Inference on the Change Point in High Dimensional Graphical Models

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Introduction

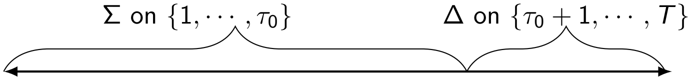
- Undirected Graphical Models are utilized in a host of applications, e.g., **Classification, Dimension reduction,** and **Inference.**
- For a random vector, $(x_1, x_2, x_3)^T$, collection of all conditional dependencies is called the Graphical model.
- Let $\Sigma = \text{Cov}(x)$ and $\Omega = \Sigma^{-1}$ be the Precision matrix.
- We know that Ω characterizes conditional dependencies.

Dynamic Graphical Model

- Consider a dynamic version Graphical models,

$$z_t = \begin{cases} w_t, & t = 1, ..., \tau^0 \\ x_t, & t = \tau^0 + 1, ..., T, \end{cases} .$$

- Here $w_t \sim i.i.d(0, \Sigma)$ and $x_t \sim i.i.d(0, \Delta)$.
- We observe p-dimensional $z_t = (z_{t1}, ...z_{tp})^T$, $t = 1, ..., T$
- Allow p to be very large, $\log p = o(T^\delta)$.



- Main objective: recover transition point τ^0.

Methods

- Define a squared loss,

$$Q(\tau,\mu,\gamma) = \sum_{t=1}^{\tau}\sum_{j=1}^{p}(z_{tj} - z_{t,-j}^T\mu_{(j)})^2 + \sum_{t=(\tau+1)}^{T}\sum_{j=1}^{p}(z_{tj} - z_{t,-j}^T\gamma_{(j)})^2,$$

- Regress the j^{th} component z_{tj} on the rest $z_{t,-j}$. Followed by an ℓ_2 aggregation of all p-regressions.

- For each $j = 1, ..., p$, define,

$$\hat{\mu}_{(j)}(\tau) = \underset{\mu_{(j)}\in\mathbb{R}^{p-1}}{\arg\min}\left\{\frac{1}{\tau}\sum_{t=1}^{\tau}\left(z_{tj} - z_{t,-j}^T\mu_{(j)}\right)^2 + \lambda_j\|\mu_{(j)}\|_1\right\},$$

- Symmetrically for $\hat{\gamma}_{(j)}(\tau)$.

- Main idea: iterate between the *Change point* and *Regression* parameters via the considered squared loss and lasso estimates.

Main Message

- Iterating twice between change point and pseudo regression parameters yields an optimal estimator for the point of transition.
- Estimator possesses good statistical properties, including ability to precisely measure uncertainty as,

$$\Big[\big(T\tilde{\tau} - ME\big),\ \big(T\tilde{\tau} + ME\big)\Big],$$

- where margin of error (ME) can be computed at any given level of uncertainty (e.g, 5%)
- All results despite high dimensionality.

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- **Full article: jmlr.org/papers/v24/22-1122.html**