

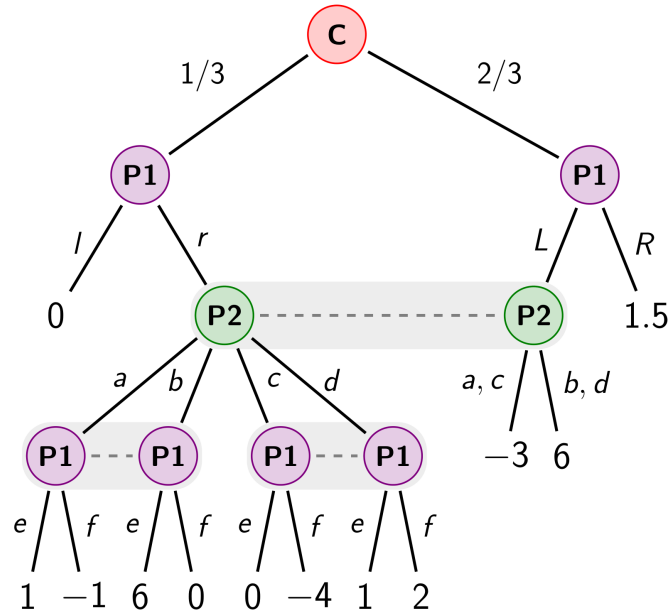
Solving Large Sequential Games with the Excessive Gap Technique

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IEOR in 2019

Extensive-Form Games



Applications - poker

Nash Equilibrium approximation used in recent breakthroughs

- Heads-Up Limit Texas Hold’Em [Bowling et al. 2015]
- Heads-Up No-Limit Texas Hold’Em [Brown and Sandholm 2017, Moravcik et al. 2017]

CFR, or variants, used to compute equilibria

BRAINS VS. ARTIFICIAL INTELLIGENCE
Be sure to tweet @WinBigRivers and @SCSatCMU using #BrainsvsAI

JANUARY 11-30 | 11AM-7PM
120,000 HANDS NO-LIMIT HOLD’EM
Each hand starts with each player having 200 big blinds.
One big blind is \$100, and one small blind is \$50.

Hands Dealt: 55,240/120,000

BRAINS : (\$676,584)	LIBRATUS : \$676,584
DONG KIM : (\$114,562)	JASON LES : (\$166,736)
LIBRATUS : \$114,562	LIBRATUS : \$166,736
JIMMY CHOU : (\$225,785)	DANIEL MCAULAY : (\$169,501)
LIBRATUS : \$225,785	LIBRATUS : \$169,501

Parentheses indicate a negative number.

Logos: CMU, TROIAN UNIVERSITY, Carnegie Mellon University School of Computer Science, RIVERS CASINO, PITTSBURGH SUPERCOMPUTING CENTER, CREATIONS! TNO, INTEL, OPTIMIZED



How compute a zero-sum Nash equilibrium

Linear programming [von Stengel 96]

Simplex and IPM too slow in practice

CFR and variants [Zinkevich et al. 07, Tammelin et al 15]

$\frac{1}{\sqrt{T}}$ in theory

Better than $\frac{1}{T}$ in practice

First-order methods, [Hoda et al 10, Kroer et al 18]

$\frac{1}{T}$ in theory

$\frac{1}{T}$ in practice

Practical Excessive Gap Technique

We introduce a practical variant of EGT

- EGT constructs smoothed approximations to the optimization problems faced by each player [Nesterov 05, Hoda et al 10, Kroer et al 18]
- We use dilated entropy DGF from [Kroer et al 18]
- Aggressive stepsizing
- Balancing of smoothing on each player
- Numerically-friendly smoothed best response computation
- GPU parallelization across different hands dealt

Experiments

Real-time subgames from Brains vs AI competition

 Last betting round of game

 43k/86k actions per player, 54M leaves

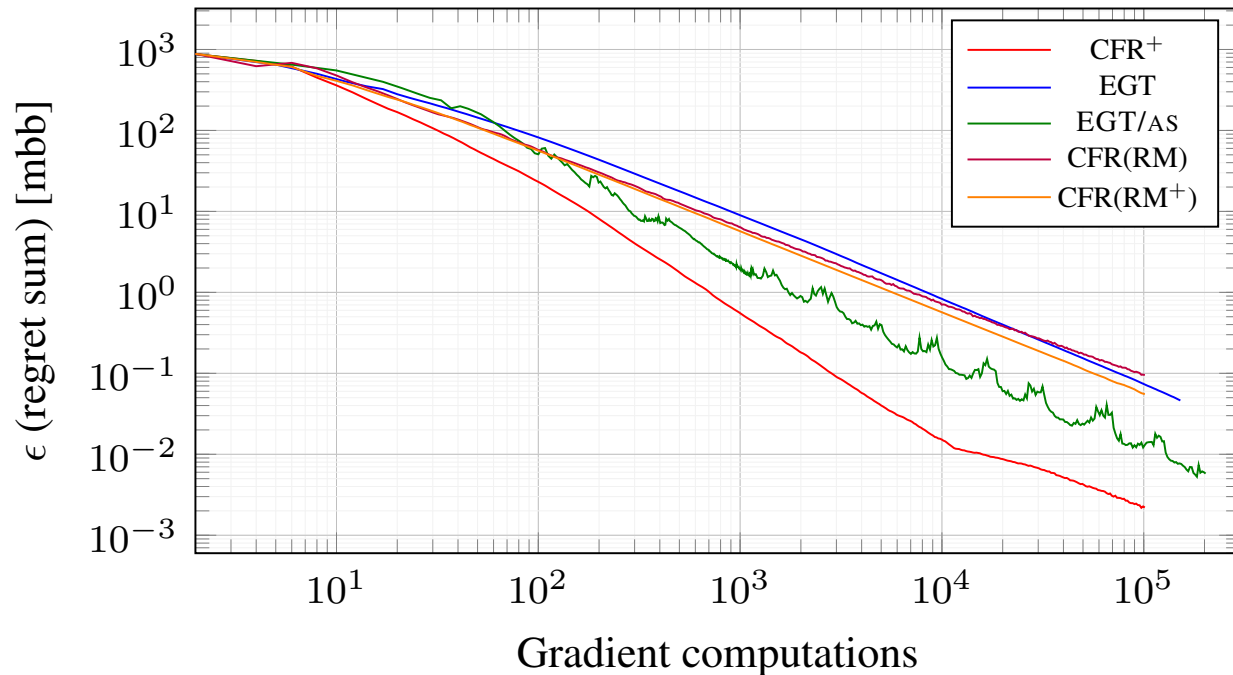
EGT with Kroer et al 18 smoothing function

Our Aggressive EGT

Three CFR variants

Comparison to existing algorithms

Endgame 7



Conclusion

- We introduce aggressive EGT variant
- Give first comparison of FOMs and CFR on real, large-scale games
- First-order methods can be made faster than all but the best practical variant of CFR

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Paper at www.christiankroer.com/publications