💥BLAST: Block-Level Adaptive Structured Matrices for Efficient Deep Neural Network Inference

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NEURAL INFORMATION PROCESSING SYSTEMS

Structured Matrix for Efficient DNN Inference

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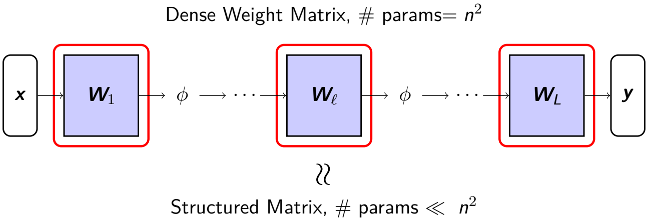

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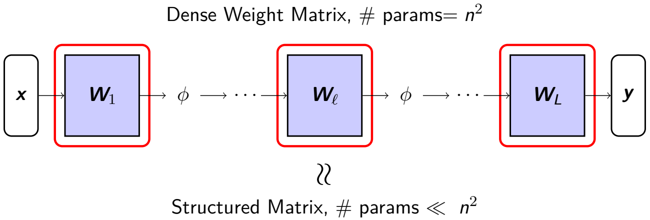

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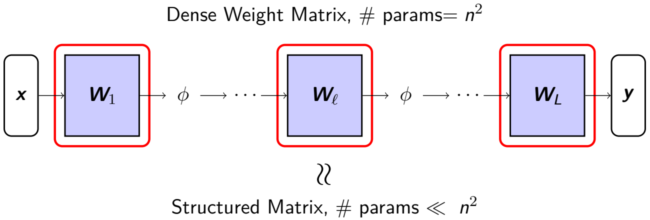
- Identifying and leveraging these underlying *structures* can significantly enhance the efficiency of a DNN without compromising accuracy.



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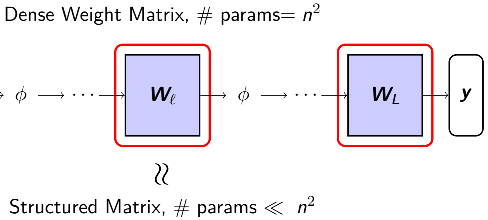
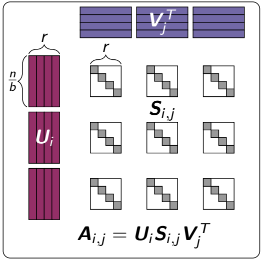
How can we uncover various low-dimensional structures in the weight matrices for accelerated inference?



BLAST: Block-Level Adaptive STructured Matrix

- **Flexible Design**: Encapsulates different structures such as low-rank, block low-rank, block-diagonal matrices, and their combinations.
- **Data-Driven Structure**: Identifies structures from data through gradient descent.
- **Factorization Algorithm**: Decomposes a dense matrix into the BLAST matrix via preconditioned gradient descent.



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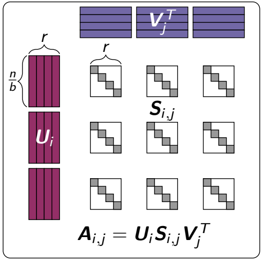
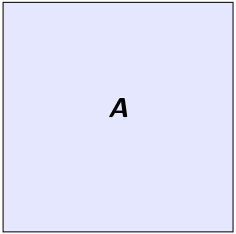
Simple integration into the deep learning pipelines:

1. Replace dense weights of a DNN with the BLAST matrices.
2. Initialize the BLAST factors randomly or through the factorization algorithm.
3. Update the BLAST factors using the training data through SGD.



Construction of BLAST Matrix



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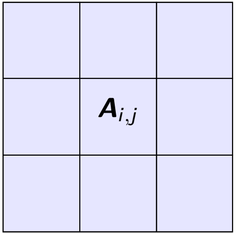
1. Partition an $n \times n$ matrix into equal-sized b^2 number of **blocks**.



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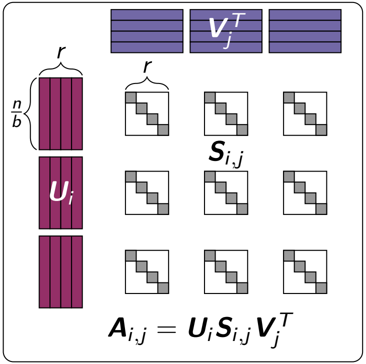

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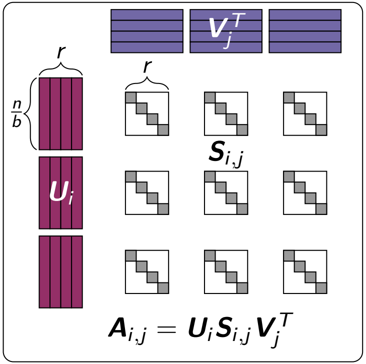

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$\boldsymbol{U}_i$, $\boldsymbol{V}_j$: Shared Bases

- Blocks at the ith row (jth column) share $\boldsymbol{U}_i$ ($\boldsymbol{V}_j$).

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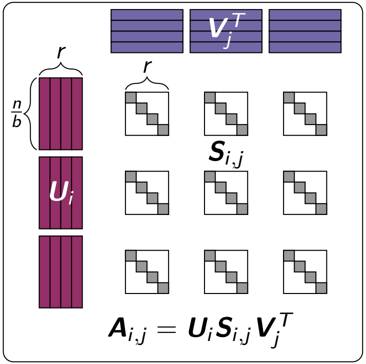

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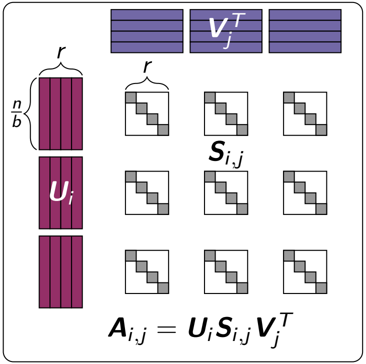

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Adaptive Structure by Diagonal Factors

- Low Rank: $\boldsymbol{S}_{i,j} = \boldsymbol{I}, \forall i, j = 1, \ldots, b$.
- Block Diagonal: $\boldsymbol{S}_{i,j} = \boldsymbol{I}$ if $i = j$, $\boldsymbol{O}$ otherwise.
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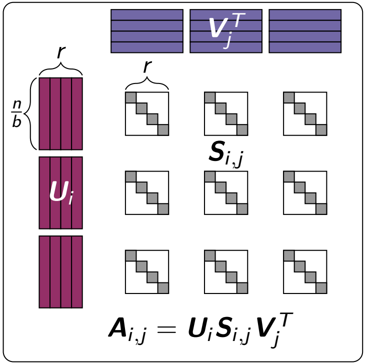

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Complexity for Matmul: $2nr + rb^2 < n^2$ multiplications.

Evaluations on Mid-Size Models

ViT-Base and GPT-2 with BLAST weight matrices trained from scratch:

Up to 73% reduction in FLOPs with *higher* accuracy.

Model	Accuracy (%)	Relative FLOPs (%)
Dense ViT-Base	78.7	100
Low-Rank	78.9	33.5
Monarch	78.9	33.5
Gaudi-GBLR	78.5	32.8
BLAST_3	**79.3**	**27.8**

Table: ImageNet validation accuracy and relative FLOPs of ViT-Base trained from scratch models with different structured weight matrices. BLAST_3 indicates the BLAST matrix with 3×3 number of blocks.

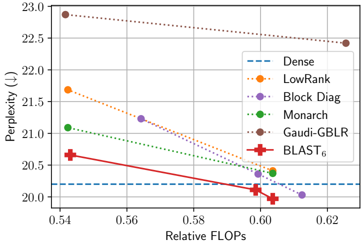

Figure: WikiText 103 test perplexity-FLOPs trade-off curves from GPT-2.

Evaluations on Foundation Model Compression

Compress DiT-XL and Llama-7B with BLAST by 50%, followed by re-training.

Minimal accuracy degradation; Up to 35% speedup on language generation on an NVIDIA A100 (40GB) GPU.

Figure: Examples of generated images using DiT-XL compressed by 50% through BLAST or low-rank matrices.

CR	Method	# Params	WikiText-2 Perplexity (↓)	0-Shot Acc. (%) (↑)
0%	Original Llama-7B	6.74B	9.37	66.07
50%	Low-Rank	3.51B	26.33 (-16.96)	48.40 (-17.67)
	Monarch	3.50B	7.53e5 (-7.53e5)	35.03 (-31.04)
	Block-Diagonal	3.50B	5.21e6 (-5.21e6)	34.86 (-31.21)
	BLAST$_{16}$	3.56B	**14.21 (-4.84)**	**56.22 (-9.84)**

CR	b	$L = 10$	$L = 100$	$L = 1000$
0%	N/A	0.41 $\pm$8e-5	3.82 $\pm$9e-4	41.23 $\pm$6e-3
20%	2	0.35 $\pm$9e-5	3.30 $\pm$2e-3	35.99 $\pm$4e-3
20%	16	0.36 $\pm$8e-5	3.36 $\pm$2e-3	36.48 $\pm$7e-3
50%	16	**0.31** $\pm$4e-4	**2.86** $\pm$1e-2	**30.35** $\pm$2e-2

Table: Top: Zero-shot performance of LLaMA-7B after compression and retraining. Bottom: Average runtime (in second) of Llama-7B with BLAST.