# Quantum algorithm for large-scale market equilibrium computation


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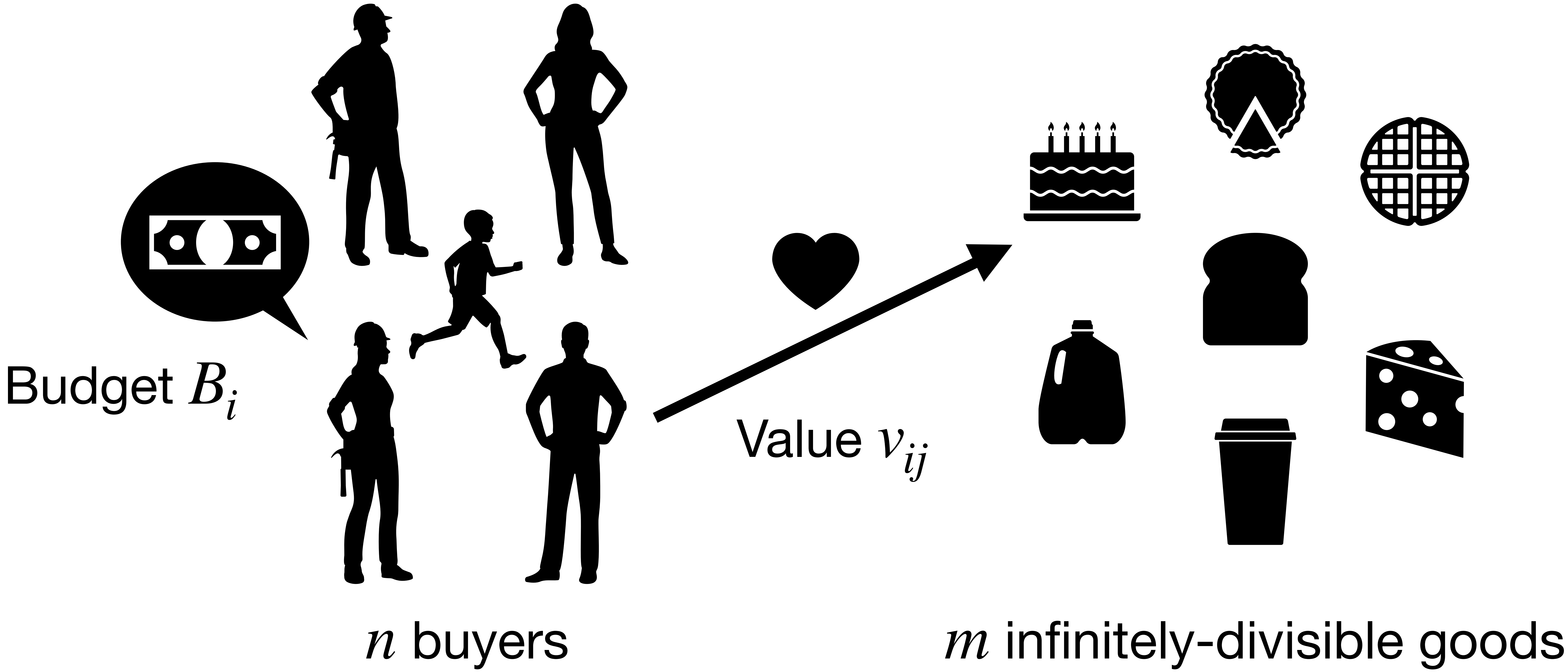
# Market equilibrium computation

## Problem statement for Fisher markets

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# Market equilibrium computation

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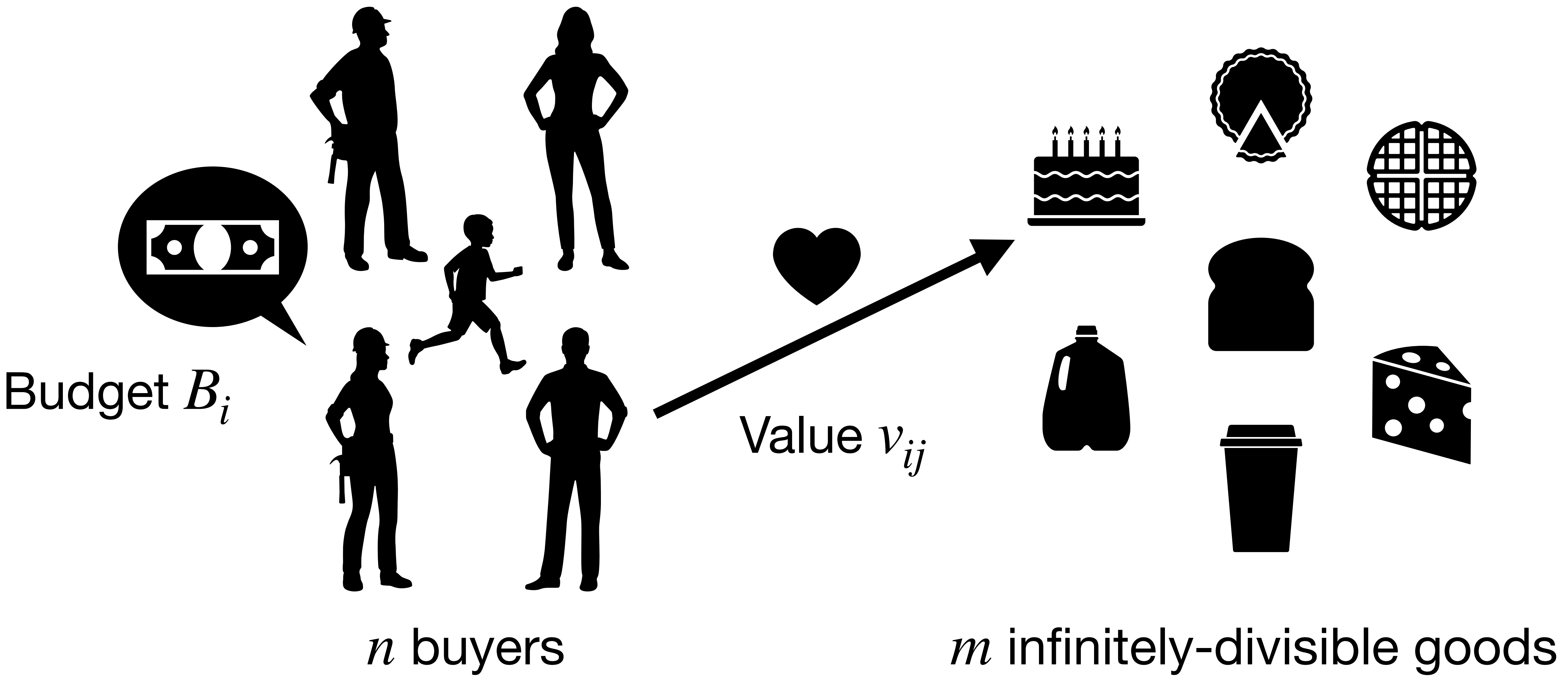


Assumption:

Utility is linear.

$$u_i = \sum_j v_{ij} x_{ij}$$

Goal: Find Allocation $x_{ij}$ and Price $p_j$ OR Bid $b_{ij}$

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# Main results

| Algorithm | Iterations | Runtime | Memory | Result Preparation |
|---|---|---|---|---|
| PR dynamics [WZ07] | $\frac{\log m}{\varepsilon}$ | $\tilde{\mathcal{O}}\left(\frac{mn}{\varepsilon}\right)$ | $\mathcal{O}(mn)$ | N/A, in RAM |
| Our work | $\frac{2\log m}{\varepsilon}$ | $\tilde{\mathcal{O}}\left(\frac{\sqrt{mn\max(m,n)}}{\varepsilon^2}\right)$ | $\mathcal{O}(m+n)$ * | QA: $\mathcal{O}\left(\text{poly}\log\frac{mn}{\varepsilon}\right)$<br>SA: $\tilde{\mathcal{O}}\left(\sqrt{mn}\right)$ |

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# Market equilibrium computation

## Optimization-based solutions

Eisenberg-Gale convex program [EG59]:

$$\max_{x\geq 0} \sum_i B_i \log u_i$$

$$s.t. \sum_j v_{ij} x_{ij} = u_i$$

$$\sum_i x_{ij} = 1$$

Proportional response dynamics [WZ07]:

$$p_j^{(t)} = \sum_i b_{ij}^{(t)}, \qquad x_{ij}^{(t)} = \frac{b_{ij}^{(t)}}{p_j^{(t)}}$$

$$u_i^{(t)} = \sum_j v_{ij} x_{ij}^{(t)}, \qquad b_{ij}^{(t+1)} = B_i \frac{v_{ij} x_{ij}^{(t)}}{u_i^{(t)}}.$$

Convergence guarantee [BDX11]:

$$\Psi(b^{(T)}) - \Psi(b^*) \leq \frac{\log m}{T}$$

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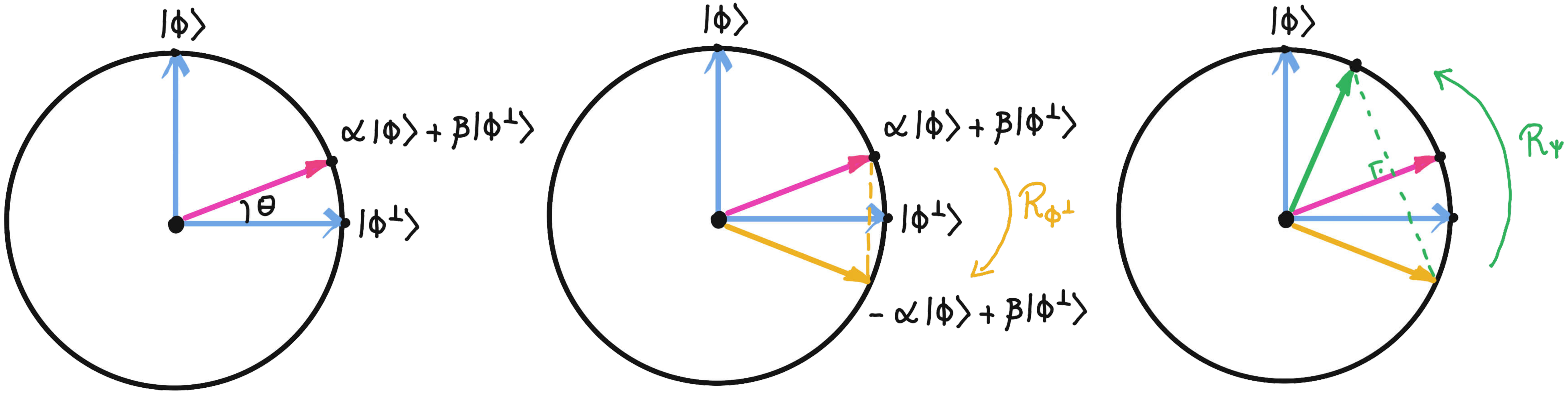
# Quantum computation

## Source of quantum speedups

Quantum amplitude amplification and estimation (QAA and QAE) [BHMT02]




https://pennylane.ai/qml/demos/tutorial_intro_amplitude_amplification/

Used for fast $\ell_1$ norm estimation and fast inner product estimation. [RHR+21]

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# Quantum computation

## Quantum data access and quantum memory

Quantum access of vector $w \in \mathbb{R}^n$

1. Quantum query access:

$$|\mathbf{j}\rangle|0\rangle \rightarrow |\mathbf{j}\rangle|w_j\rangle$$

2. Quantum sample access:

$$|0\rangle \rightarrow \sum_{j=0}^{n-1} \sqrt{\frac{w_j}{\|w\|_1}}|\mathbf{j}\rangle$$

Quantum random access memory (QRAM):

Memory unit that allows quantum query and sample access to vector $w \in \mathbb{R}^n$.

Access time: $\mathcal{O}(\text{poly} \log n)$

One time construction cost: $\tilde{\mathcal{O}}(n)$

Construction cost is non-trivial.

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# Faulty proportional response (FPR) dynamics

## Making the algorithm robust to estimation errors

Modified updates:

$$\tilde{p}_j^{(t)} = \left(\sum_i \hat{b}_{ij}^{(t)}\right)(1 \pm \varepsilon_p), \quad (\Pi_p^{(t)})_j = \prod_{k=0}^{t} \tilde{p}_j^{(k)}, \quad \hat{x}_{ij}^{(t)} = \frac{\hat{b}_{ij}^{(t)}}{\tilde{p}_j^{(t)}} = \frac{B_i^{t+1} v_{ij}^{t}}{m(\Pi_p^{(t)})_j(\Pi_\nu^{(t-1)})_i}$$

$$\tilde{\nu}_i^{(t)} = \left(\sum_j v_{ij}\hat{x}_{ij}^{(t)}\right)(1 \pm \varepsilon_\nu), \quad (\Pi_\nu^{(t)})_i = \prod_{k=0}^{t} \tilde{\nu}_i^{(k)}, \quad \hat{b}_{ij}^{(t+1)} = B_i \frac{v_{ij}\hat{x}_{ij}^{(t)}}{\tilde{\nu}_i^{(t)}} = \frac{B_i^{t+2} v_{ij}^{t+1}}{m(\Pi_p^{(t)})_j(\Pi_\nu^{(t)})_i}.$$

Convergence guarantee:

If $\varepsilon_p \leq \frac{\log m}{8T}, \varepsilon_\nu \leq \frac{\log m}{6T}, t^* = \arg\max_{t \in [T]} \sum_i B_i \tilde{\nu}_i^{(t)}$, then $\Psi(b^{(t^*)}) - \Psi(b^*) \leq \frac{2 \log m}{T}$

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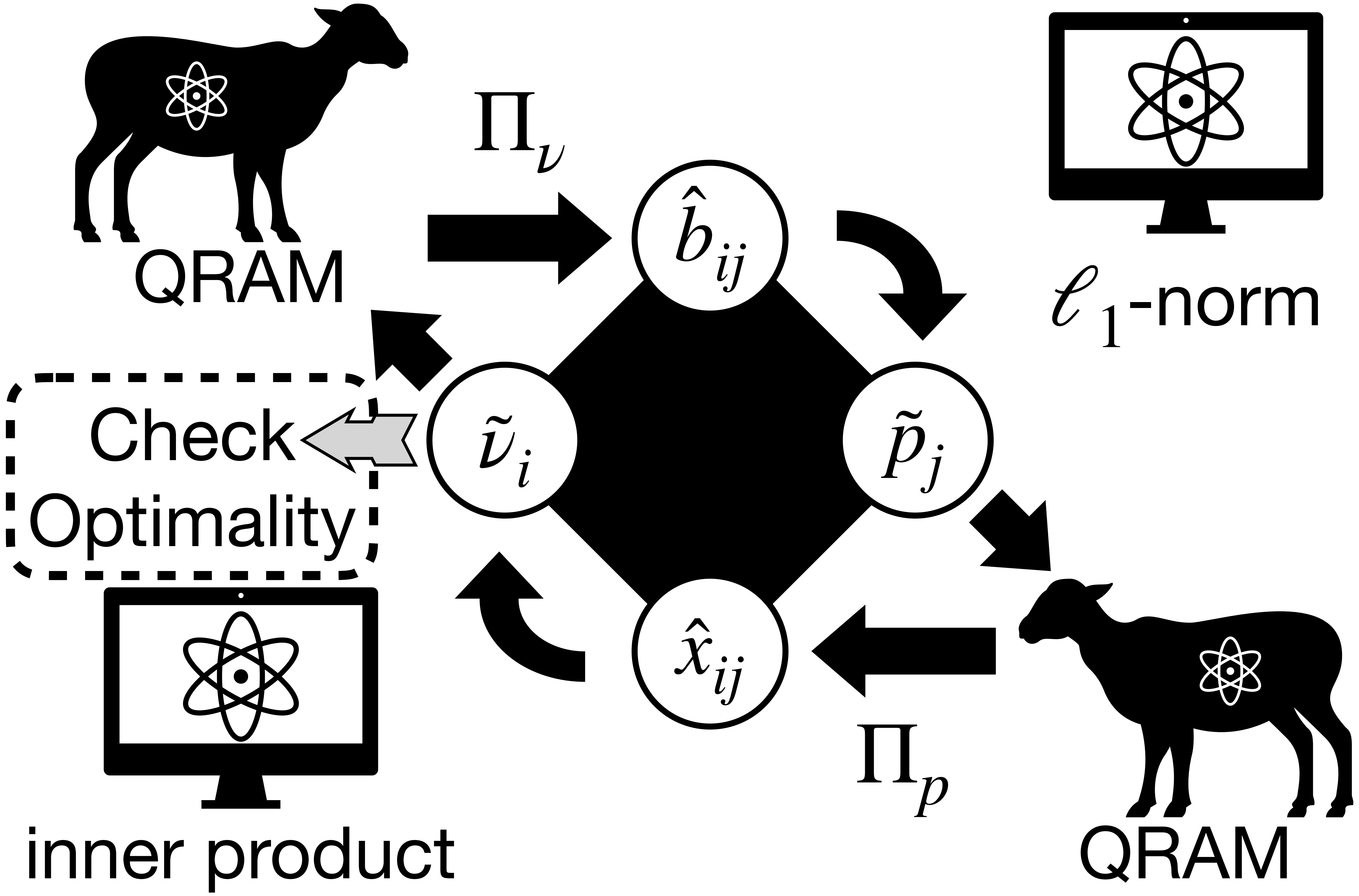
# Quantum algorithm

## Combining quantum subroutines with FPR updates




Runtime analysis:

$\ell_1$-norm: $\mathcal{O}\left(\frac{\sqrt{n}}{\varepsilon_p}\log\frac{mT}{\delta}\right)\times mT$

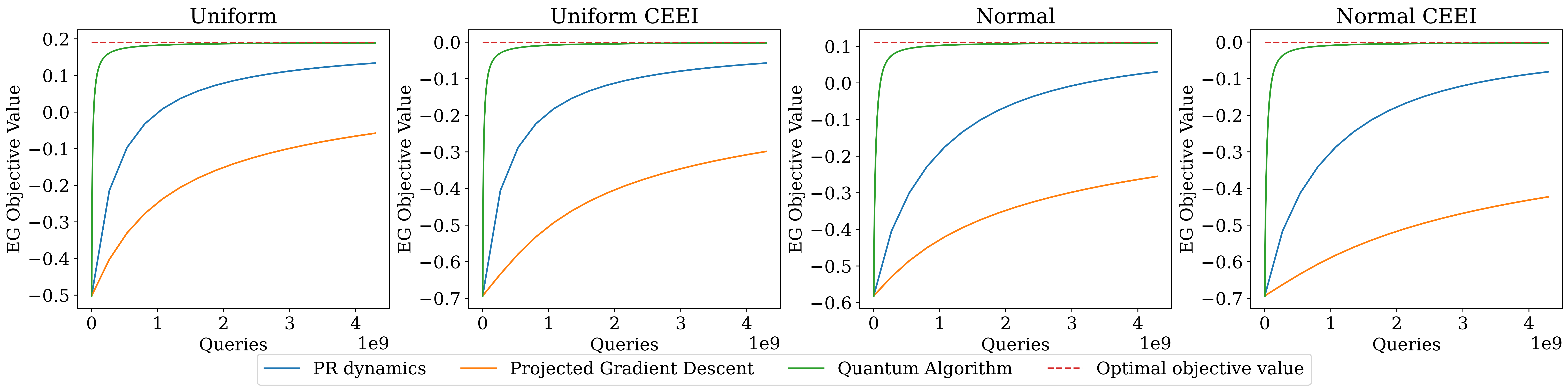
Inner prod: $\mathcal{O}\left(\frac{\sqrt{m}}{\varepsilon_\nu}\log\frac{nT}{\delta}\right)\times nT$

QRAM construction: $\tilde{\mathcal{O}}((m+n)T)$

Optimality check: $\mathcal{O}(nT)$

# Simulation results

## Proof of concept simulations on a classical computer

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- [WZ07] Wu and Zhang, in STOC'07, 354–363.
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- [RHR+21] Rebentrost, Hamoudi, Ray, Wang, Yang, and Santha, Phys. Rev. A 103, 012418.


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