Validating Climate Models with Spherical Convolutional Wasserstein Distance

Robert C. Garrett, Trevor Harris, Bo Li, Zhuo Wang

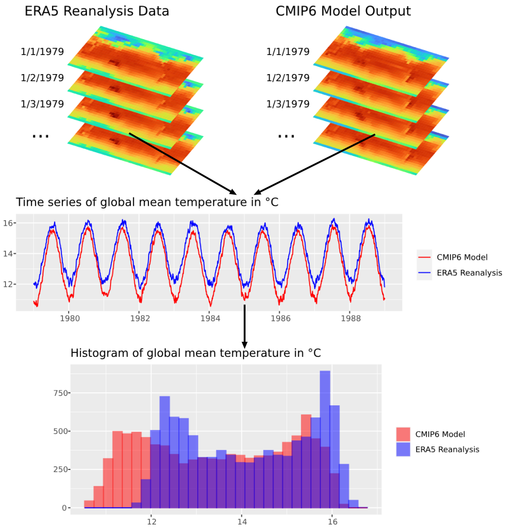
University of Illinois Urbana-Champaign

Objective: Develop a new criteria to evaluate climate model performance

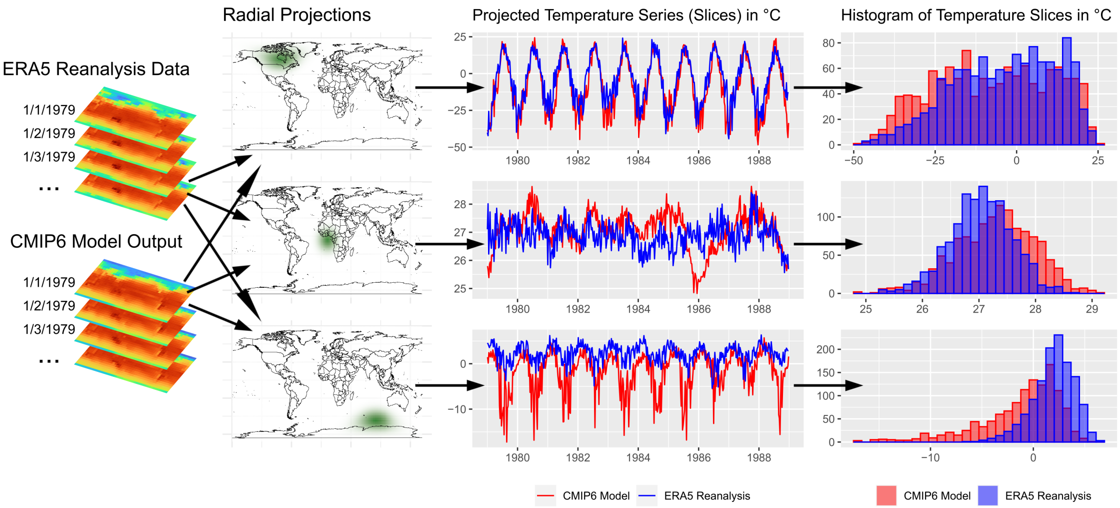
- Spherical Convolutional Wasserstein Distance (SCWD)
  - Compares distributions of spatial fields
  - Finds regions where models struggle to create realistic climatologies
- Models: Physics-based simulations of the climate
  - Coupled Model Intercomparison Project (CMIP) goal: understand sources and impacts of climate changes
  - 45 models from CMIP6, 33 models from CMIP5
  - Historical experiment, global coverage, daily frequency
- Observational data: Historical weather records
  - ERA5 Reanalysis for surface temperature
  - GPCP observations for total precipitation
  - NCEP Reanalysis included to test our method

Previous approach: global mean Wasserstein distance (GMWD)

- Method from Vissio et al., 2020
- Convert climate fields for each day into one spatial mean
  - *Cannot directly compare these time series!*
- Compare distributions of global means using Wasserstein distance
- **Problem:** global mean does not describe the spatial variability in climate fields
  - Localized extreme events are smoothed out
  - Positive and negative biases in different regions cancel out



Proposal: incorporate more spatial perspectives via convolution



Intuition: Spatial fields as functional data

- Proposal: view spatial fields as functions of space $f(s)$
  - $\mathbb{S}^2$ is the set of latitude-longitude coordinates
  - $L^2(\mathbb{S}^2)$ is a convenient space of functions from $\mathbb{S}^2 \to \mathbb{R}$
- Convolution Sliced WD (Nguyen and Ho, 2022) designed for image data
  - Generate one-dimensional slices using $k \times k$ kernels
  - Not designed to deal with continuous & spherical geometry of $L^2(\mathbb{S}^2)$
- Spherical CNNs (Cohen et al., 2018) adapts CNNs to continuous spherical data
  - We adopt a similar spherical convolution operation to generate slices
- Use Wendland RBF as compact alternative to Gaussian kernel
  - Range parameter in our analysis: 1,000km

Definition: Convolution Slicer

For a given location $s \in \mathbb{S}^2$, the convolution slicer takes a function in $L^2(\mathbb{S}^2)$ and takes a weighted mean of data around location s. This local mean is an observation in $\mathbb{R}$.

Convolution slicer $c_s(f)$

For $s \in \mathbb{S}^2$, define the convolution slicer centered at s, $c_s : f \in L^2(\mathbb{S}^2) \to \mathbb{R}$ as:

$$c_s(f) = \int_{\mathbb{S}^2} f(u)\phi(|s-u|)du,$$

where ϕ is a radial kernel function.

Definition: Spherical Convolutional Wasserstein Distance

Local (univariate) distributions are obtained via the slicing operation. WD is calculated for each pair of local distributions. SCWD is calculated as the global mean of the local WDs.

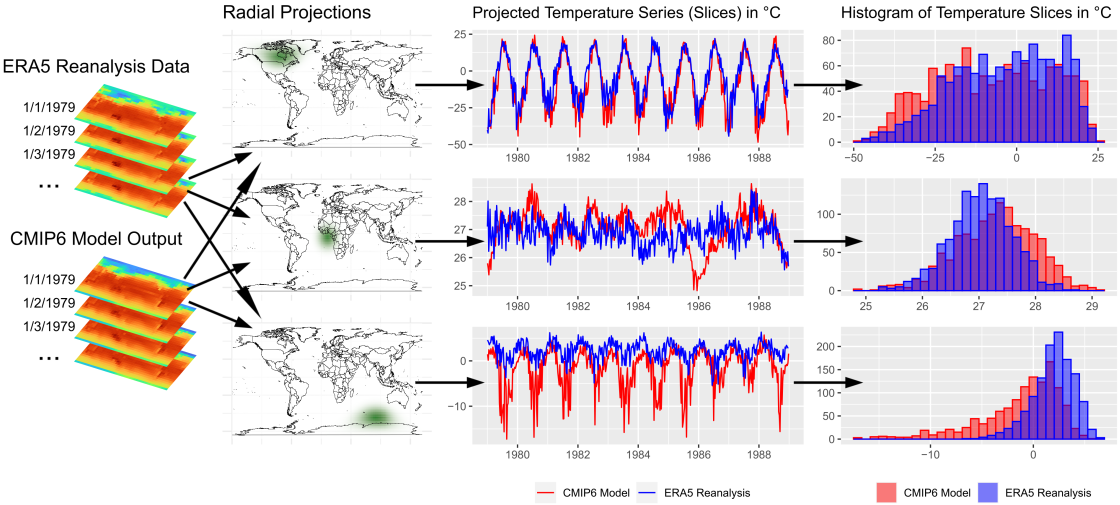
Spherical Convolutional Wasserstein Distance (SCWD)

Let $P, Q \in \mathcal{P}(L^2(\mathbb{S}^2))$, the set of probability measures on $L^2(\mathbb{S}^2)$. We define the spherical convolutional WD as follows:

$$SCW(P, Q) = \left(\int_{\mathbb{S}^2} W(c_s \# P, c_s \# Q)^2 ds \right)^{1/2},$$

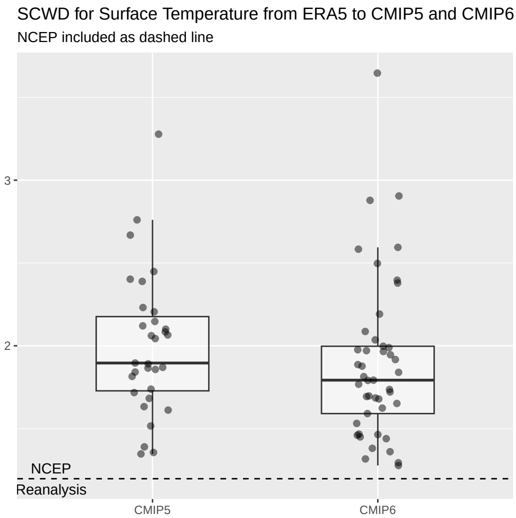
where W is the ordinary Wasserstein distance and $c_s \# P, c_s \# Q$ are the pushforward measures that represent the distribution of sliced data at each location s.

Proposal: incorporate more spatial perspectives via convolution

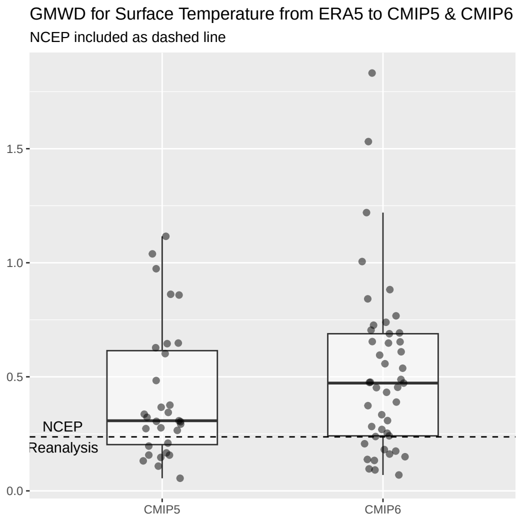


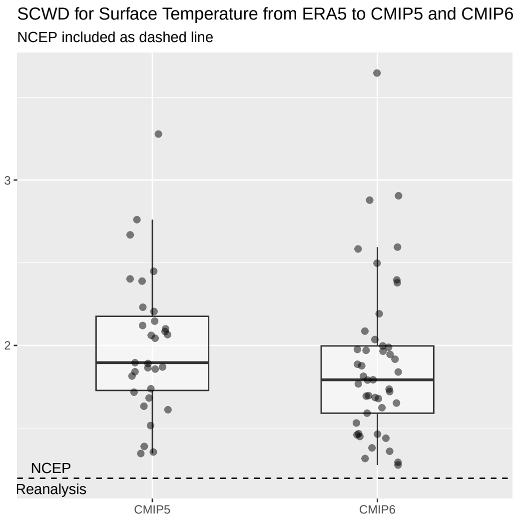
Surface Temperature Results: SCWD

- Reference model: ERA5 Reanalysis
  - Daily data from 1979-2005
- Calculate SCWD from each CMIP model output to ERA5
  - Lower SCWD = more similar to ERA5
- CMIP6 models have (subtly) lower SCWD than CMIP5 models
- NCEP has lower SCWD than all models
  - Shows that no model perfectly represents local distributions
  - Compare to previous approach (GMWD)?



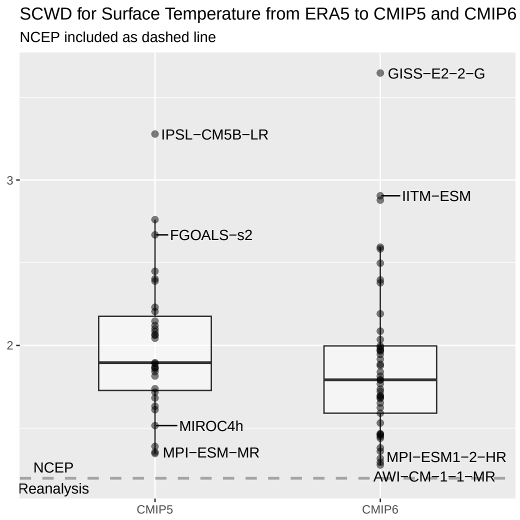
Surface Temperature Results: GMWD vs. SCWD



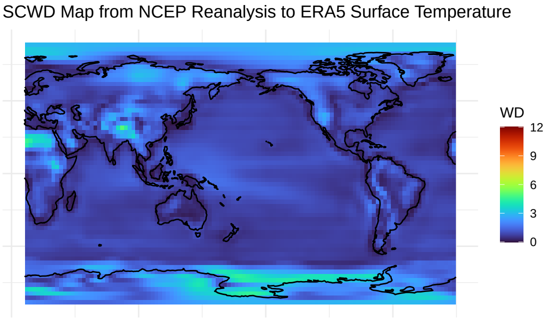


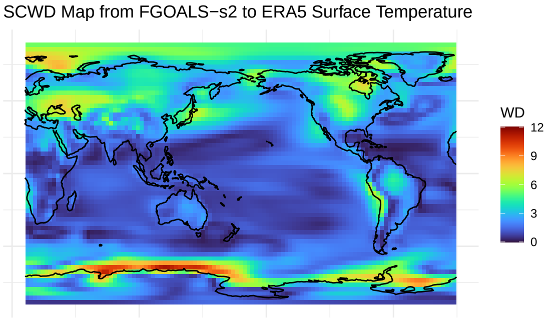
Surface Temperature Results: SCWD

- GMWD shows models can replicate variability in global mean up to a reasonable expectation of error
- SCWD shows that at the local level, there is still ongoing improvement
- SCWD is calculated as a mean over location-wise WDs
  - View map for NCEP reanalysis and FGOALS-s2 model



SCWD maps: NCEP and FGOALS vs. ERA5





- Overall: smooth maps are a good sign for our implementation
- NCEP: relatively low distance everywhere, higher near poles and mountains
- FGOALS-s2: very high distance off the coast of Antarctica
  - Follow-up reveals extreme lows in the winter due to issue with sea ice extent

SCWD conclusions

- Key points:
  - Our rankings incorporate many spatial features to improve upon previous climate model evaluation criteria
  - See paper for comparison to other criteria from climate and ML
  - Our maps can assist climate modelers in diagnosing model errors
- Future work:
  - Further comparisons of climate models, additional scenarios such as future projections
  - Machine-assisted climate model tuning
  - Similar to square images (Nguyen and Ho, 2022), can use SCWD to train generative models for 360^o images
  - Paper includes a more general functional sliced WD framework which can be used for color transfer/texture mapping on 3D models