

Addressing Spatial-Temporal Heterogeneity: General **Mixed Time Series** Analysis via Latent Continuity Recovery and Alignment

Jiawei Chen, Chunhui Zhao

State Key Laboratory of Industrial Control Technology, College of Control Science and Engineering,
Zhejiang University, China

Time Series Analysis

➤ Application



Industry



Energy



Climate



Traffic

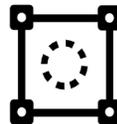
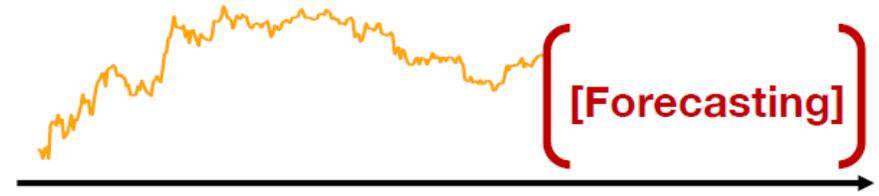


Health care



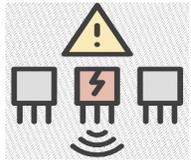
Finance

➤ Tasks



Mixed Time Series

- **Mixed Time Series** encompass both **continuous variables (CVs)** and **discrete variables (DVs)** are frequently encountered in practice, e.g., finance, health care, industry, weather.
- Due to **external factors**, many **intrinsically continuous signals** are often **recorded with discrete forms**, e.g., meteorological data, stock returns, and health states.



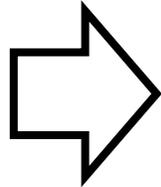
Measurement
Limitations



Storage
Requirements



Transmission
interference



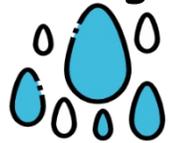
Temperature



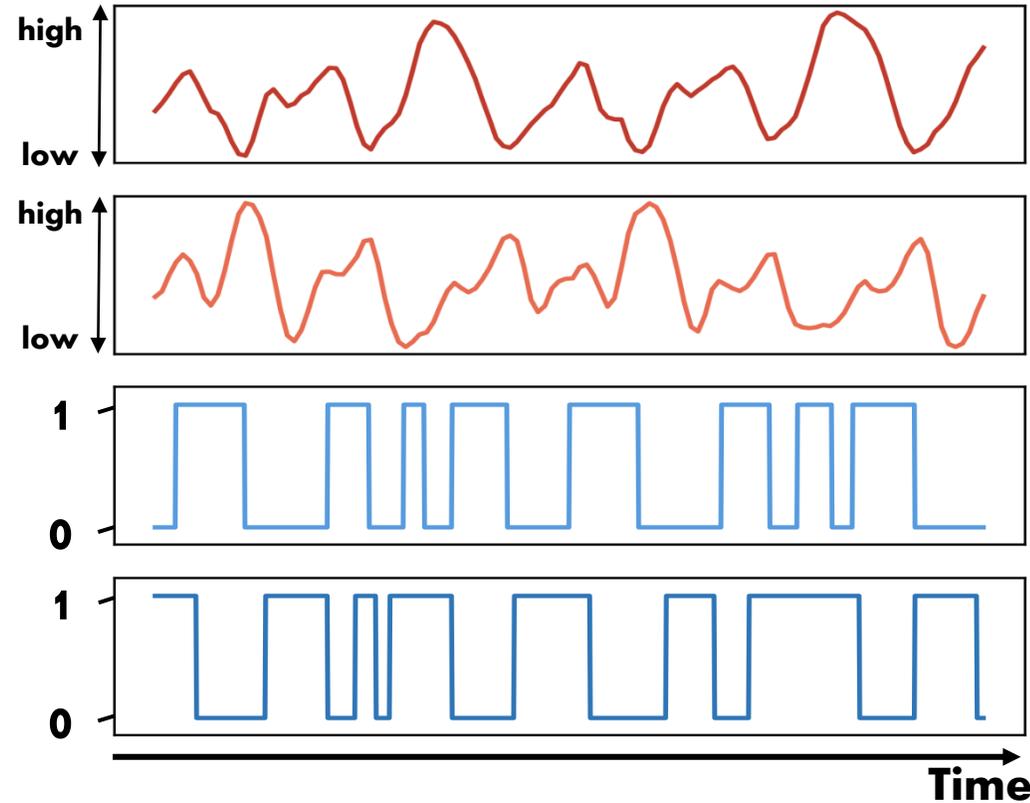
Humidity



Cloudage



Rainfall

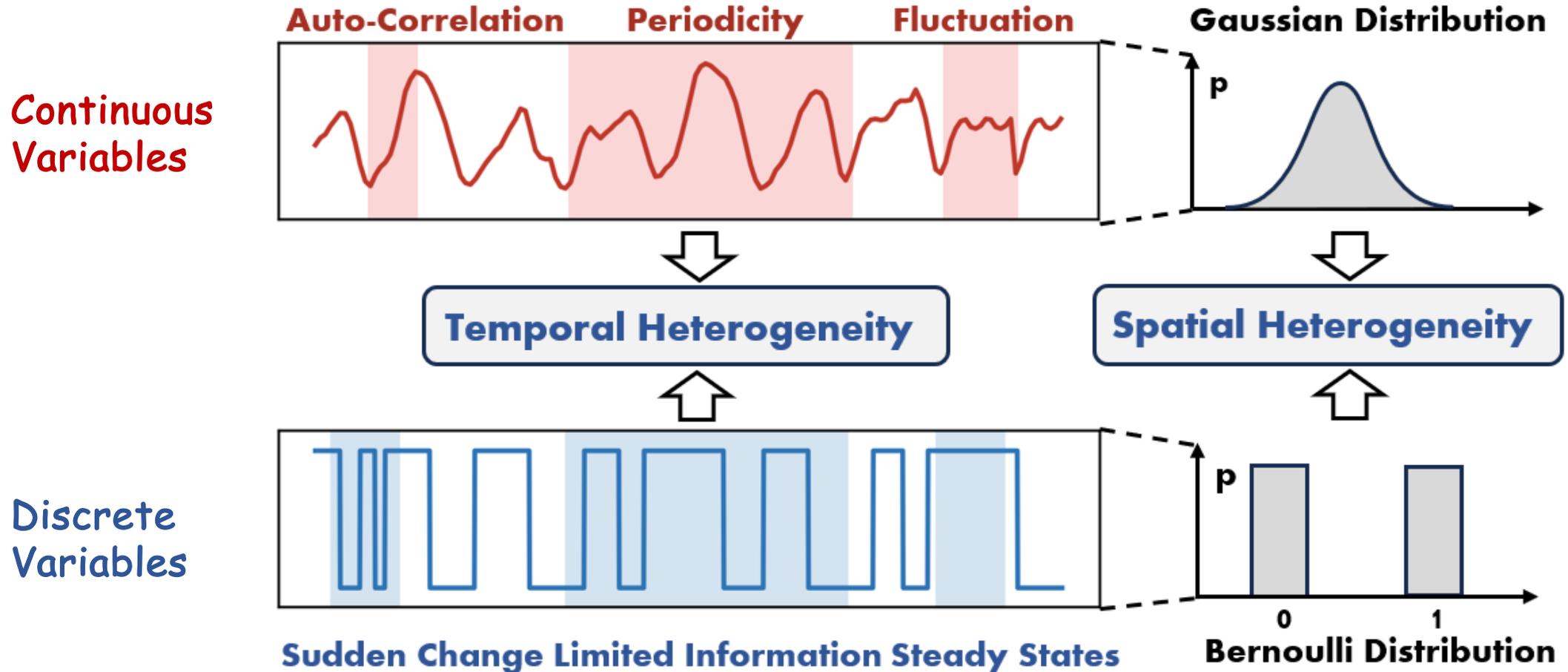


Mixed Time Series

External Factors

Spatial-Temporal heterogeneity

- The discrepancies in **temporal variation properties** and **distribution types** between CVs and DVs cause Spatial-Temporal heterogeneity challenge.

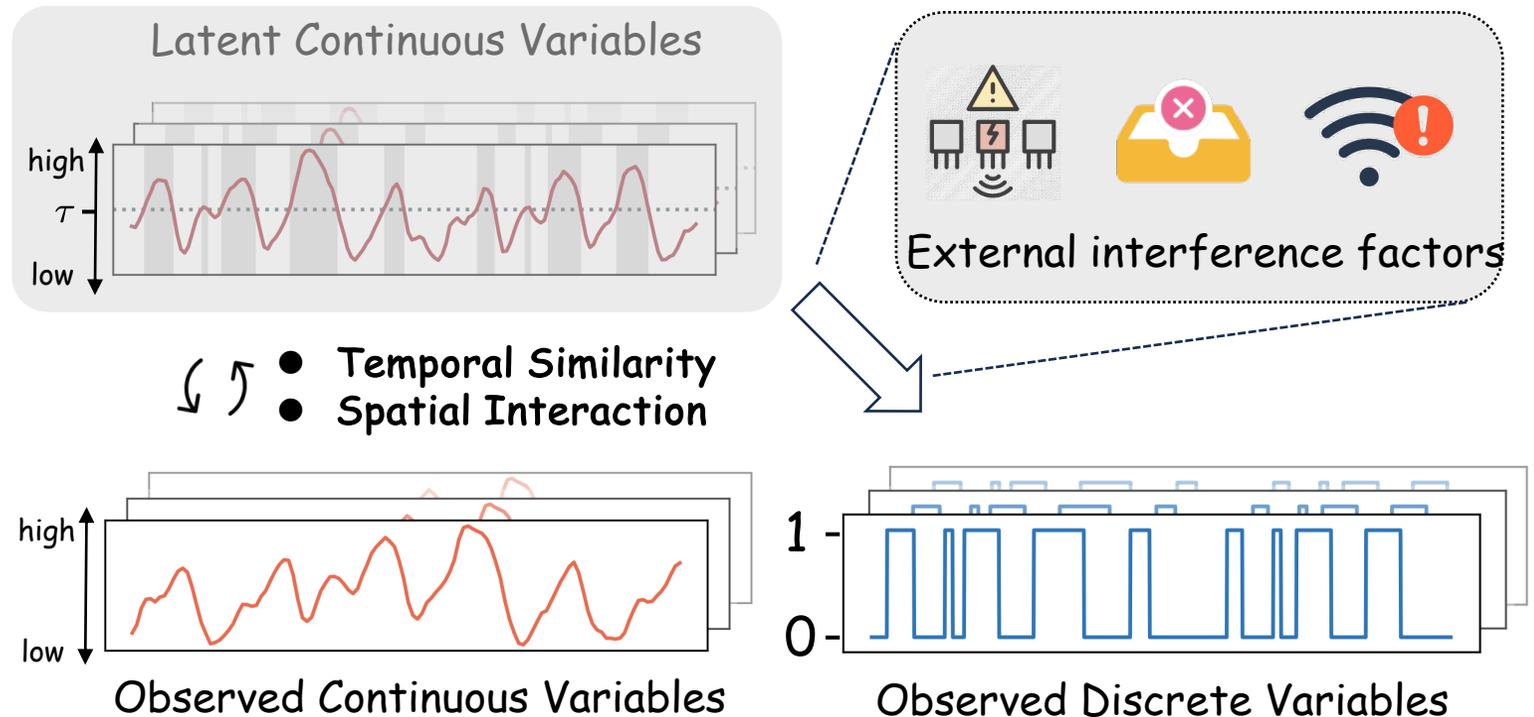


Insights and Motivations

🌟 Key insights:

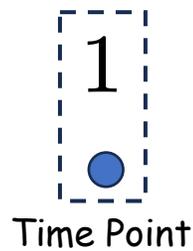
- 1) DVs may **originate from intrinsic latent continuous variables (LCVs)**, which lose fine-grained information due to extrinsic discretization;
- 2) LCVs and CVs share **similar temporal patterns and interact spatially**.

- **(Temporal Similarity):** The LCVs share similar temporal variation patterns with the observed CVs .
- **(Spatial Interaction):** LCVs and CVs exhibit information interaction and inter-variable spatial correlations.

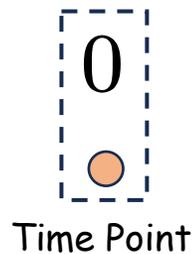
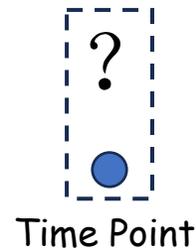
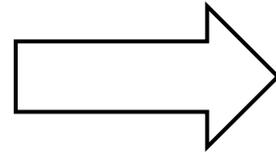


Latent Continuity Recovery

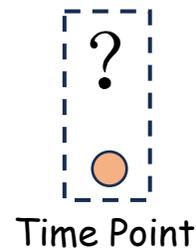
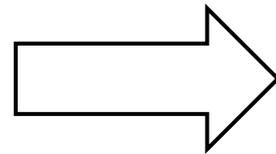
🤔 How to restore/map a discrete value point with a value of {0 or 1} to a [0~1] continuous value ?



Recovery

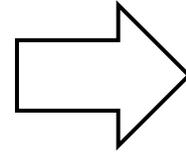


Recovery

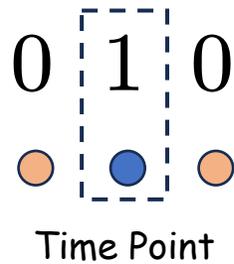


Latent Continuity Recovery

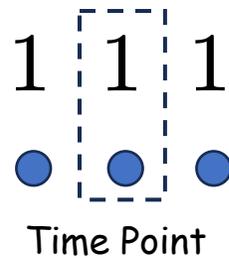
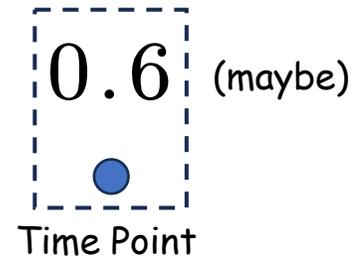
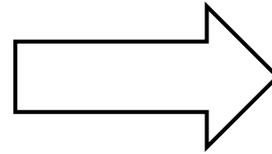
• Natural **autocorrelation / temporal adjacency** properties of time series



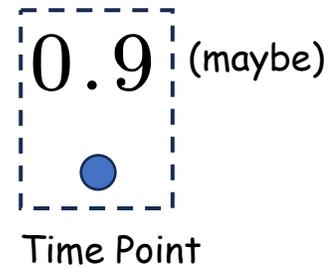
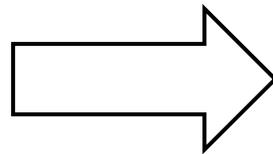
• Model's **inductive bias**



Recovery



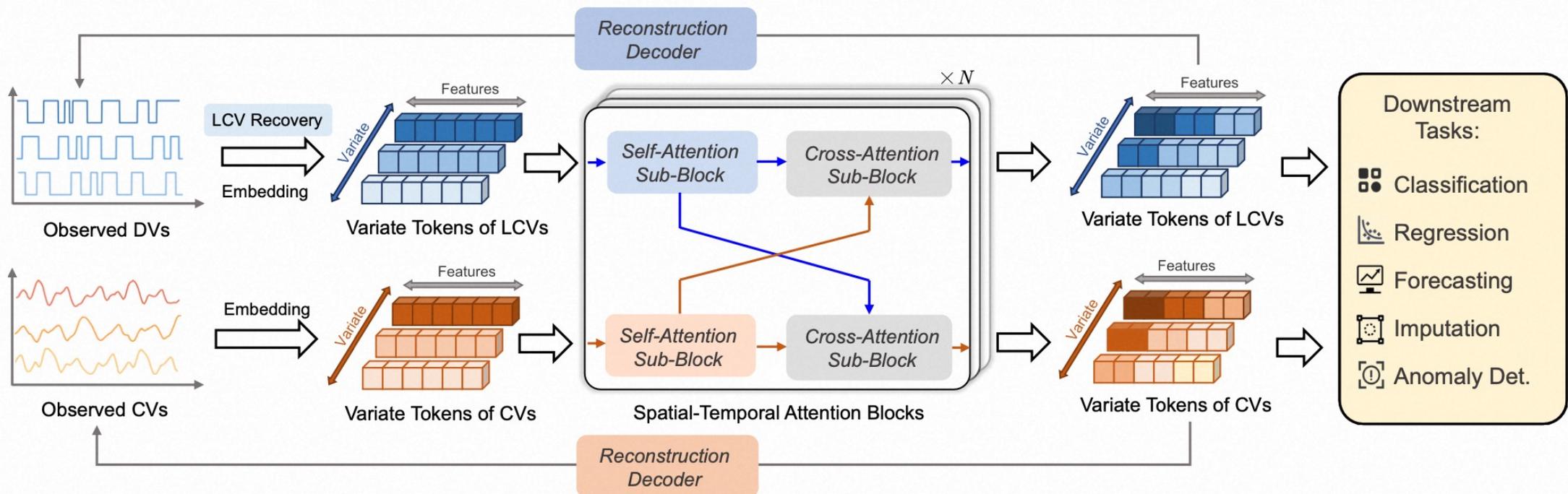
Recovery



MiTSformer

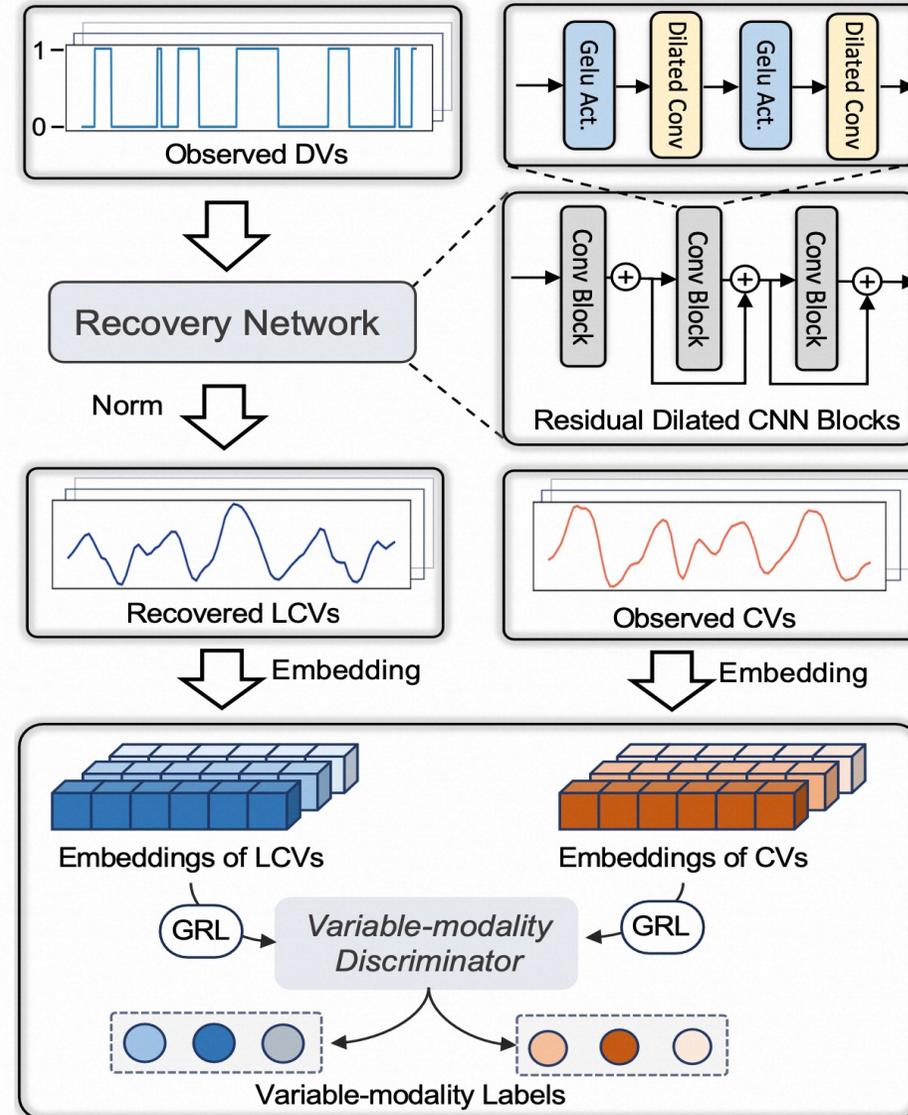
🚀 Network Module:

- ❑ **Latent Continuity Recovery:** adaptively and hierarchically aggregating multi-scale adjacent context information, facilitating to recover the LCVs for DVs.
- ❑ **Spatial-Temporal Attention:** Captures complete and balanced spatial-temporal dependencies within and across LCVs and CVs via cascaded self-attention and cross-attention blocks



MiTSformer

- **Latent Continuity Recovery:** Inspired by **auto-correlation** nature, we adopt **residual dilated CNNs** to **adaptively and hierarchically aggregate multi-scale adjacent context information**, facilitating to recover the LCVs for DVs.

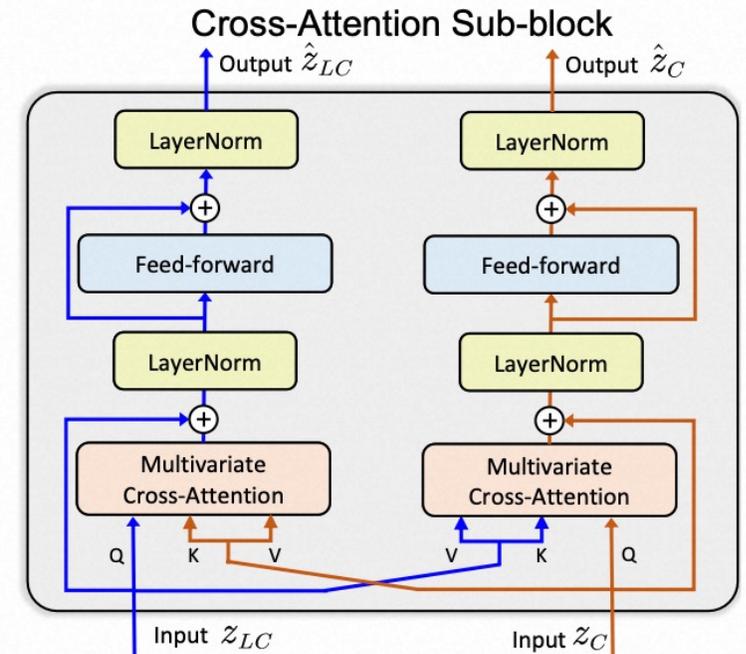
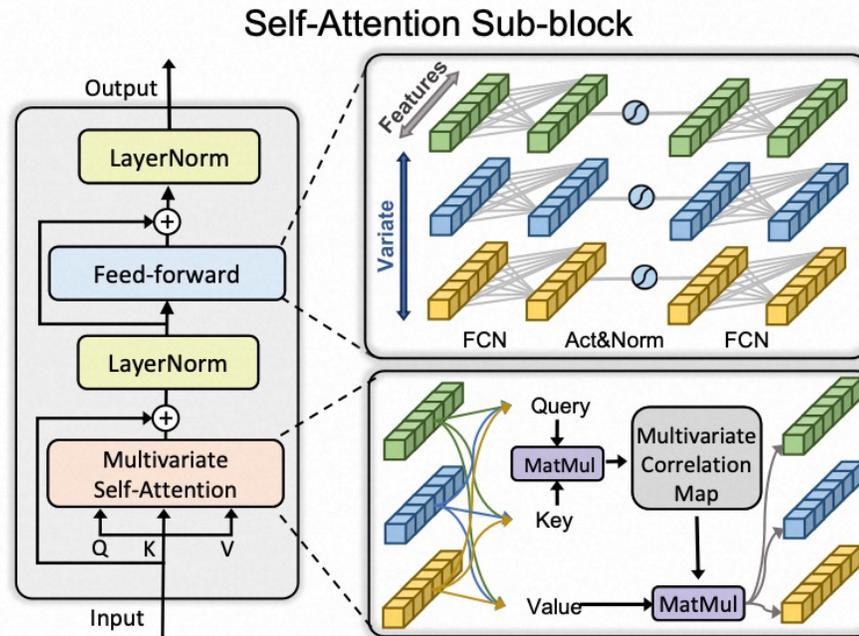
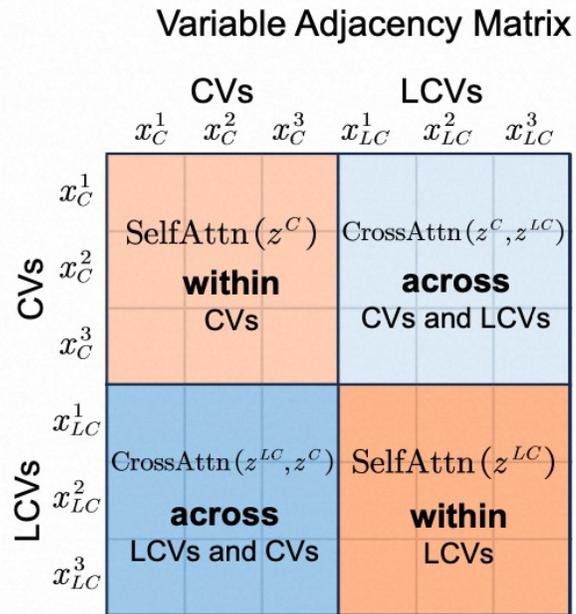


MiTSformer

- Spatial-Temporal Attention Blocks:** Captures complete and balanced **spatial-temporal dependencies within and across LCVs and CVs** via cascaded **self-attention and cross-attention blocks, respectively.**

$$\hat{z}_l^C = \text{LN} \left(z_l^C + \text{Self-Attn} \left(\left[\mathbf{Q}_l^C, \mathbf{K}_l^C, \mathbf{V}_l^C \right] \right) \right), \hat{z}_l^C = \text{LN} \left(\hat{z}_l^C + \text{FFN} \left(\hat{z}_l^C \right) \right) \quad z_{l+1}^C = \text{LN} \left(\hat{z}_l^C + \text{Cross-Attn} \left(\left[\mathbf{Q}_l^C, \mathbf{K}_l^{LC}, \mathbf{V}_l^{LC} \right] \right) \right), z_{l+1}^C = \text{LN} \left(z_{l+1}^C + \text{FFN} \left(z_{l+1}^C \right) \right)$$

$$\hat{z}_l^{LC} = \text{LN} \left(z_l^{LC} + \text{Self-Attn} \left(\left[\mathbf{Q}_l^{LC}, \mathbf{K}_l^{LC}, \mathbf{V}_l^{LC} \right] \right) \right), \hat{z}_l^{LC} = \text{LN} \left(\hat{z}_l^{LC} + \text{FFN} \left(\hat{z}_l^{LC} \right) \right) \quad z_{l+1}^{LC} = \text{LN} \left(z_{l+1}^{LC} + \text{Cross-Attn} \left(\left[\mathbf{Q}_l^{LC}, \mathbf{K}_l^C, \mathbf{V}_l^C \right] \right) \right), z_{l+1}^{LC} = \text{LN} \left(z_{l+1}^{LC} + \text{FFN} \left(z_{l+1}^{LC} \right) \right)$$



MiTSformer

Learning Objectives:

- ✓ Temporal Adjacent **Smoothness** Constraint of LCVs

$$\mathcal{L}_{\text{smooth}} = \|\text{Abs}(\mathbf{S}x^D) \otimes (\mathbf{S}x^{LC})\|_2^2$$

- ✓ Adversarial **Variable-Modality Discrimination**

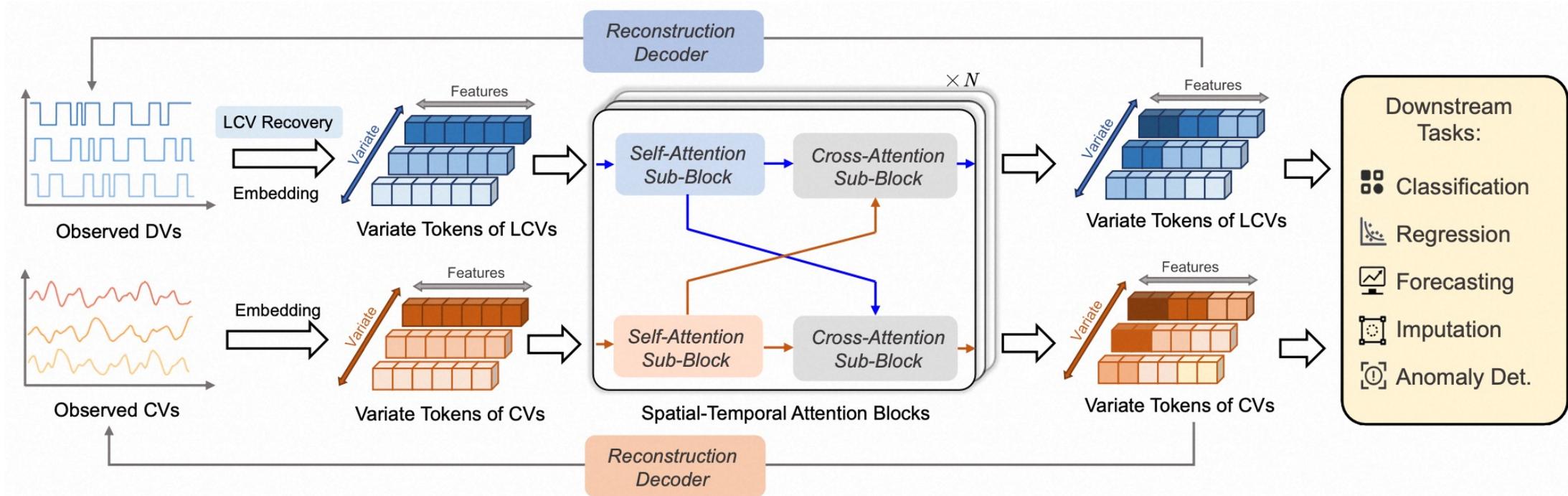
$$\text{argmin}_{\theta_{\text{Dis}}} \left(\max_{\theta_{\text{Rec}}, \theta_{\text{Emb}}} (\mathcal{L}_{\text{Dis}} = \mathbb{E}[\log(\text{Dis}(z^C))] + \mathbb{E}[\log(1 - \text{Dis}(z^{LC}))]) \right)$$

- ✓ **Self-Reconstruction** of DVs and CVs

$$\mathcal{L}_{\text{Rec}} = \sum_{i=1}^{p-n} \text{MSE}(\text{Rec-Decoder}_C(z_{L,i}^C), x_i^C) + \sum_{i=1}^n \text{CE}(\text{Rec-Decoder}_{LC}(z_{L,i}^{LC}), x_i^D)$$

- ✓ **Task-** Supervision (e.g., cross-entropy loss)

$$\mathcal{L}_{\text{All}} = \underbrace{\lambda_1 \mathcal{L}_{\text{Smooth}} + \lambda_2 \mathcal{L}_{\text{Rec}} + \lambda_3 \mathcal{L}_{\text{Dis}}}_{\text{Self-Supervision}} + \underbrace{\mathcal{L}_{\text{Task}}}_{\text{Task-Supervision}}$$



Pipeline: Classification

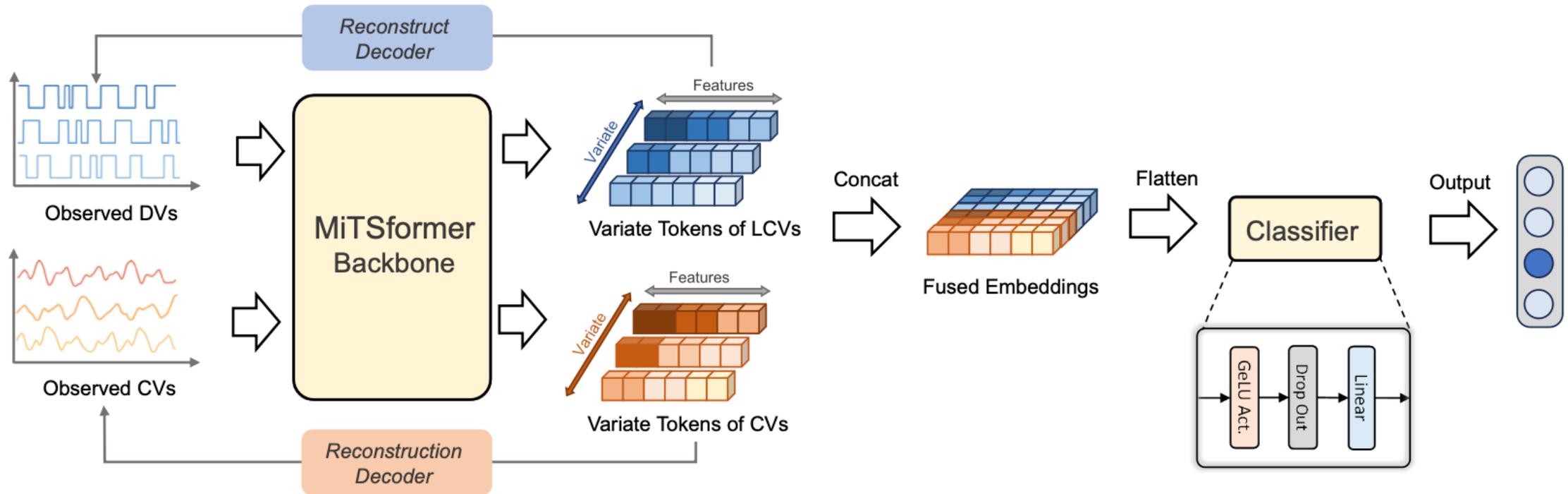


Figure 10: Overall pipeline of MiTSformer-based classification. The embeddings of LCVs and CVs are concatenated, flattened, and fed into the classifier for classification.

Pipeline: Extrinsic Regression

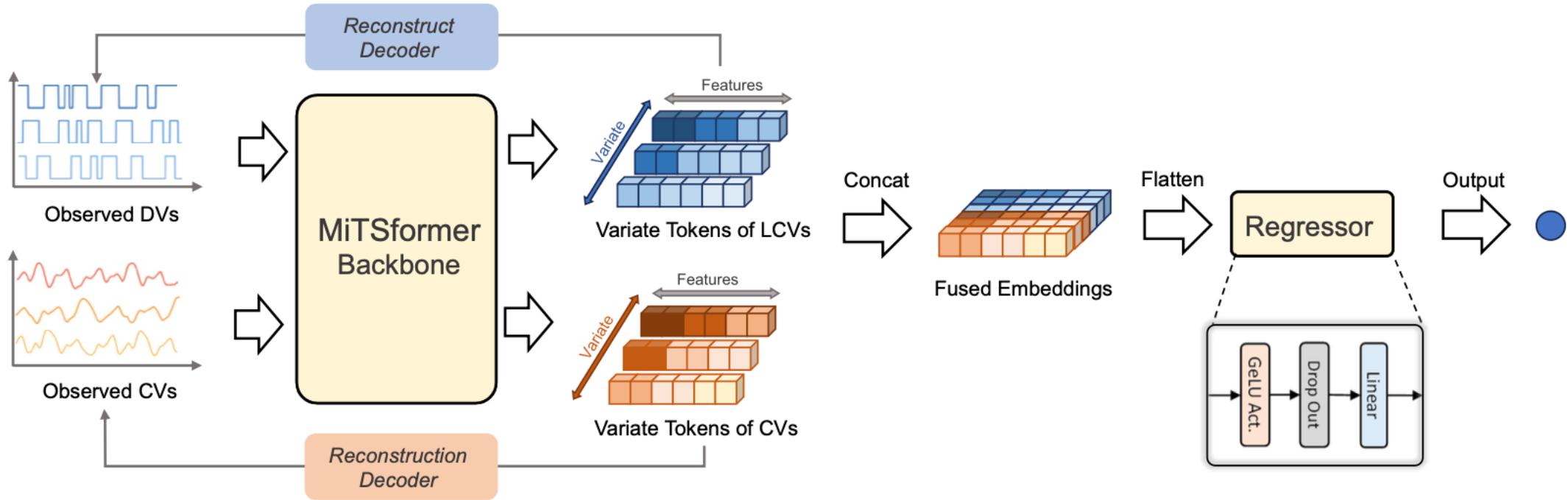


Figure 11: Overall pipeline of MiTSformer-based extrinsic regression. The embeddings of LCVs and CVs are concatenated, flattened, and fed into the regressor for regression.

Pipeline: Imputation

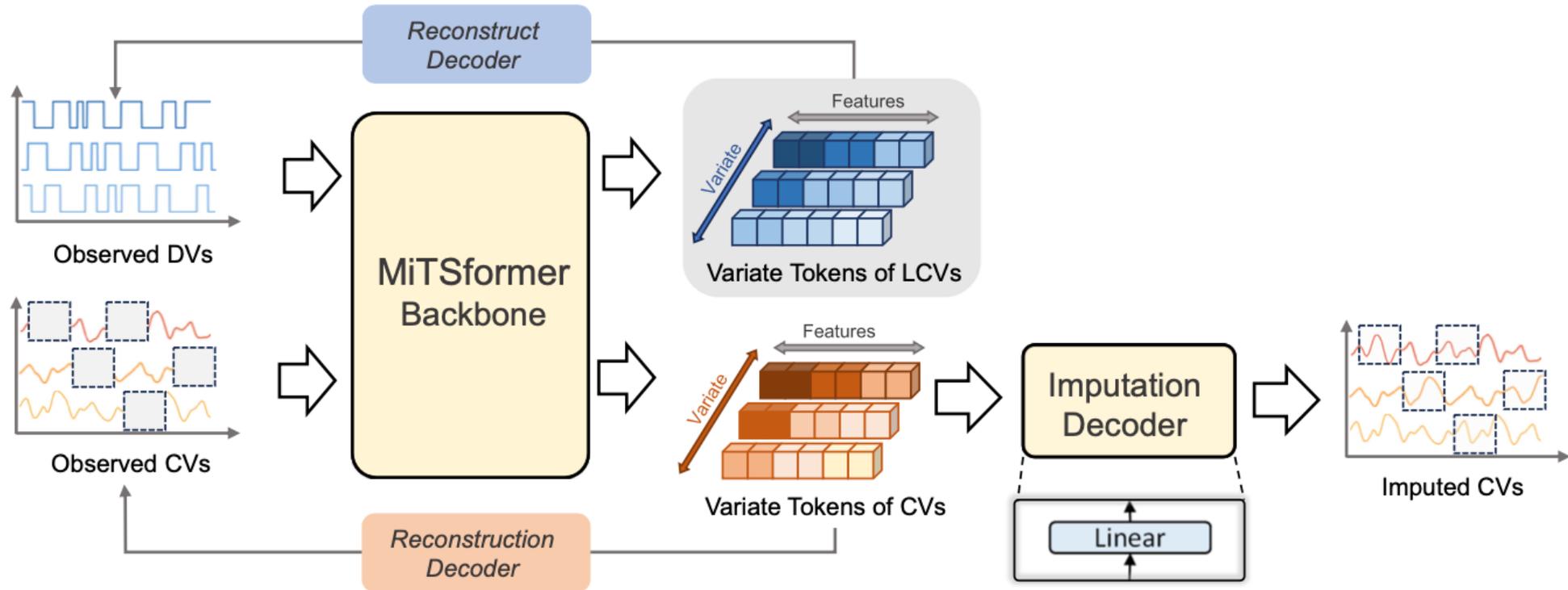


Figure 12: Overall pipeline of MiTSformer-based imputation. The embeddings of CVs are individually fed into the imputation decoder to impute missing values of CVs.

Pipeline: Long-term Forecasting

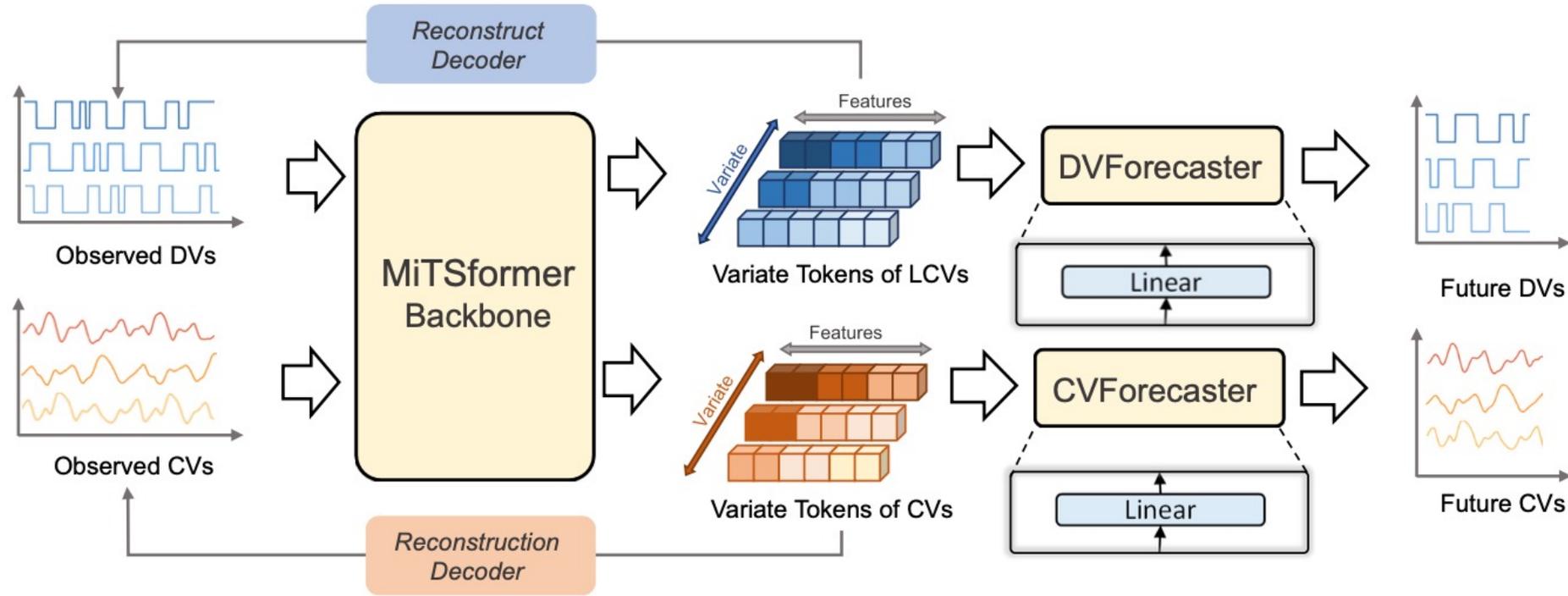


Figure 13: Overall pipeline of MiTSformer-based long-term forecasting. The embeddings of LCVs are individually fed into the DVForecaster to predict the future value of corresponding DVs, and the embeddings of CVs are individually fed into the DVForecaster to predict the future value of corresponding CVs.

Pipeline: Anomaly Detection

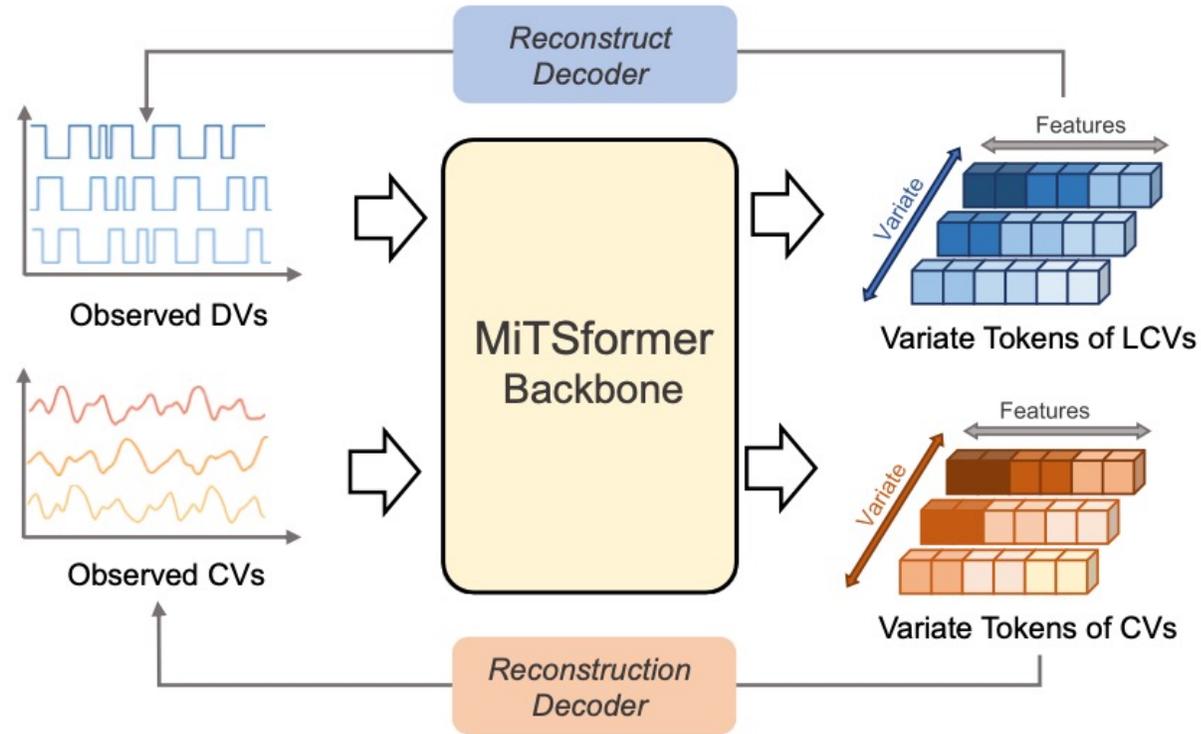


Figure 14: Overall pipeline of MiTSformer-based anomaly detection. The anomaly detection tasks only rely on self-reconstruction and thus no task head is attached.

Experiments

Table 1: Summary of experiment benchmarks. For each dataset, we randomly select $n = \lfloor 0.5p \rfloor$ variables as DVs, whose values are first MinMax normalized and then discretized into the value of 0 or 1 with the threshold 0.5 as $\text{int}(\text{MinMax}(x) > 0.5)$. See Table 5 for more details.

| Tasks | Benchmarks | Metrics | Series-length | #Variables (p) |
|-----------------------|---|-----------------------------|------------------------|--------------------|
| Classification | UEA (10 subsets) | Accuracy | 29~1751 | 3~963 |
| Extrinsic Regression | UCR (10 subsets) | MAE, RMSE | 24~1140 | 4~24 |
| Imputation | ETT (4 subsets), Electricity, Weather | MSE, MAE | 96 | 7~321 |
| Anomaly Detection | SMD, MSL, SMAP, SWaT, PSM | Precision, Recall, F1-Score | 100 | 25~55 |
| Long-term Forecasting | ETT (4 subsets), Electricity, Traffic, Weather, Exchange, ILI | MSE, MAE | 96~720 (ILI: 24~60) | 7 ~ 862 |

Implementations Table 1 summarizes the experiment benchmarks. For each dataset, we randomly select *half the variables* and discretize them as DVs to generate MiTS data. More information about datasets and experimental platforms, hyperparameters and experimental configurations, and algorithm implementations can be found in Appendix A.1, A.2, A.3, respectively. The pipelines of different mixed time series analysis tasks can be found in Appendix A.5~A.8.

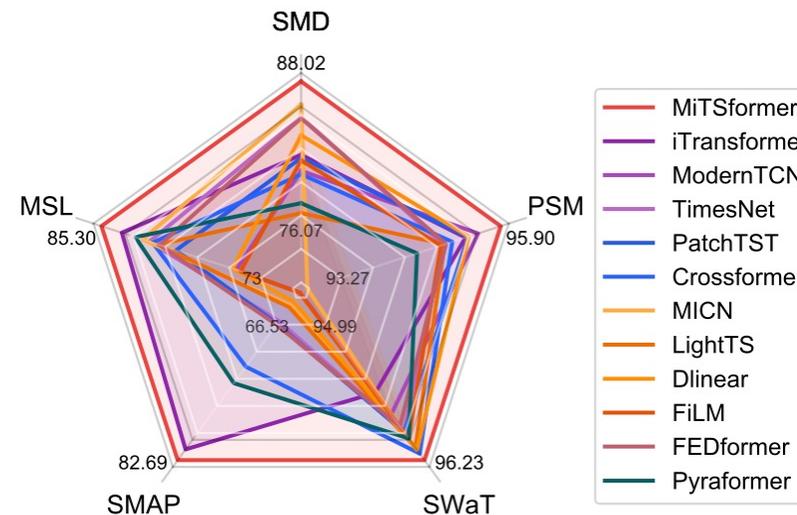
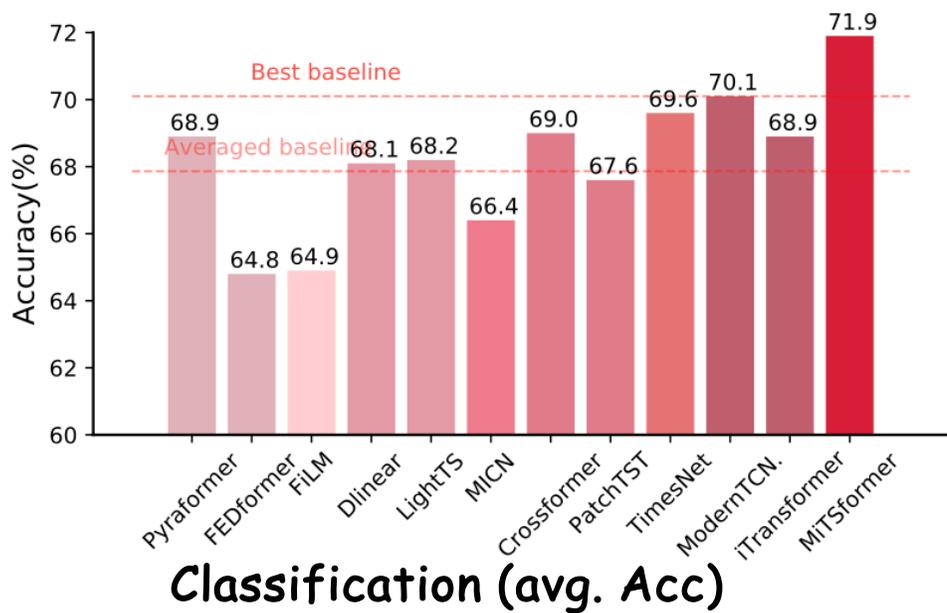
Baselines We extensively compare MiTSformer with the latest and advanced models in the time series community, including CNN-based models: ModernTCN (2024), TimesNet (2023) and MICN (2023); Transformer-based models: iTransformer (2024), PatchTST (2023), Crossformer (2023), FEDformer (2022) and Pyraformer (2022); MLP-based models: LightTS (2023), DLinear (2023) and FiLM (2022). To guarantee fairness, we keep the original backbone for each method as the feature extractor, and we adopt universal task-specific heads and loss functions consistently for all methods.

Experiments

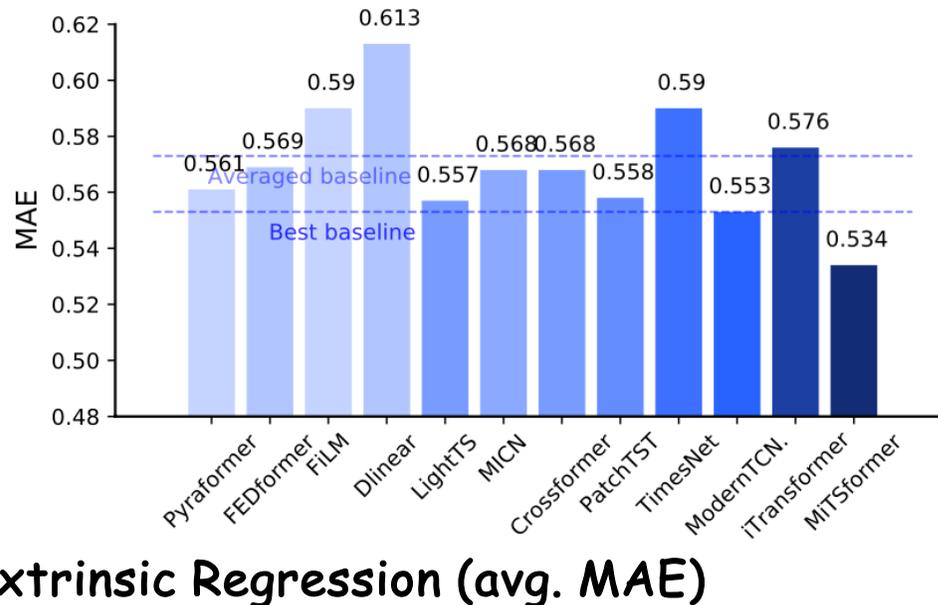
MiTSformer **establishes SOTA performance**

on five mixed time series analysis tasks:

- ✓ **Classification,**
- ✓ **Extrinsic regression**
- ✓ **Imputation**
- ✓ **Anomaly detection**
- ✓ **Long-term forecasting**



Anomaly Detection (F1-score)



Extrinsic Regression (avg. MAE)

Experiments

MiTSformer **establishes SOTA performance** on five mixed time series analysis tasks:

- ✓ Classification,
- ✓ Extrinsic regression
- ✓ Imputation
- ✓ Anomaly detection
- ✓ Long-term forecasting

Table 2: Imputation Task. The best results are **bolded** and the second-best results are underlined. The same goes for Table 3. See Table 14 for full results.

| Models | MiTSformer (Ours) | iTrans. (2024) | M-TCN. (2024) | TimesNet (2023) | PatchTST (2023) | Cross. (2023) | MICN (2023) | LightTS (2023) | Dlinear (2023) | FiLM (2022) | FED. (2022) | Pyra. (2022) |
|----------|--------------------|--------------------|--------------------|--------------------|--------------------|---------------|-------------|----------------|----------------|-------------|-------------|--------------|
| Metric | MAE MSE | MAE MSE | MAE MSE | MAE MSE | MAE MSE | MAE MSE | MAE MSE | MAE MSE |
| ETTm1 | 0.156 0.049 | 0.169 0.057 | 0.135 0.037 | <u>0.139 0.039</u> | 0.164 0.057 | 0.160 0.051 | 0.160 0.052 | 0.178 0.064 | 0.193 0.080 | 0.194 0.080 | 0.170 0.060 | 0.199 0.081 |
| ETTm2 | 0.116 0.036 | 0.186 0.080 | 0.188 0.077 | 0.170 0.065 | <u>0.145 0.055</u> | 0.216 0.103 | 0.245 0.131 | 0.209 0.095 | 0.253 0.147 | 0.258 0.152 | 0.238 0.123 | 0.265 0.141 |
| ETTh1 | <u>0.223 0.096</u> | 0.241 0.116 | 0.215 0.092 | 0.237 0.112 | 0.251 0.129 | 0.246 0.121 | 0.234 0.107 | 0.266 0.145 | 0.262 0.145 | 0.268 0.152 | 0.244 0.112 | 0.247 0.113 |
| ETTh2 | 0.186 0.083 | <u>0.250 0.134</u> | 0.309 0.213 | 0.319 0.239 | 0.260 0.148 | 0.304 0.198 | 0.318 0.220 | 0.314 0.209 | 0.312 0.211 | 0.342 0.262 | 0.350 0.258 | 0.367 0.253 |
| Electric | 0.186 0.076 | 0.212 0.093 | 0.207 <u>0.088</u> | 0.211 0.094 | <u>0.203 0.115</u> | 0.209 0.097 | 0.229 0.103 | 0.223 0.099 | 0.257 0.130 | 0.257 0.128 | 0.260 0.130 | 0.274 0.150 |
| Weather | 0.062 0.031 | 0.091 0.038 | <u>0.081 0.033</u> | 0.149 0.065 | 0.088 0.037 | 0.115 0.043 | 0.135 0.055 | 0.107 0.041 | 0.121 0.048 | 0.124 0.184 | 0.139 0.057 | 0.100 0.042 |

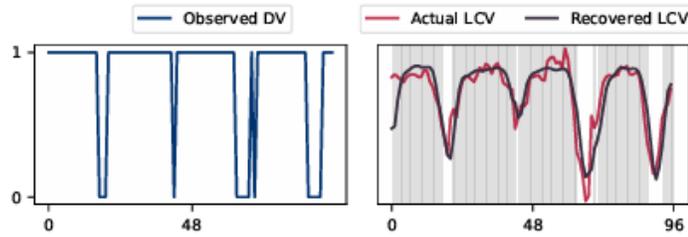
Imputation (avg. missing rate of 12.5%,25%,37.5%,50%)

Table 3: Long Term Forecasting of CVs. “-” denotes out of memory. See Table 16 for full results.

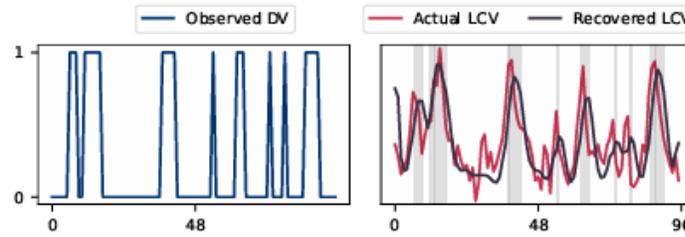
| Models | MiTSformer (Ours) | iTrans. (2024) | M-TCN. (2024) | TimesNet (2023) | PatchTST (2023) | Cross. (2023) | MICN (2023) | LightTS (2023) | Dlinear (2023) | FiLM (2022) | FED. (2022) | Pyra. (2022) |
|-----------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|----------------|--------------------|-------------|-------------|--------------|
| Metric | MAE MSE | MAE MSE | MAE MSE | MAE MSE | MAE MSE | MAE MSE |
| ETTm1 | 0.376 0.328 | 0.385 0.340 | 0.380 0.334 | 0.403 0.357 | <u>0.379 0.330</u> | 0.407 0.358 | 0.385 0.331 | 0.394 0.348 | 0.381 0.331 | 0.397 0.357 | 0.425 0.371 | 0.484 0.470 |
| ETTm2 | 0.365 0.363 | 0.371 0.373 | <u>0.366 0.371</u> | 0.376 0.395 | 0.368 <u>0.364</u> | 0.878 1.056 | 0.528 0.599 | 0.529 0.578 | 0.512 0.568 | 0.376 0.389 | 0.386 0.385 | 0.896 1.732 |
| ETTh1 | 0.414 0.373 | 0.427 0.393 | <u>0.415 0.388</u> | 0.446 0.412 | 0.416 0.381 | 0.425 0.376 | 0.448 0.404 | 0.478 0.465 | 0.417 <u>0.376</u> | 0.443 0.430 | 0.437 0.390 | 0.526 0.543 |
| ETTh2 | 0.430 0.449 | 0.442 0.472 | 0.442 0.480 | 0.453 0.490 | <u>0.440 0.464</u> | 0.917 1.422 | 0.692 1.000 | 0.729 1.063 | 0.675 0.982 | 0.450 0.490 | 0.476 0.508 | 1.304 2.548 |
| Weather | 0.326 0.268 | 0.334 0.279 | <u>0.328 0.269</u> | 0.348 0.296 | 0.332 0.276 | 0.338 0.257 | 0.356 0.278 | 0.354 0.277 | 0.346 0.274 | 0.353 0.291 | 0.386 0.322 | 0.357 0.274 |
| Exchange | 0.445 0.398 | 0.452 0.412 | 0.448 0.403 | 0.503 0.498 | 0.453 0.417 | 0.596 0.632 | <u>0.412 0.323</u> | 0.459 0.402 | 0.409 0.318 | 0.449 0.398 | 0.564 0.599 | 0.650 0.679 |
| ILI | 0.779 1.482 | 0.995 2.132 | 0.922 <u>1.957</u> | <u>0.891 2.015</u> | 0.973 2.140 | 1.140 2.962 | 1.358 2.360 | 1.734 5.432 | 1.340 3.197 | 1.188 2.702 | 1.267 3.003 | 1.096 2.747 |
| Electric. | 0.260 0.168 | 0.293 0.207 | <u>0.270 0.174</u> | 0.288 0.187 | 0.295 0.207 | 0.326 0.237 | 0.295 0.185 | 0.339 0.240 | 0.314 0.220 | 0.316 0.234 | 0.351 0.248 | 0.400 0.319 |
| Traffic | 0.312 0.499 | 0.372 <u>0.593</u> | 0.366 0.635 | <u>0.354 0.803</u> | 0.360 0.603 | - - | 0.355 0.692 | 0.465 0.824 | 0.421 0.742 | - - | 0.416 0.774 | 0.456 0.945 |

Long-term Forecasting (avg. horizon of 96,192,336,720)

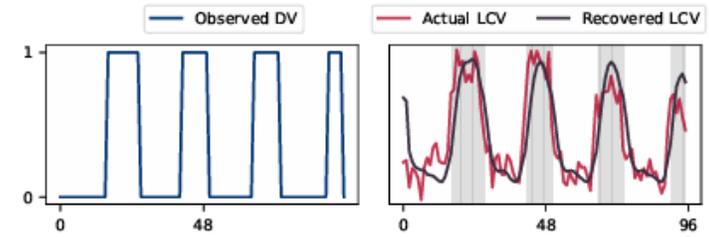
Visualization



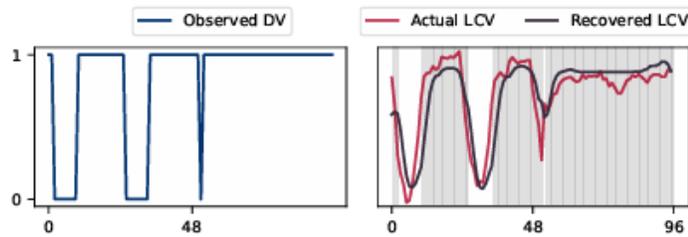
(a) ETTh1 case#1



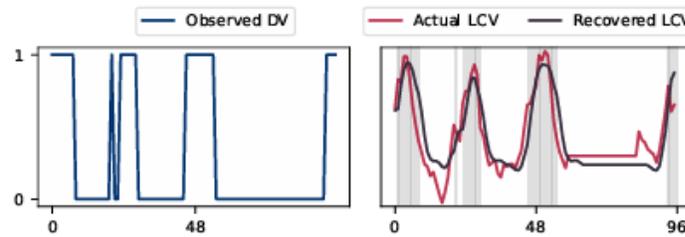
(b) ETTh1 case#2



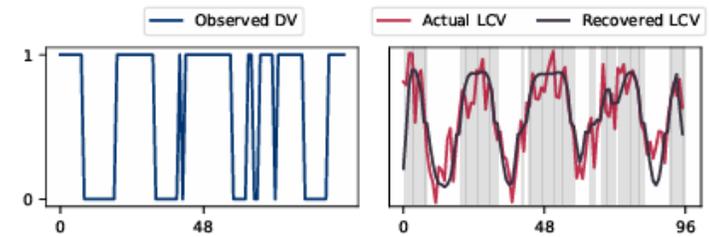
(c) ETTh1 case#3



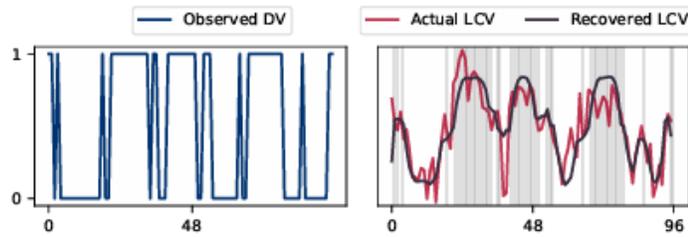
(d) ETTh1 case#4



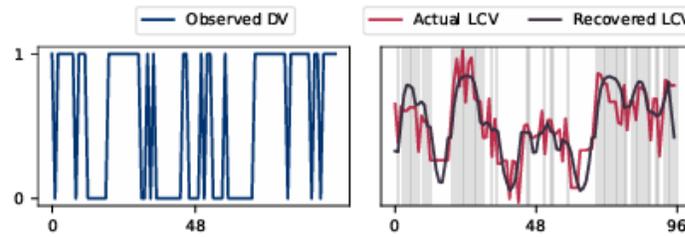
(e) ETTh1 case#5



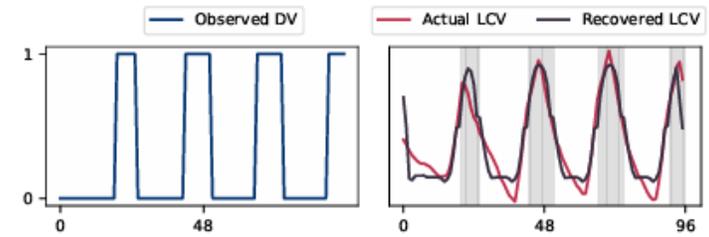
(f) ETTh2 case#1



(g) ETTh2 case#2



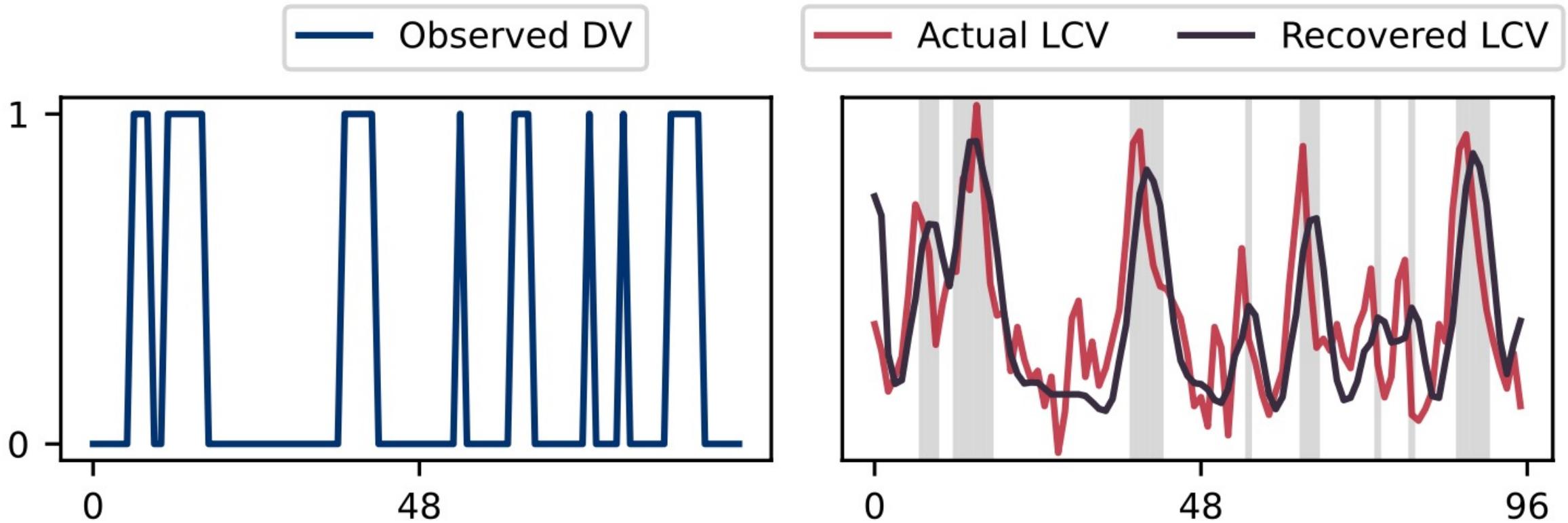
(h) ETTh2 case#3



(i) ETTh2 case#4

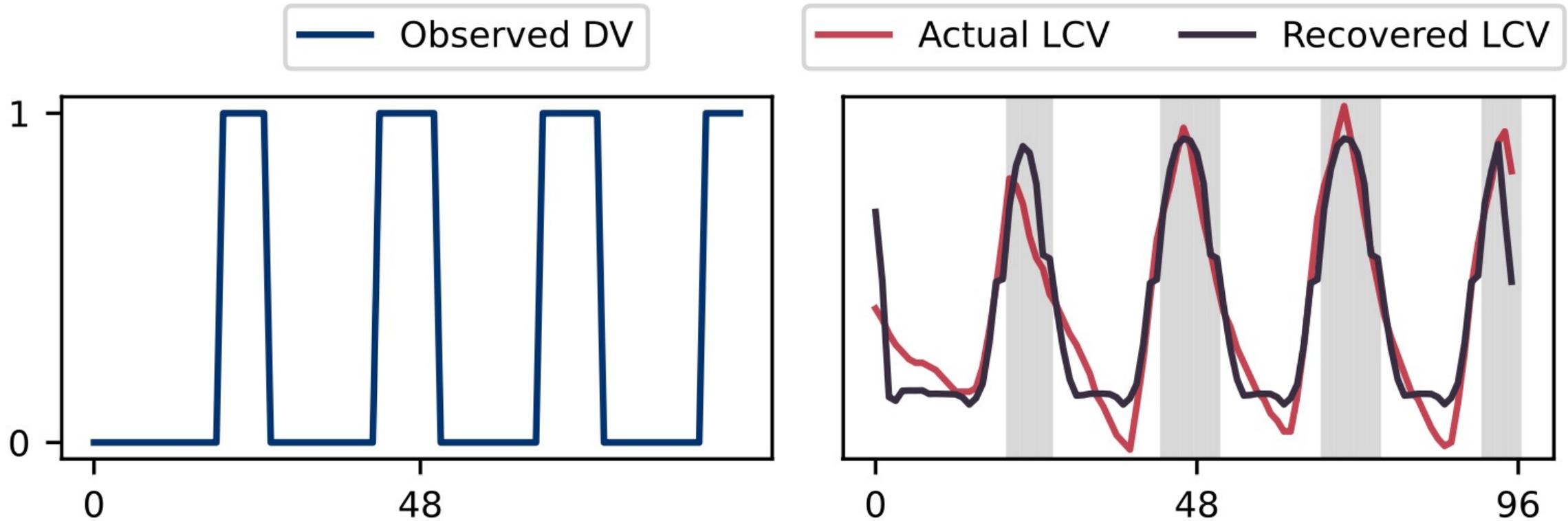
MiTSformer **accurately recovers the LCVs for DVs under the guidance of CVs.**

Visualization



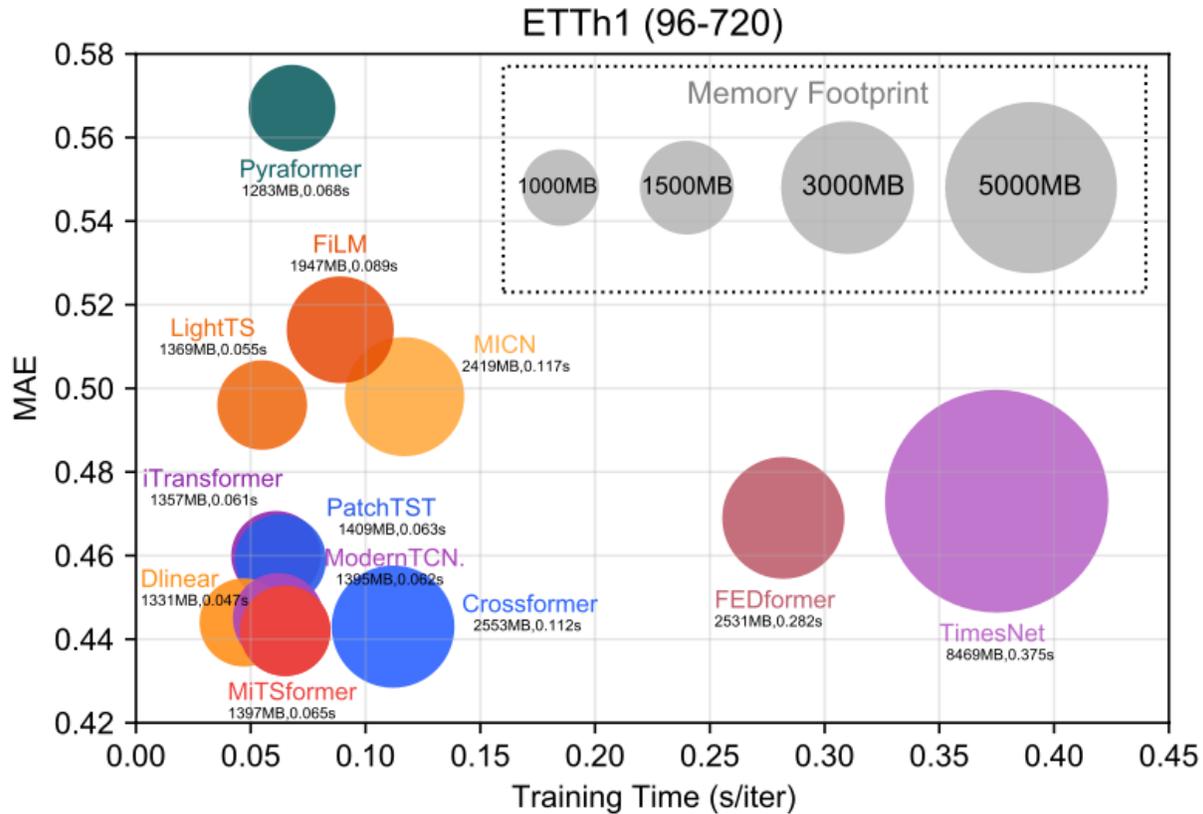
MiTFormer **accurately recovers the LCVs for DVs under the guidance of CVs.**

Visualization

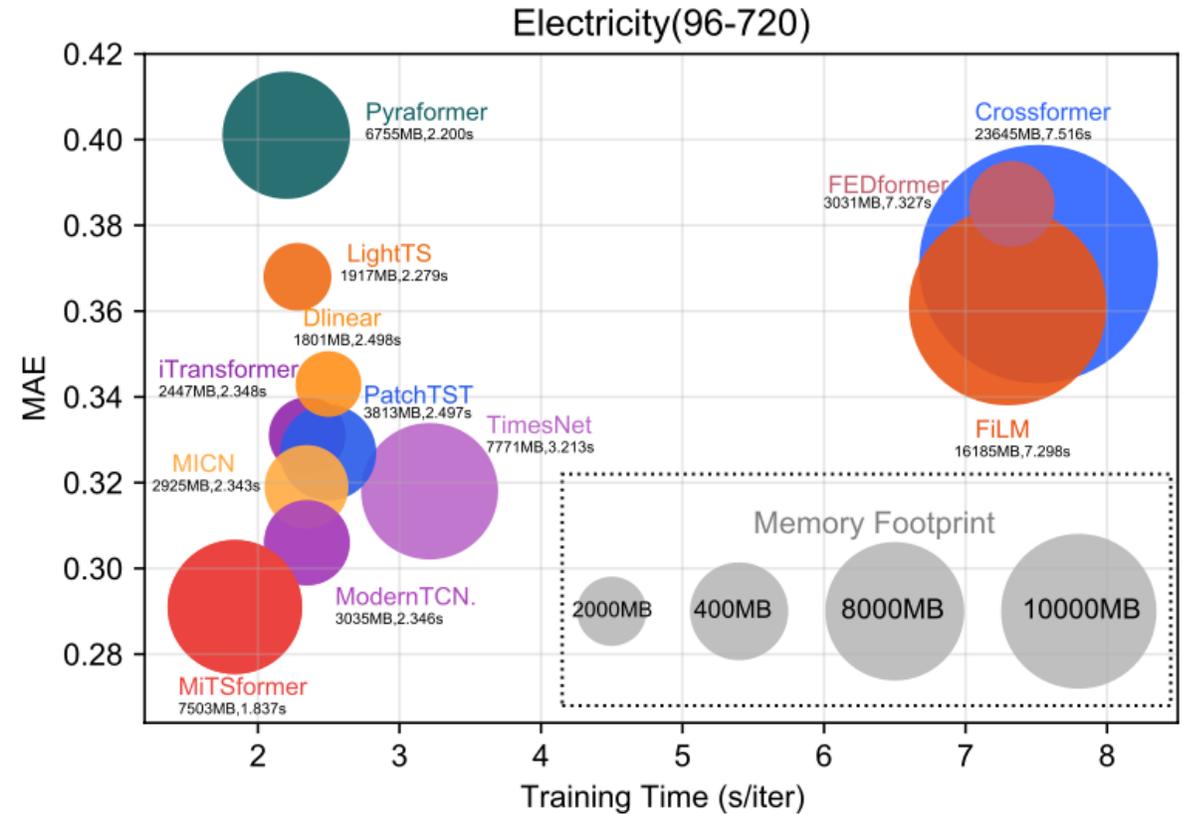


MiTSformer **accurately recovers the LCVs for DVs under the guidance of CVs.**

Computational Efficiency



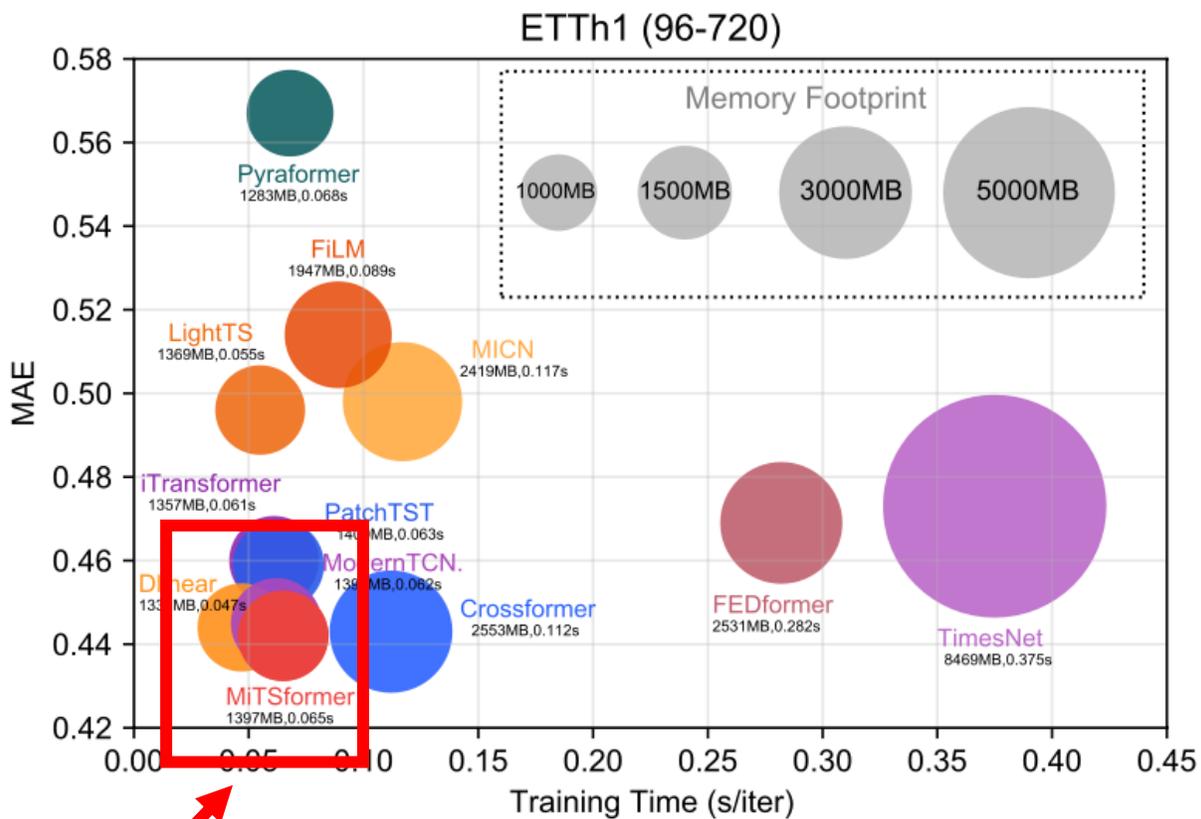
(a) ETTh1 (3DVs, 4CVs)



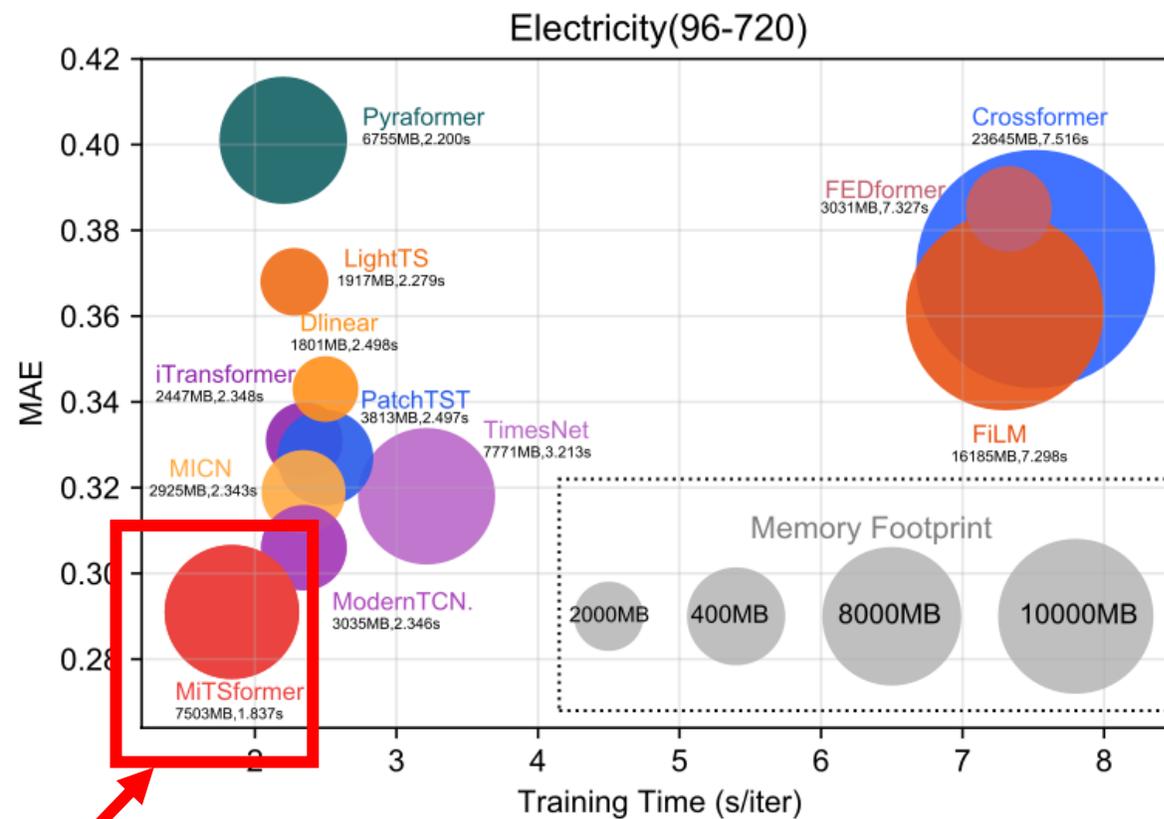
(b) Electricity (160DVs, 161CVs)

MiTSformer maintains **great performance and efficiency** against most advanced baselines.

Computational Efficiency



(a) ETTh1 (3DVs, 4CVs)



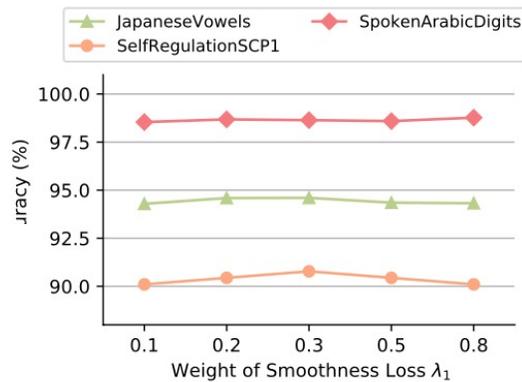
(b) Electricity (160DVs, 161CVs)

MiTSformer maintains **great performance and efficiency** against most advanced baselines.

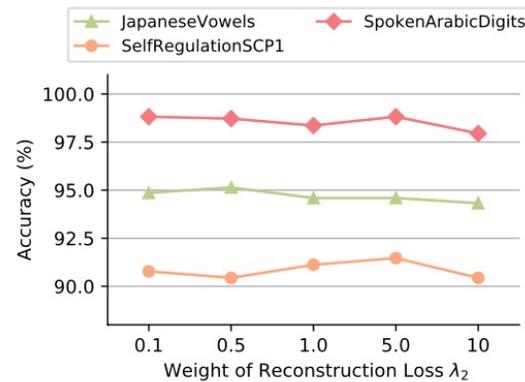
Hyperparameter Sensitivity

Sensitivity of loss weights

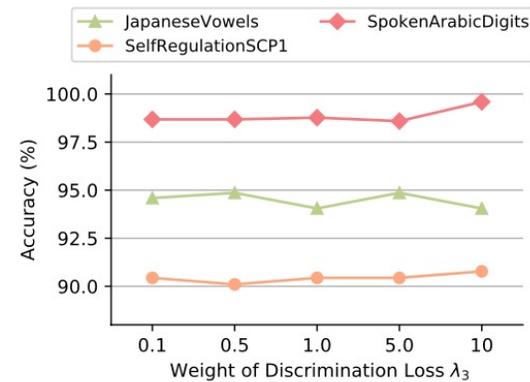
$$\mathcal{L}_{\text{All}} = \underbrace{\lambda_1 \mathcal{L}_{\text{Smooth}} + \lambda_2 \mathcal{L}_{\text{Rec}} + \lambda_3 \mathcal{L}_{\text{Dis}}}_{\text{Self-Supervision}} + \underbrace{\mathcal{L}_{\text{Task}}}_{\text{Task-Supervision}}$$



(a) Sensitivity of λ_1

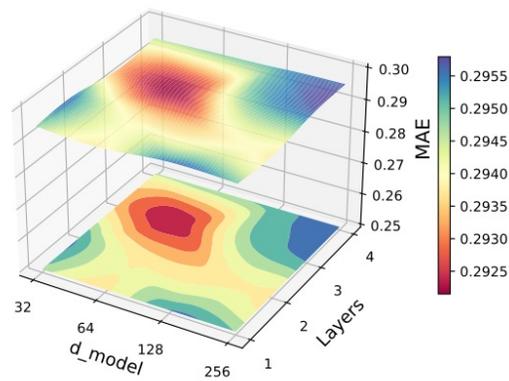


(b) Sensitivity of λ_2

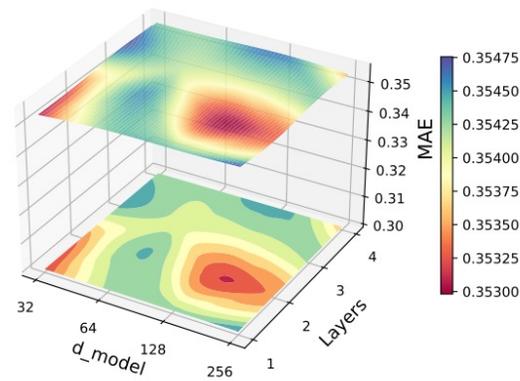


(c) Sensitivity of λ_3

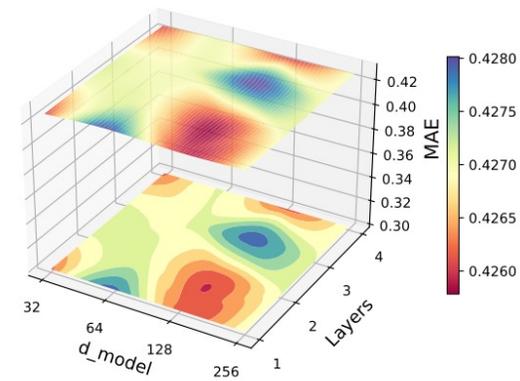
Sensitivity of model capacity (layer number and d_model)



(j) Weather 96-192



(k) Weather 96-336



(l) Weather 96-720

MiTSformer is **quite robust to hyper-parameters**: (1). the weights of loss items and (2). model capacity.

Resources



Paper



Code & Data



WeChat

<https://github.com/chunhuiz/MiTSformer>