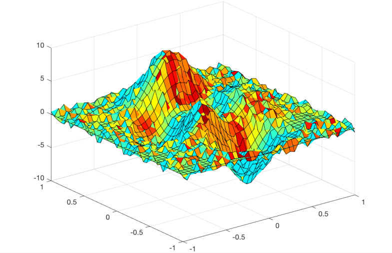
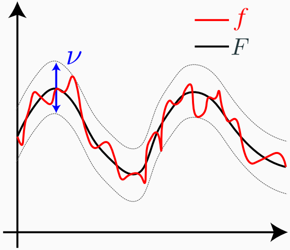




Unifrom convergence guarantees $\sup_\theta |R(\theta) - \hat{R}_n(\theta)| \le O(1/\sqrt{n})$.

Results



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Central Questions:

1. **What algorithm can achieve this**?
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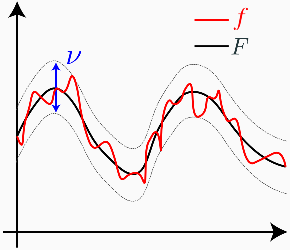

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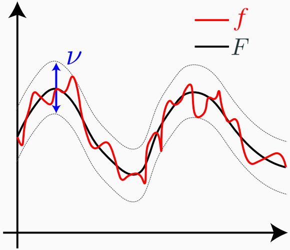

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This Work: Perturbed SGD on a "smoothed" version of f if $\nu \leq \epsilon^{1.5}/d$.

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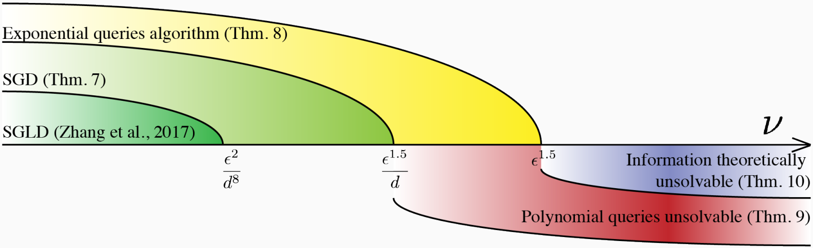

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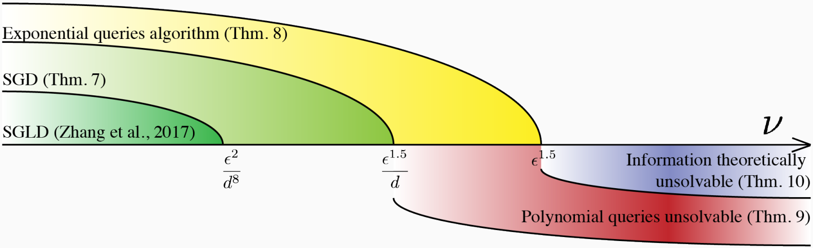

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Poster: Wed 5-7 PM, #43. Thanks!