A Non-Parametric Direct Learning Approach to Heterogeneous Treatment Effect Estimation under Unmeasured Confounding

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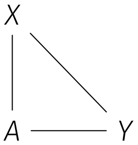
No Unmeasured Confounding

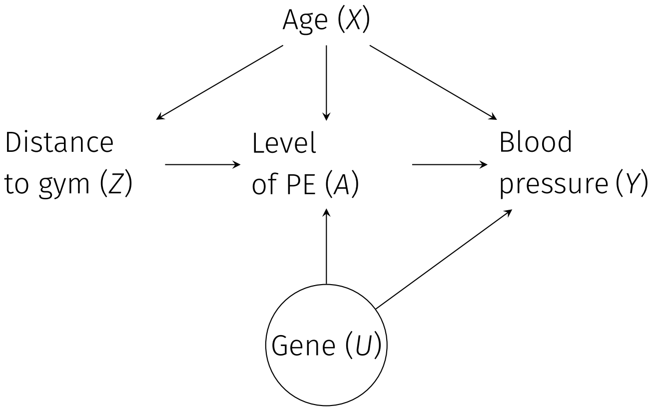
- Goal: Conditional Average Treatment Effect (CATE)
  $\Delta(X) = \mathbb{E}[Y(1) - Y(0)|X]$
- Y continuous outcome.
- $A \in \{-1, 1\}$ binary treatment.
- $Y(a), \quad \forall a \in \mathcal{A}$ potential outcome under intervention.
- X pre-treatment covariates
- Unconfoundedness: $Y(1), Y(0) \perp\!\!\!\perp A|X$.



Unmeasured Confounding

Instrumental Variable (IV) Approach

- $Z \not\perp\!\!\!\perp A | X$.
- $Z \perp\!\!\!\perp U | X$.
- $Y(z, a) = Y(z', a)$.



Direct Learning using Instrumental Variables

Identification

$$\Delta(x) = \frac{\mathbb{E}[Y|Z=1, X=x] - \mathbb{E}[Y|Z=-1, X=x]}{\mathrm{P}[A=1|Z=1, X=x] - \mathrm{P}[A=1|Z=-1, X=x]} \tag{1}$$

$$= \mathbb{E}\left[\frac{ZY}{\delta(x)\pi_Z(Z, x)}\middle| X = x\right], \tag{2}$$

where $\pi_Z(z, x) = \mathrm{P}[Z = z|X = x]$ and $\delta(x) = \mathrm{P}[A = 1|Z = 1, X = x] - \mathrm{P}[A = 1|Z = -1, X = x]$.

IV-DL

$$\Delta \in \operatorname*{argmin}_f \mathbb{E}\left[\pi_Z^{-1}(Z, X)\left(\frac{2ZY}{\delta(X)} - f(X)\right)^2\right]$$

Robust Direct Learning using Instrumental Variables – first approach

IV-RDL1

$$\Delta \in \underset{f}{\operatorname{argmin}}\, \mathbb{E}\left[\pi_Z^{-1}(Z,X)\left(\frac{2Z(Y-g(X))}{\delta(X)} - f(X)\right)^2\right]$$

- optimal $g^*(x) = \frac{\mu_1^Y(x)+\mu_{-1}^Y(x)}{2}$, where $\mu_z^Y(x) = \mathbb{E}[Y|Z=z, X=x]$.
- double robustness: Consistent if either π_Z or g is correctly modeled.

Robust Direct Learning using Instrumental Variables – second approach

IV-RDL2

$$\Delta \in \operatorname*{argmin}_f \mathbb{E}\left[\pi_Z^{-1}(Z,X)\left(\frac{2Z(Y-h(X,A,Z))}{\delta(X)} - f(X)\right)^2\right]$$

optimal h

- $h_1^*(x,a,z) = \mu_1^Y(x) + \Delta(x)\big(a - \mu_1^A(x) - z\delta(x)\big)/2$
- $h_2^*(x,a,z) = \mu_{-1}^Y(x) + \Delta(x)\big(a - \mu_{-1}^A(x) - z\delta(x)\big)/2$
- $h_3^*(x,a,z) = \overline{\mu}^Y(x) + \Delta(x)\big(a - \overline{\mu}^A(x) - z\delta(x)\big)/2$

multiply robustness

- π_Z, Δ
- π_Z, δ
- μ_1^Y, μ_1^A, Δ
- μ_{-1}^Y, μ_{-1}^A, Δ
- $\overline{\mu}^Y$, $\overline{\mu}^A$, Δ
- $\overline{\mu}^Y$, $\overline{\mu}^A$, δ.

Simulation results

- Evaluation: Mean Squared Error, Accuracy rate, and Empirical Value. mean$\times 10^{-2}$(SE$\times 10^{-2}$).
- Benchmark methods: Bayesian additive regression trees (BART; Chipman et al., 2010); Robust direct learning (RD; Meng and Qiao, 2022); Multiply robust weighted learning (IPW-MR; Cui and Tchetgen Tchetgen, 2021); Causal forest (CF; Athey et al., 2019); MRIV method (Frauen and Feuerriegel, 2022).

		BART	RD	IPW-MR	CF	MRIV	IV-DL	IV-RDL1	IV-RDL2
1	MSE	121(3.4)	97.6(2.9)	NA	89.6(1.8)	66.3(2.3)	55.5(3.8)	**40.5(2.9)**	42.5(3.3)
	AR	66.3(0.7)	71.4(0.5)	84.1(0.7)	79.1(0.7)	78.3(0.6)	81.4(1)	**84.6(0.8)**	83.7(1)
	Value	75.4(0.8)	81.6(0.5)	84.1(0.7)	85.4(0.7)	84.5(0.7)	87.1(1)	**89.9(0.9)**	88.9(1.1)
2	MSE	449(11.1)	397(9.8)	NA	149(2.9)	150(5.6)	164(8.9)	**140(7.7)**	142(7.4)
	AR	57.6(0.6)	60.8(0.7)	55.5(0.3)	70.1(1)	68.8(0.9)	77.1(0.9)	**77.9(0.8)**	77.2(0.9)
	Value	53.1(1.6)	61(1.6)	81.6(0.2)	83.5(1.2)	81.6(1)	89.9(1)	**90.6(0.9)**	90(1)