A General Method for Testing Bayesian Models with Neural Data

Gabor Lengyel*[1], Sabyasachi Shivkumar*[1,2] & Ralf Haefner[1]

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Department of Brain and Cognitive Sciences
University of Rochester

[2]Zuckerman Institute
Columbia University

*Equal contribution

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UNIVERSITY of ROCHESTER

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Bayesian models

are successful in explaining **behavior**

- Perception (e.g., Knill & Richards 1996, Kersten et al. 2004)
- Cognition (e.g., Gold & Shadlen 2007, Lange et al. 2021)
- Perceptual learning (e.g., Michel & Jacobs 2007, Fiser & Lengyel 2019)
- Statistical learning (e.g., Orban et al. 2008, Fiser & Lengyel 2022)
- Concept learning (e.g., Huth et al. 2016, Lake et al. 2015, 2017)
- Motor learning (e.g., Kording & Wolpert 2004, Heald et al. 2021)

Bayesian models

are also successful in explaining some **neural activity**

- Primary visual cortex (e.g., Berkes et al. 2011, Banyai et al. 2018, Walker et al. 2020)
- Dorsal medial superior temporal area (e.g., Fetsch et al. 2011)
- Inferior temporal cortex (e.g., Tajima et al. 2016)
- Primary auditory cortex (e.g., Kumar et al. 2007, 2011)
- Hippocampus (e.g., Ujfalussy et al. 2022)
- Lateral intraparietal cortex (e.g., Beck et al. 2008, Hou et al. 2019)
- Posterior parietal cortex (e.g., Funamizu et al. 2016)

Testing Bayesian models with neural data

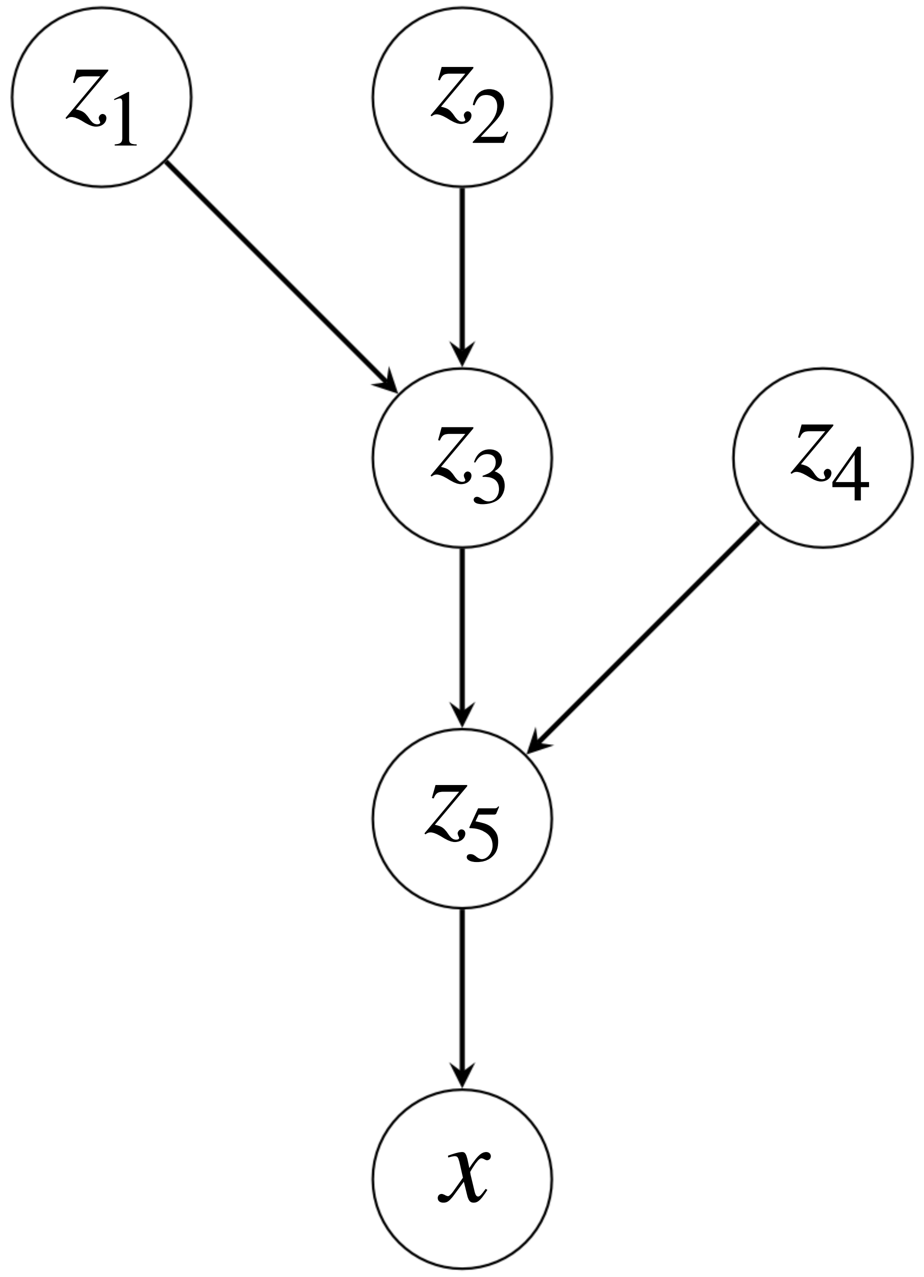
- Two key ingredients

Testing Bayesian models with neural data

- Two key ingredients

  1. Generative model

$p(z_1, z_2, \ldots\ z_5\ \ x)$



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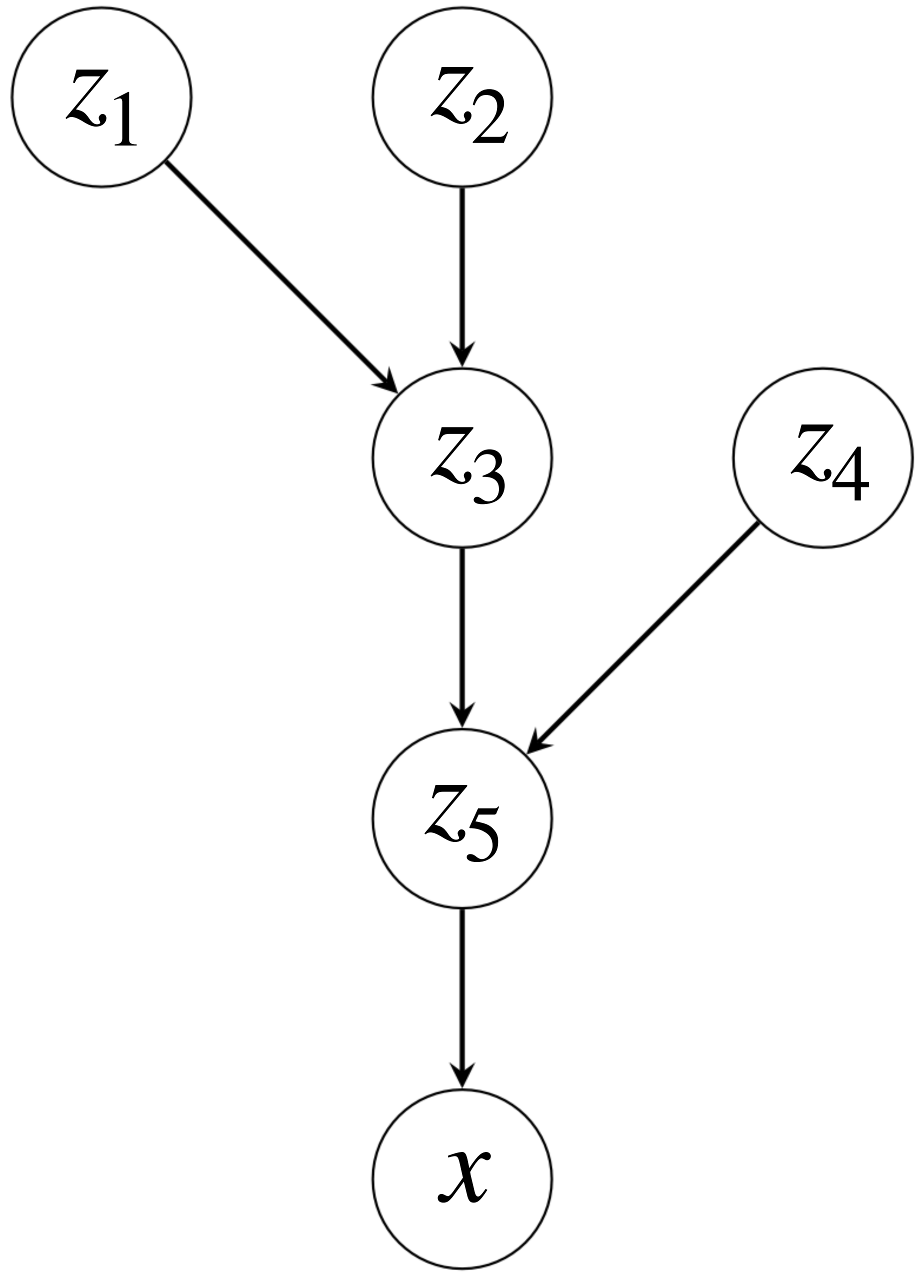
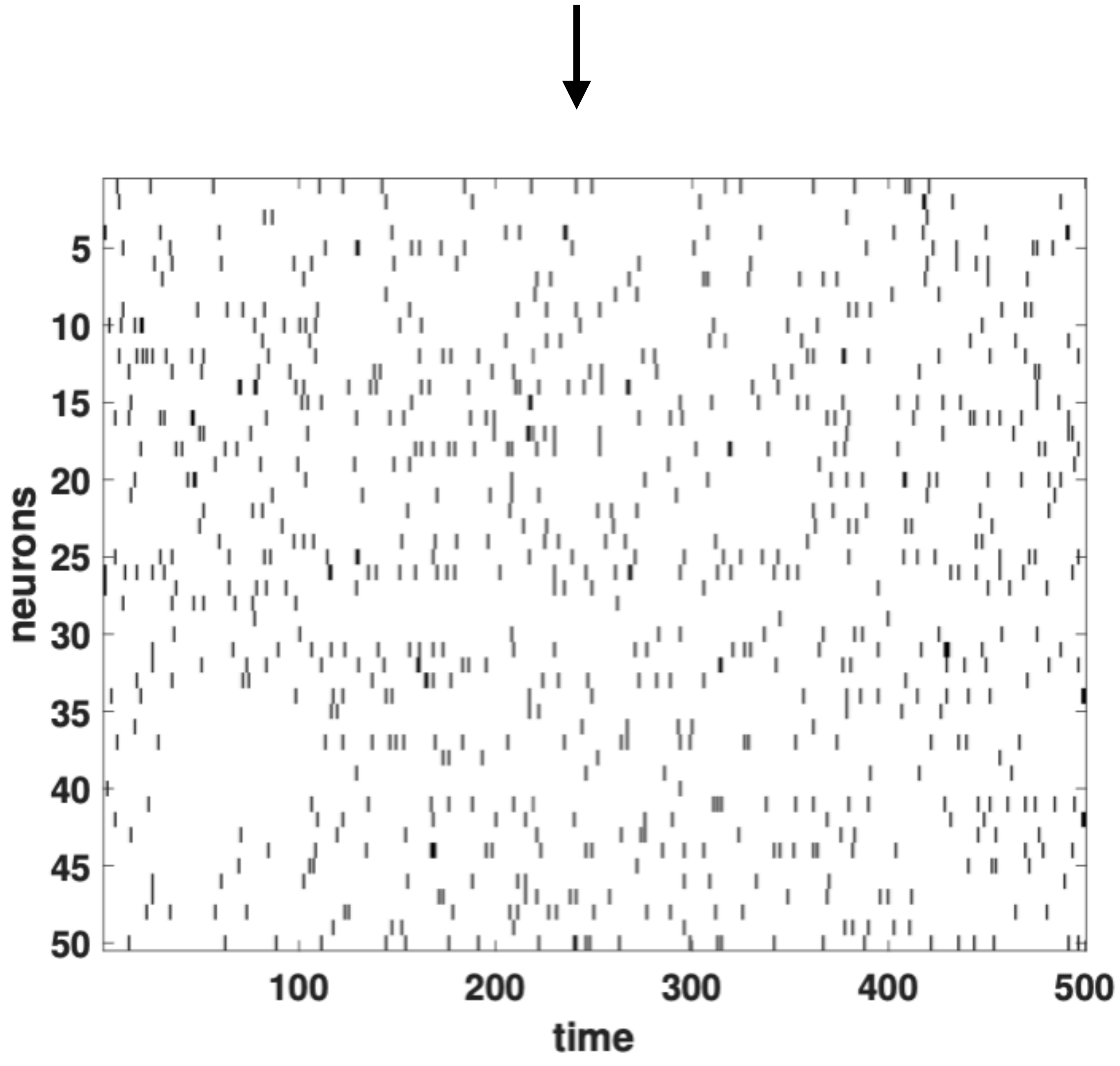


2. Encoding model

$p(z_1, z_2, \dots\ z_5\ \ x)$



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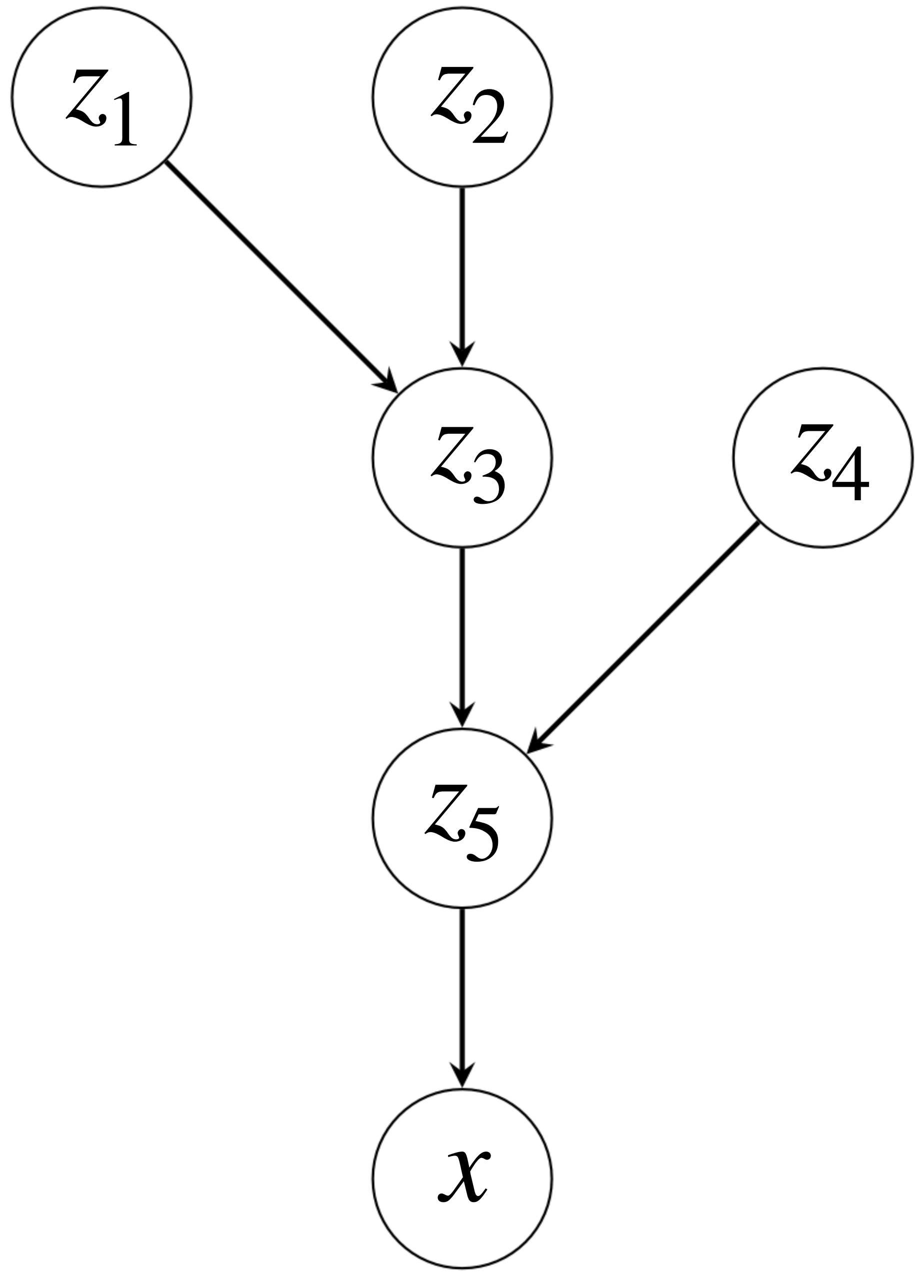

2. Encoding model

- Probabilistic Population Codes (PPCs)
  (e.g., Ma et al. 2006, Beck et al. 2012)
- Distributed Distributional Codes (DDCs)
  (e.g., Vertes & Sahani 2018, 2019)
- Neural sampling
  (e.g., Orban et al. 2016, Ujfalussy et al. 2022)

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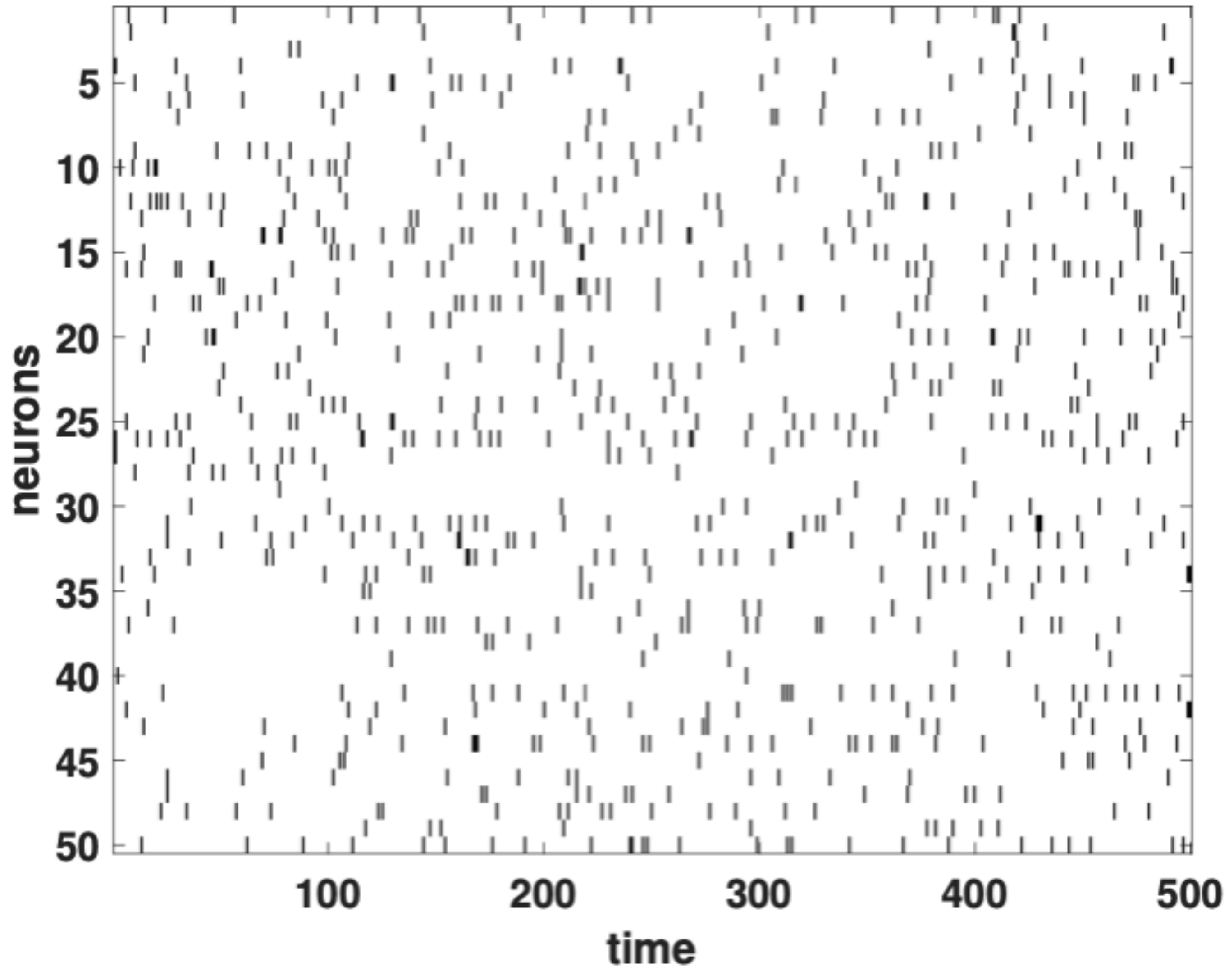

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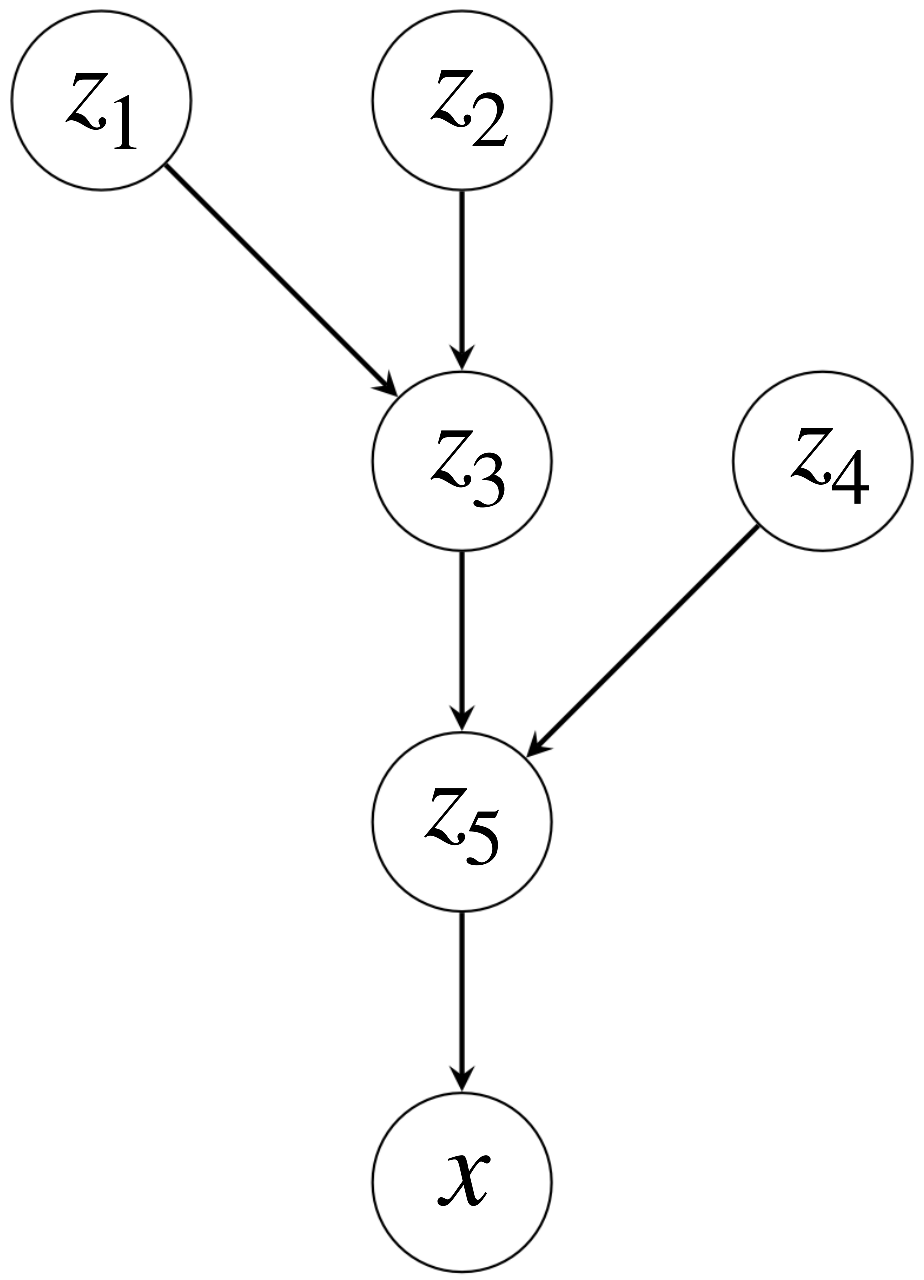

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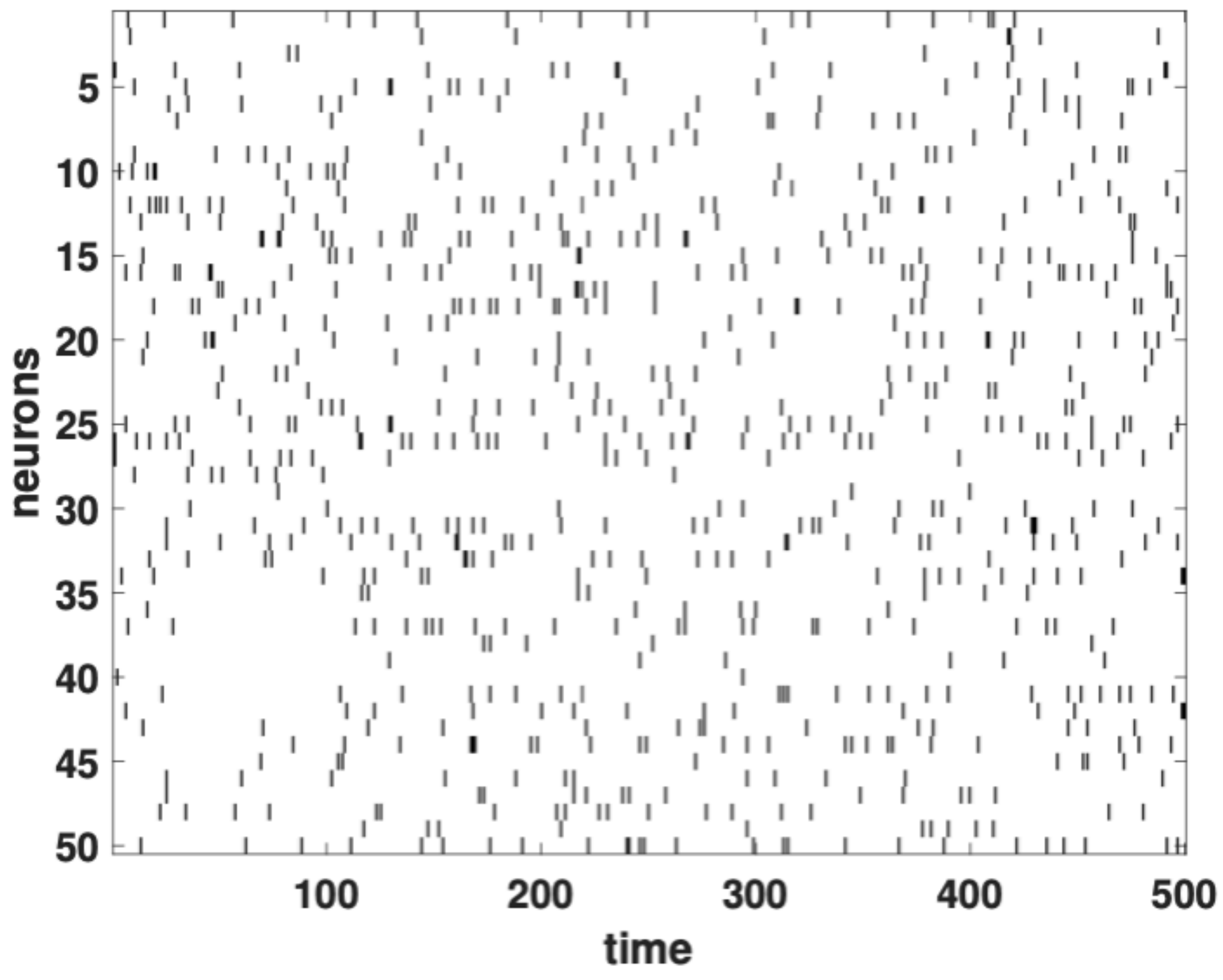

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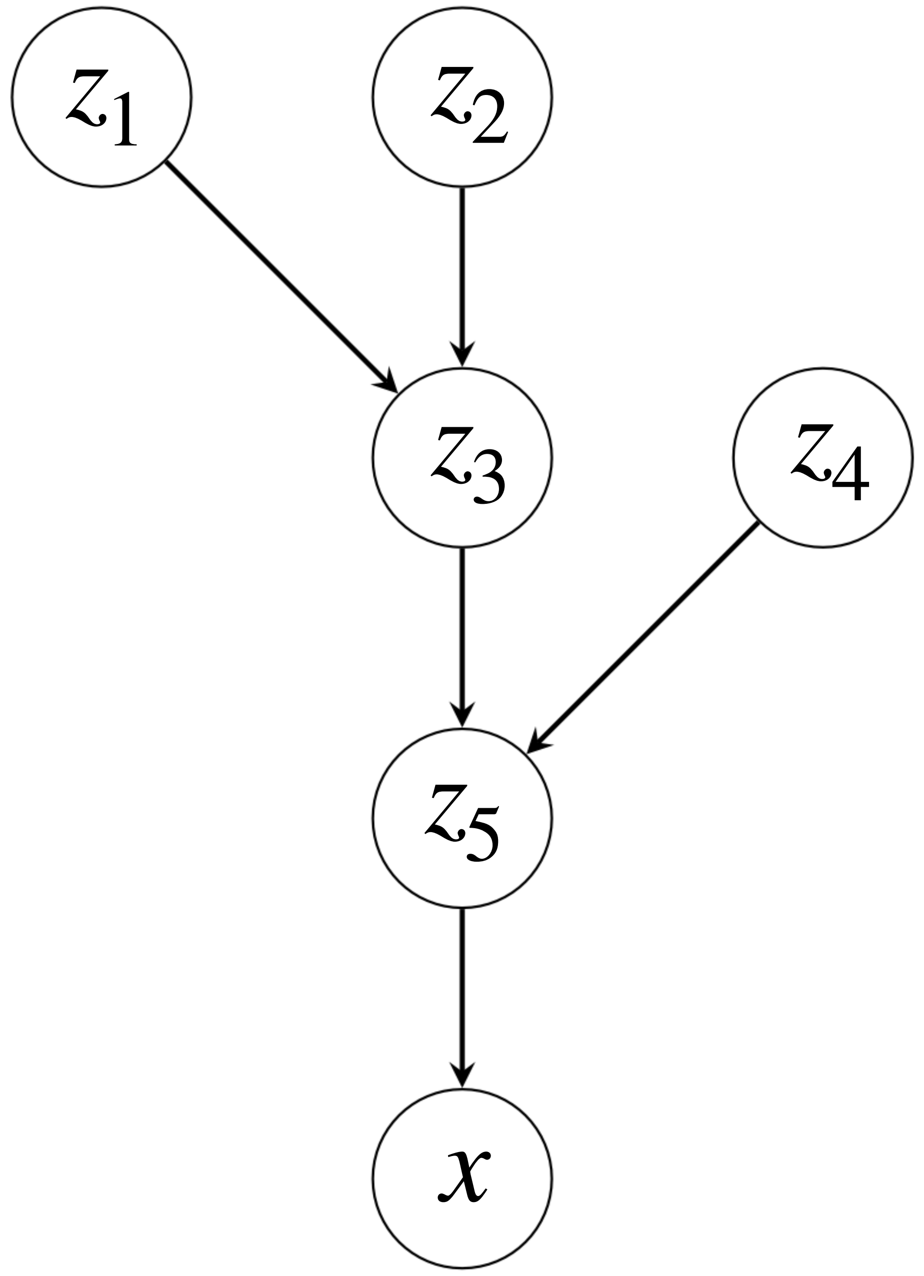

Testing a generative model, M_1

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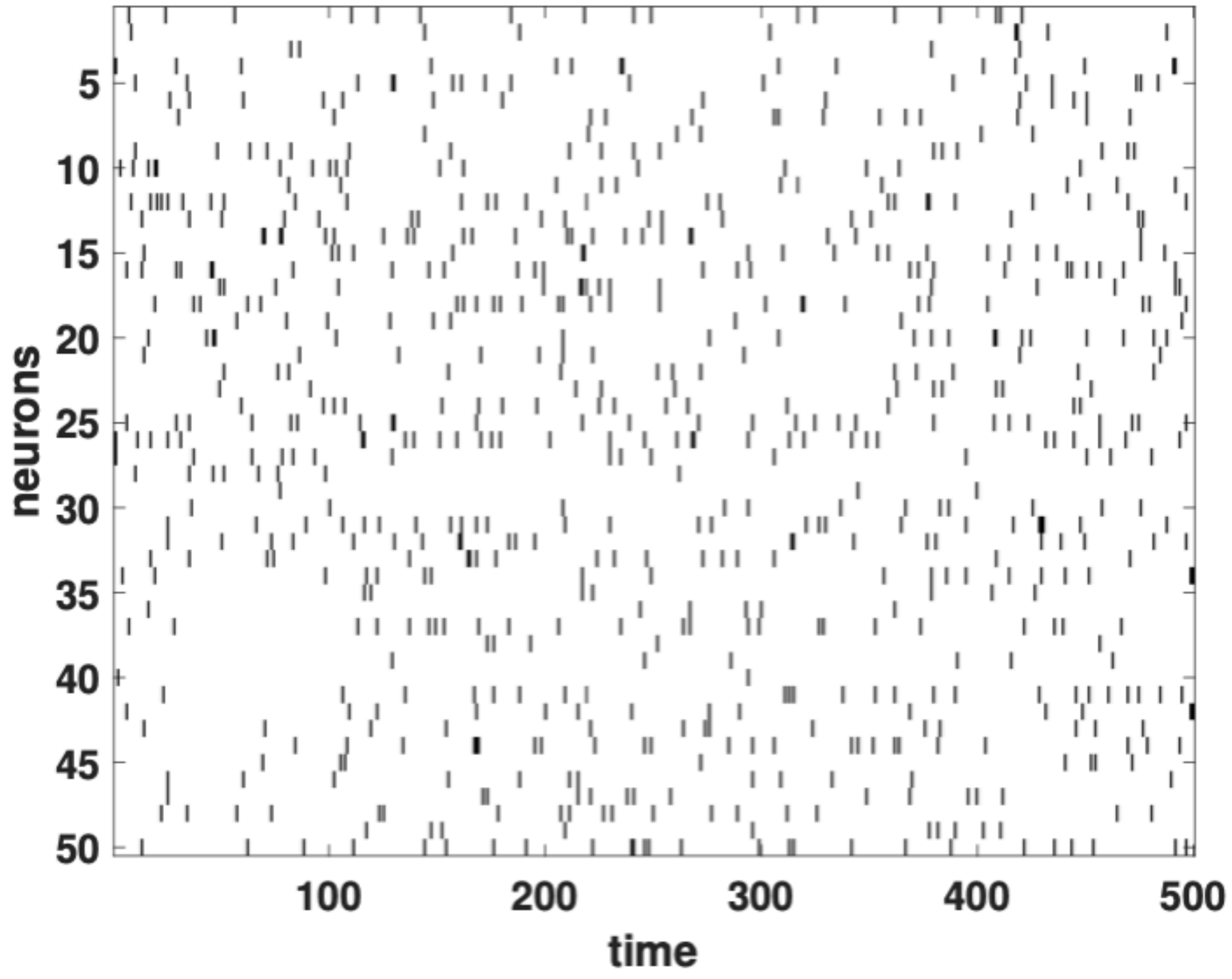

Testing Bayesian models with neural data

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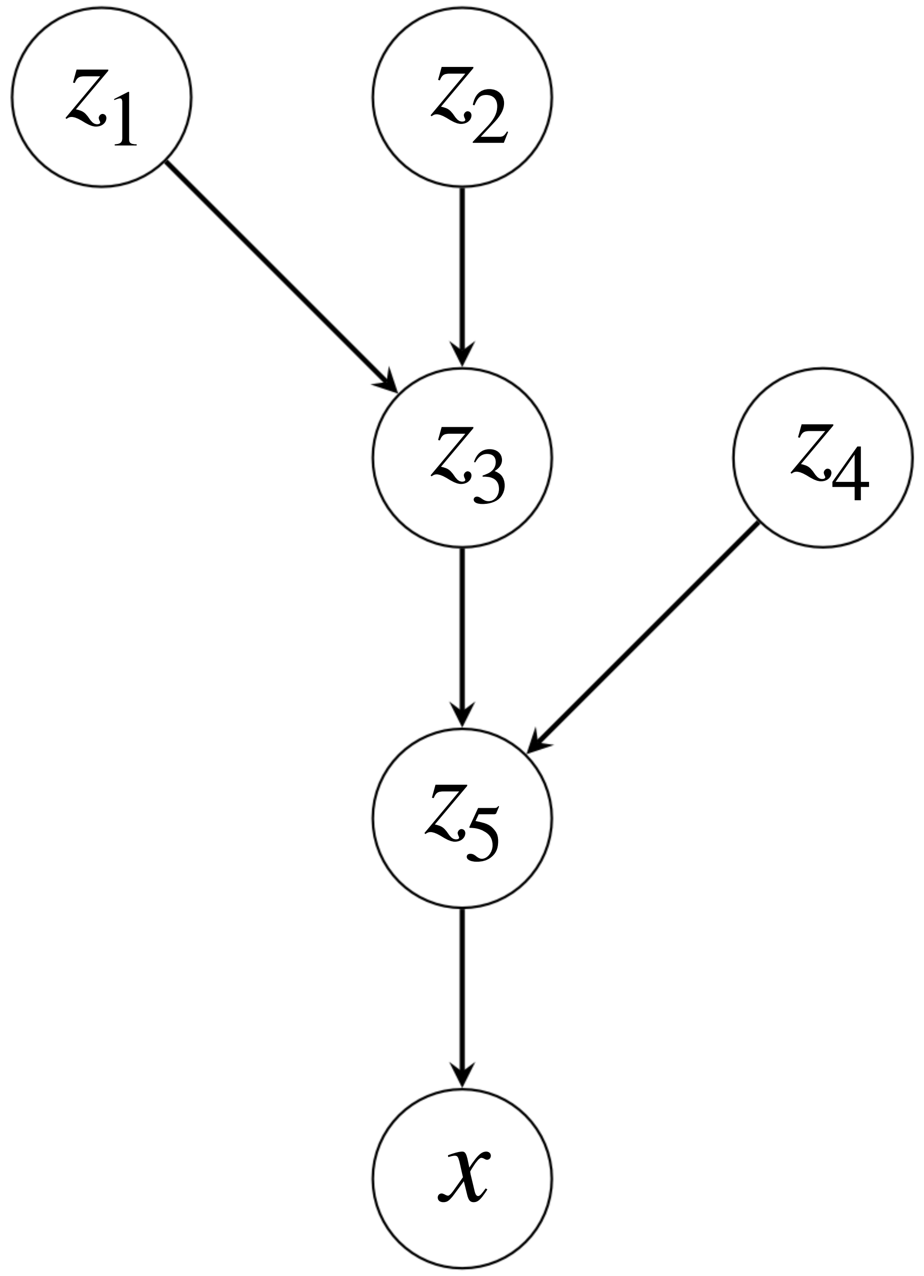

Testing a generative model, M_1

Assessing the predictions generated from all possible encodings

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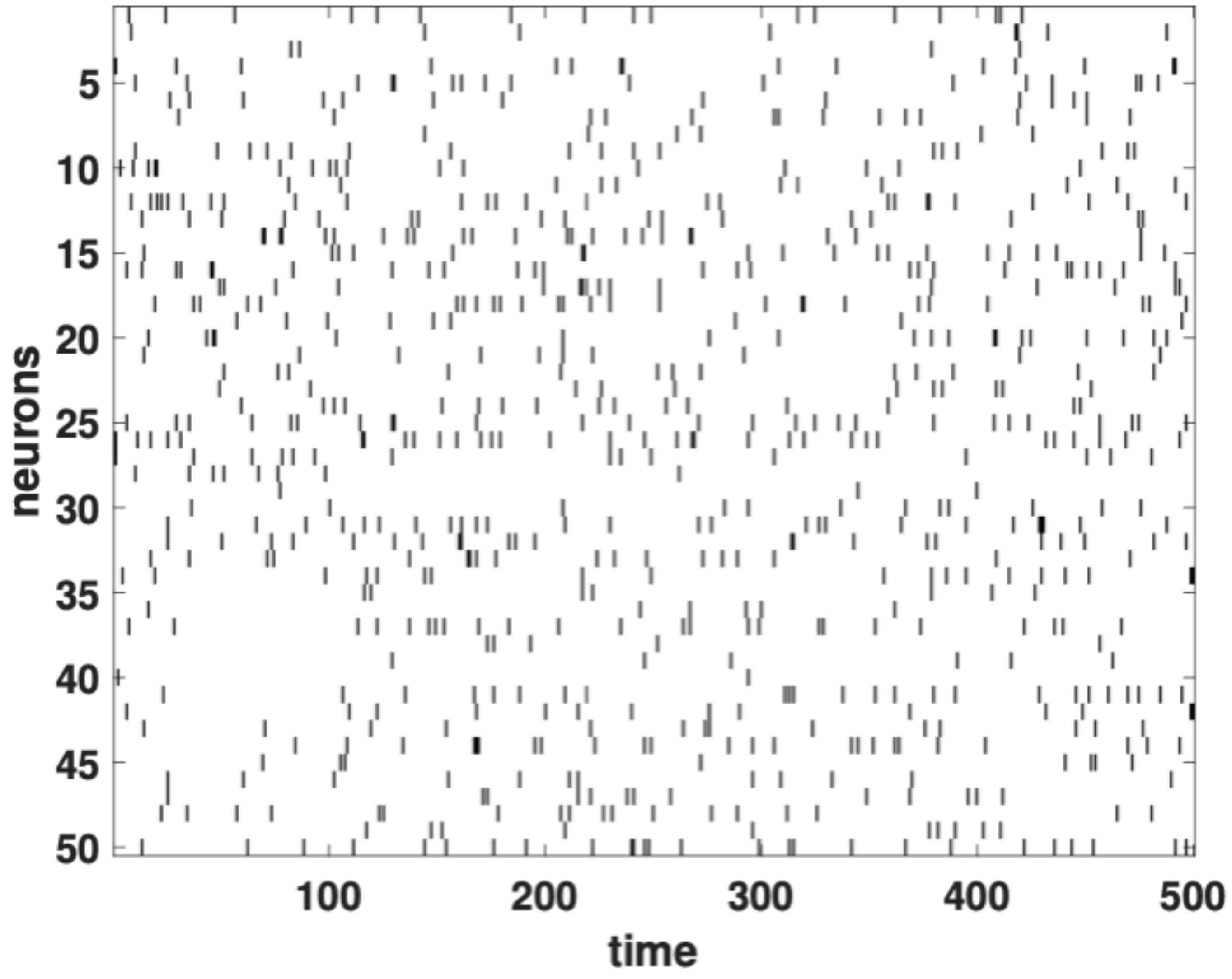

Testing Bayesian models with neural data

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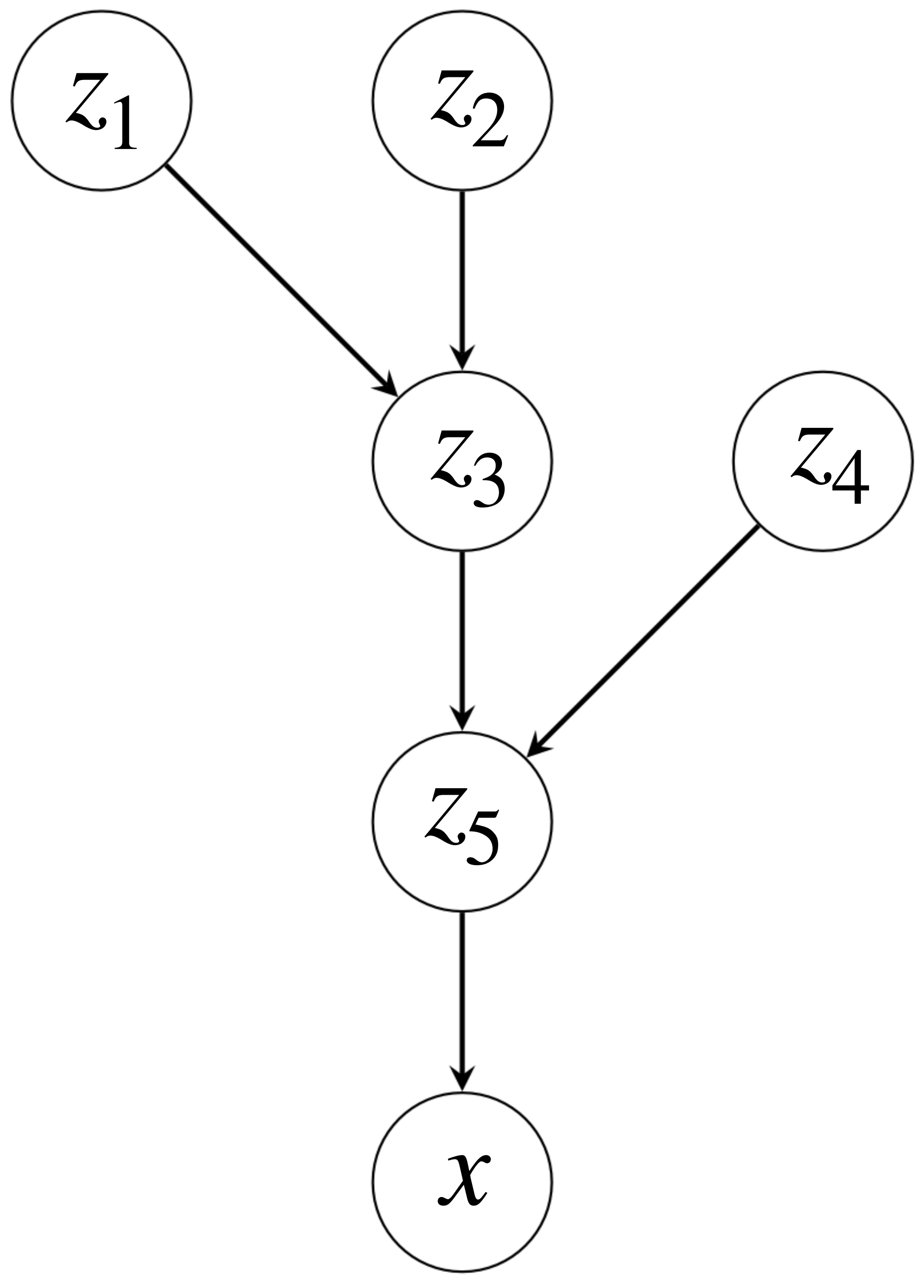

Testing a generative model, M_1

Assessing the predictions generated from all possible encodings

E.g., Ujfalussy & Orban, 2022

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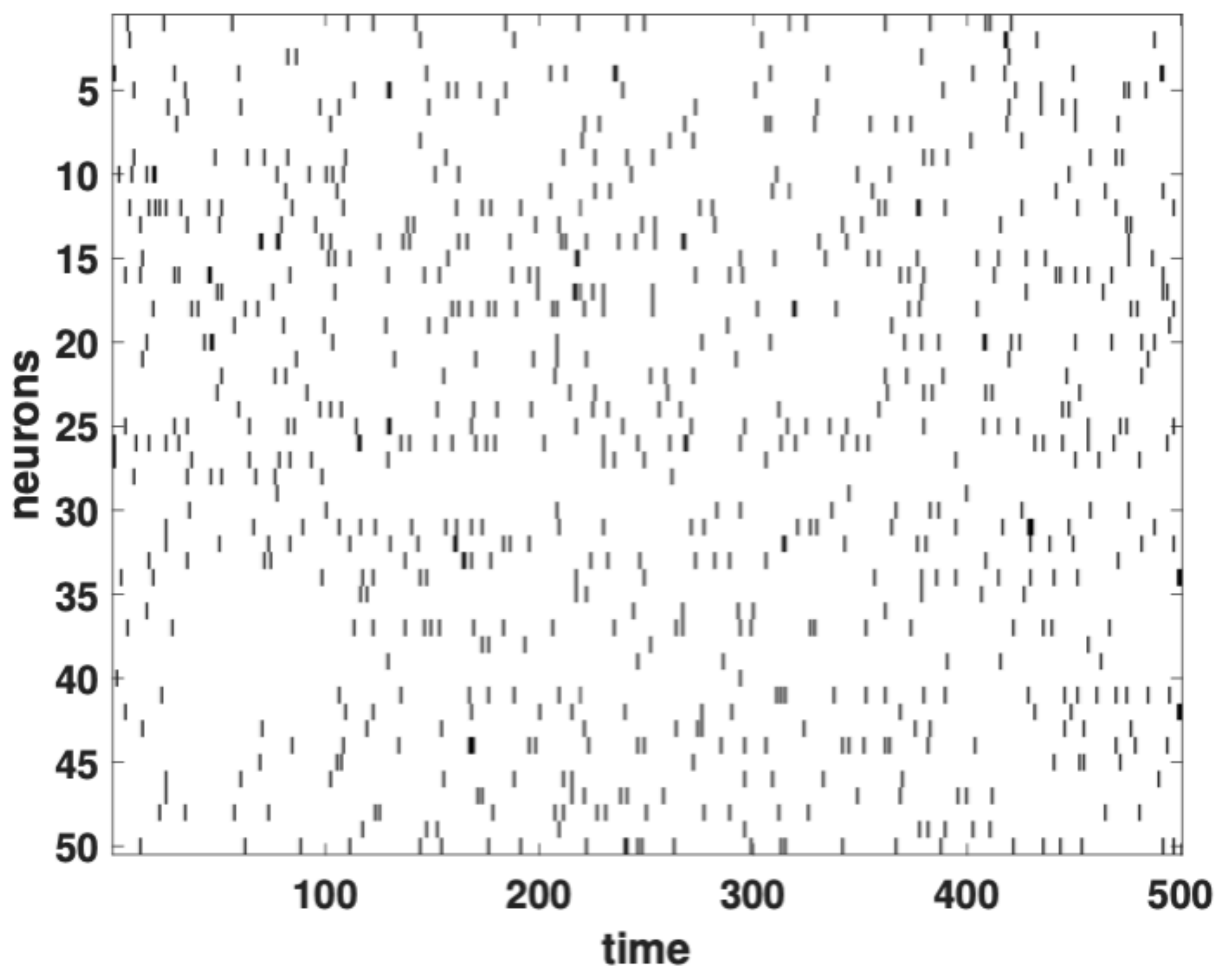

Testing Bayesian models with neural data

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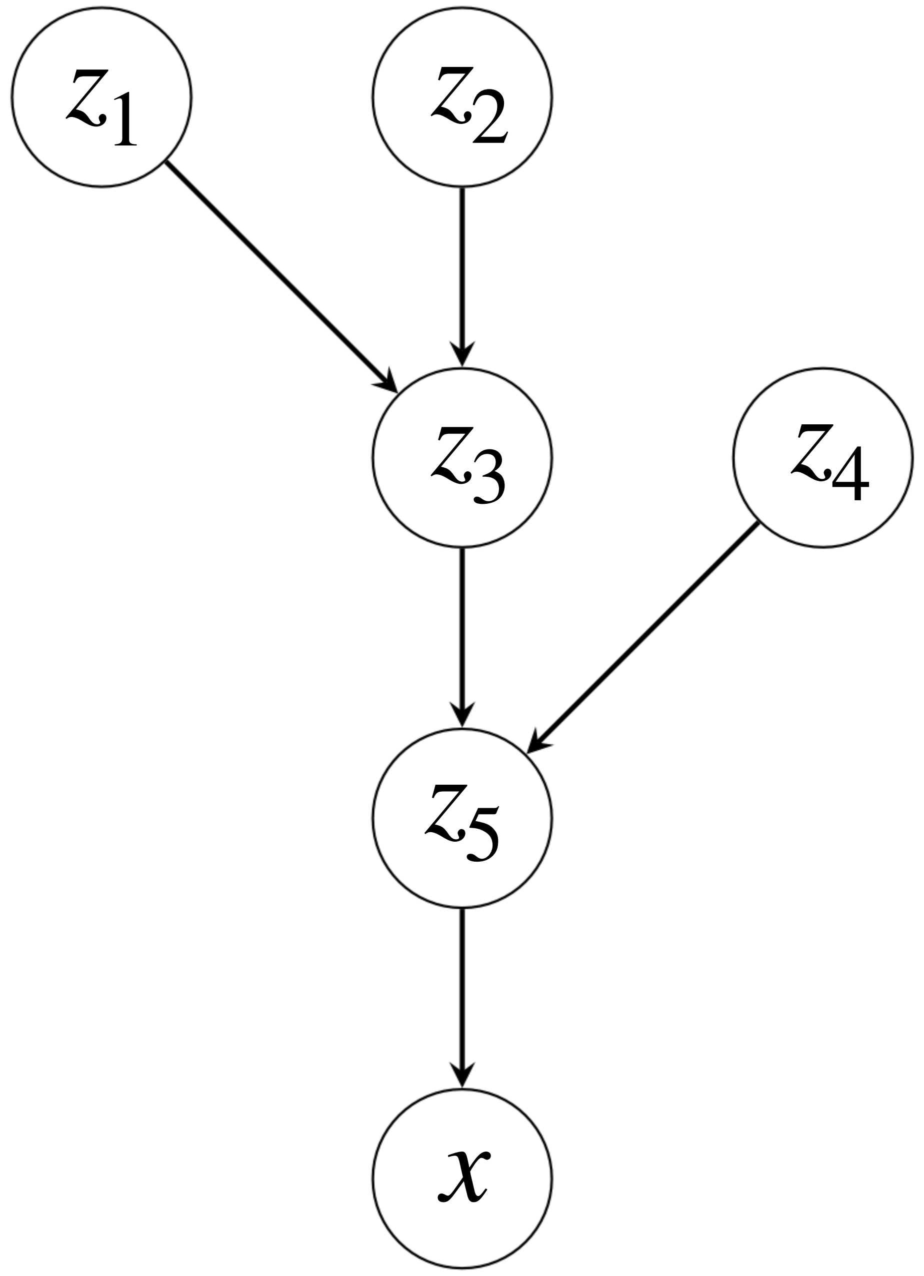

Testing multiple generative models,

$M_1, \dots\ M_m$

…

Assessing the predictions generated from all possible encodings

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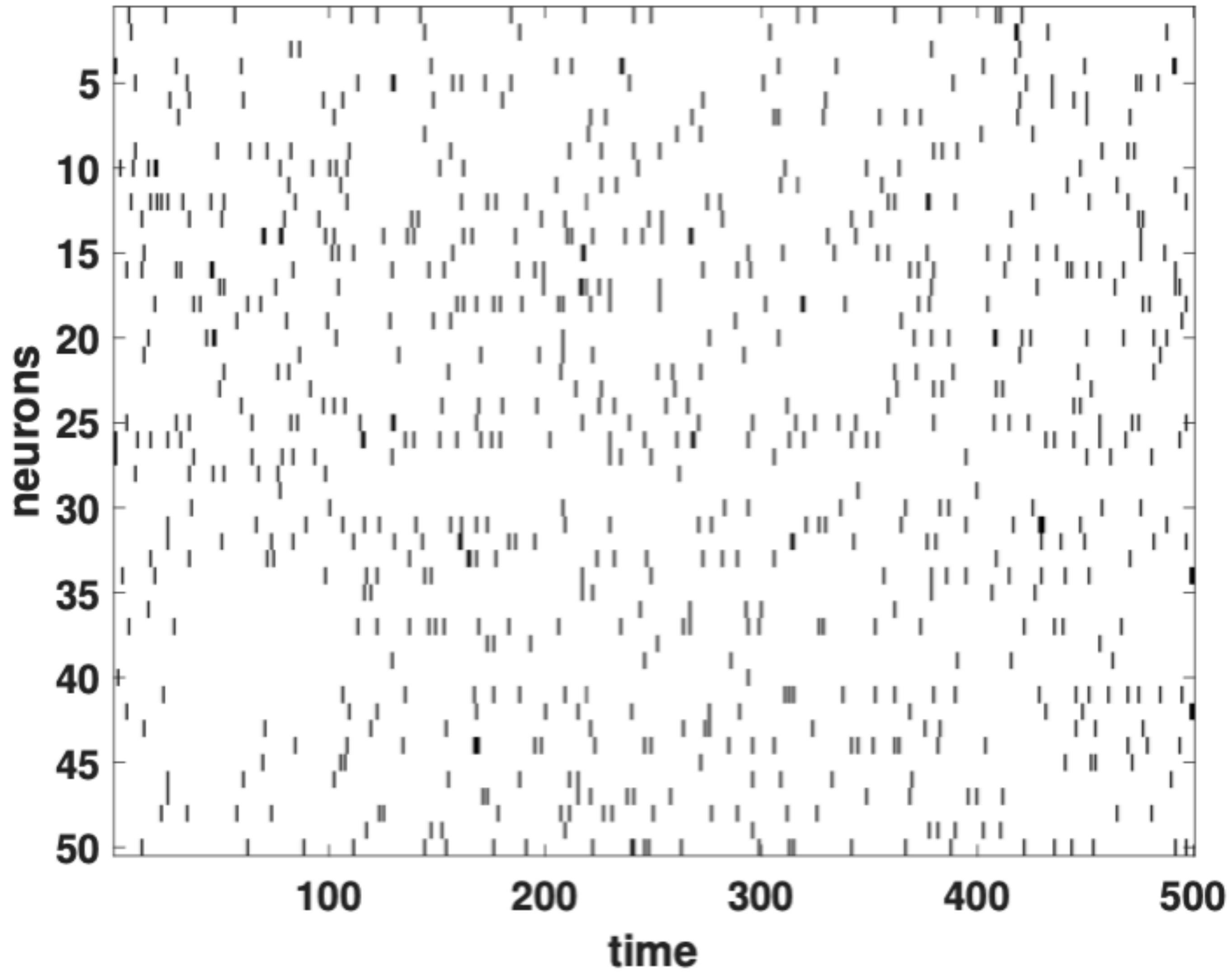

Testing Bayesian models with neural data

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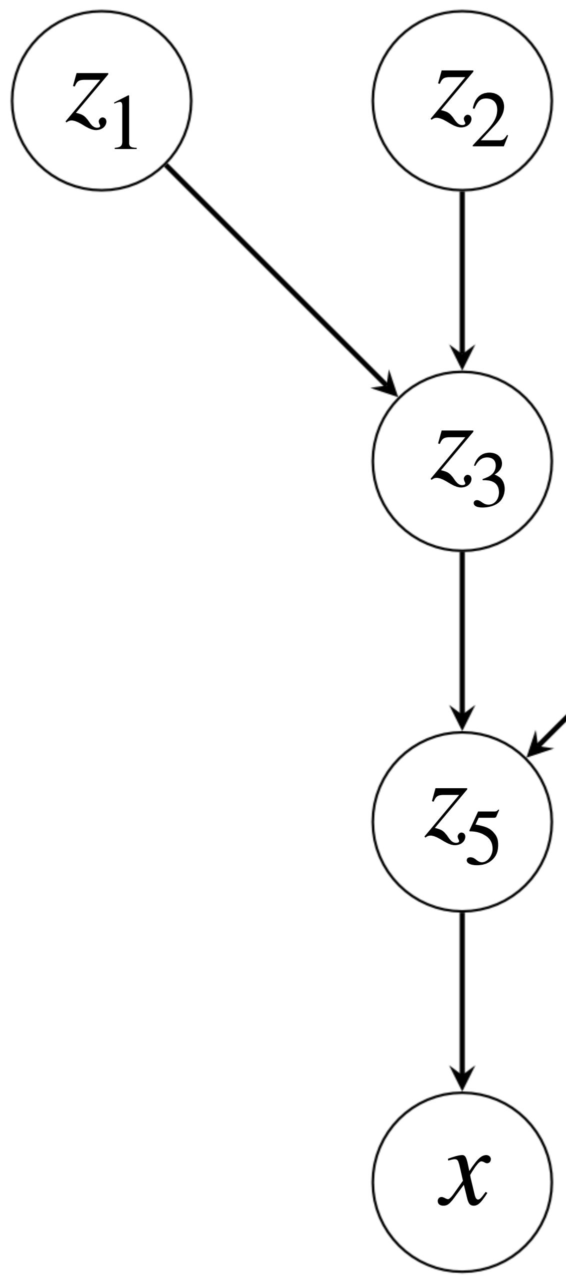

Testing multiple generative models, $M_1, \dots\ M_m$

Aim: to develop a method for testing different generative models that is invariant to the details of the encoding

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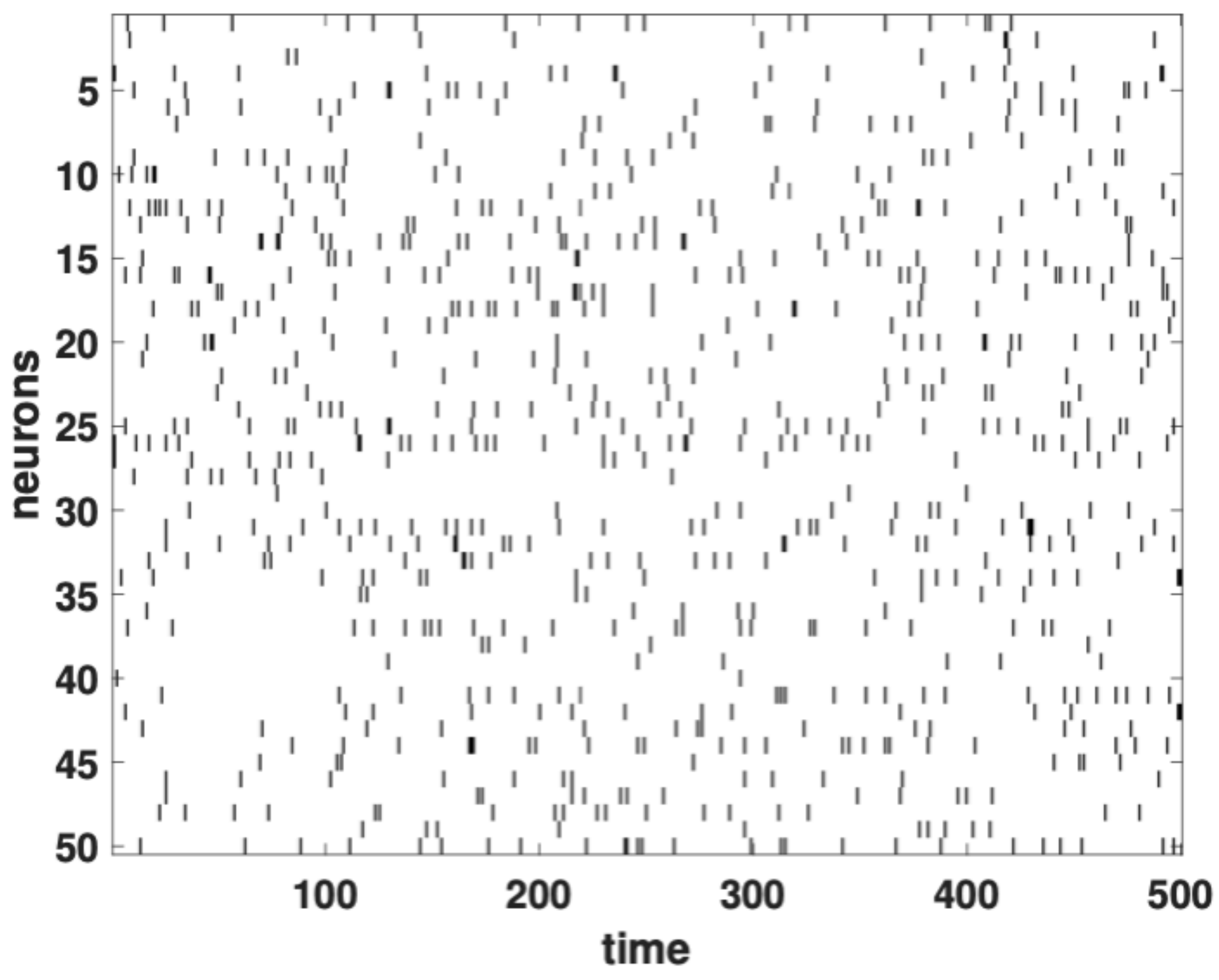

Testing Bayesian models with neural data

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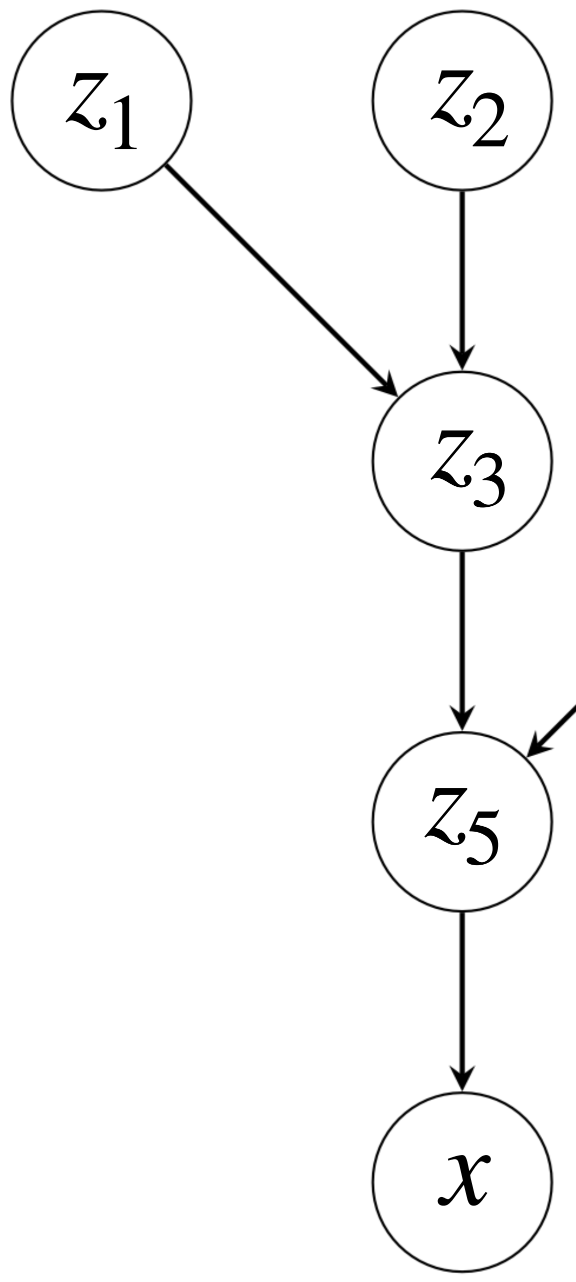

Testing multiple generative models, $M_1, \dots\ M_m$

Aim: to develop a method for testing different generative models that is invariant to the details of the encoding

It will be possible to conclude which generative model best explains data

2. Encoding model

No consensus about the encoding

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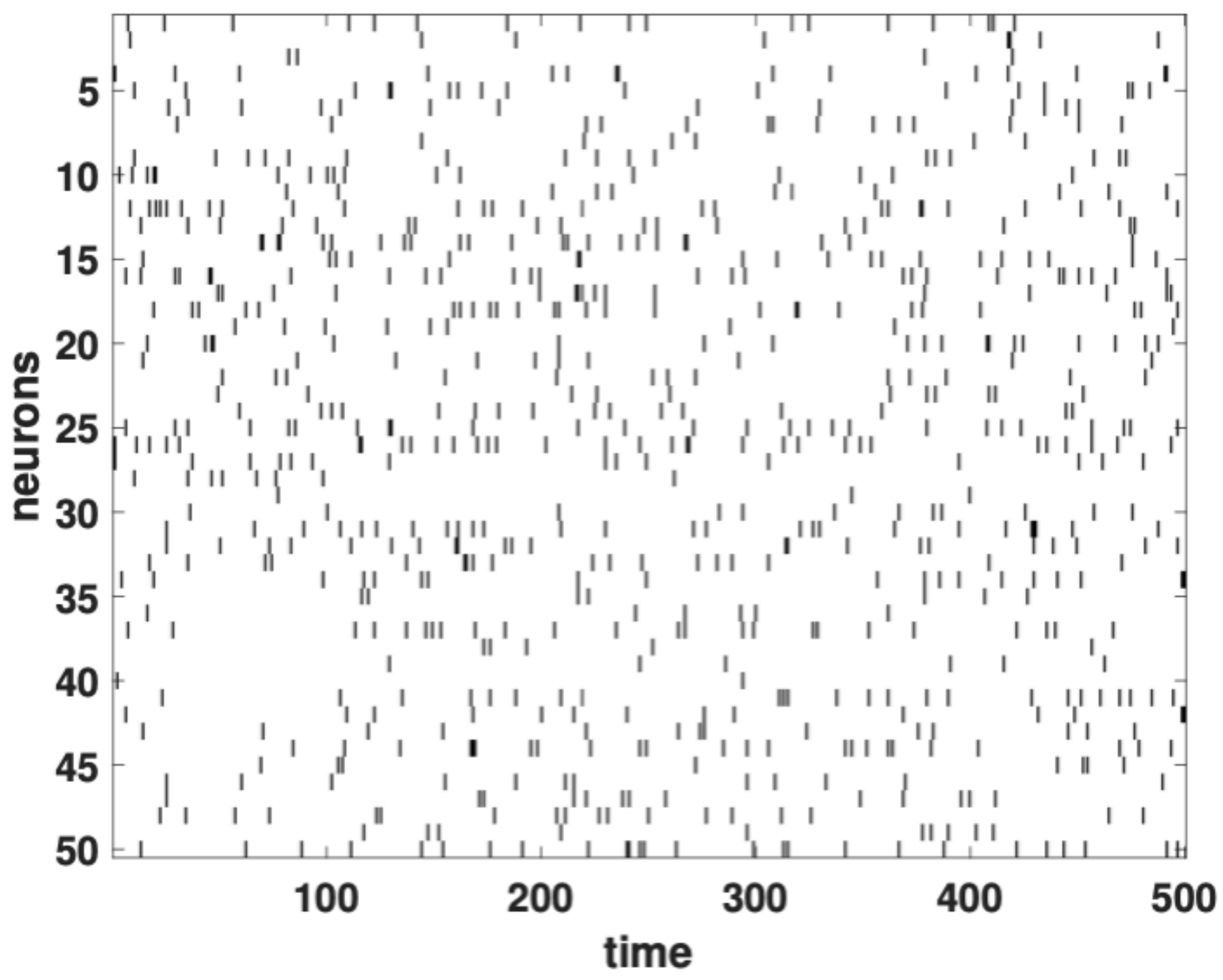

The general method for testing Bayesian models

- Assumption
  - Our method is invariant to many encodings
    - Linear Distributional Codes (LDCs) (Lange & Haefner 2022)

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  - Is not invariant to
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- Method
  - Akin to Representational Similarity Analysis (RSA, Kriegeskorte et al. 2008)

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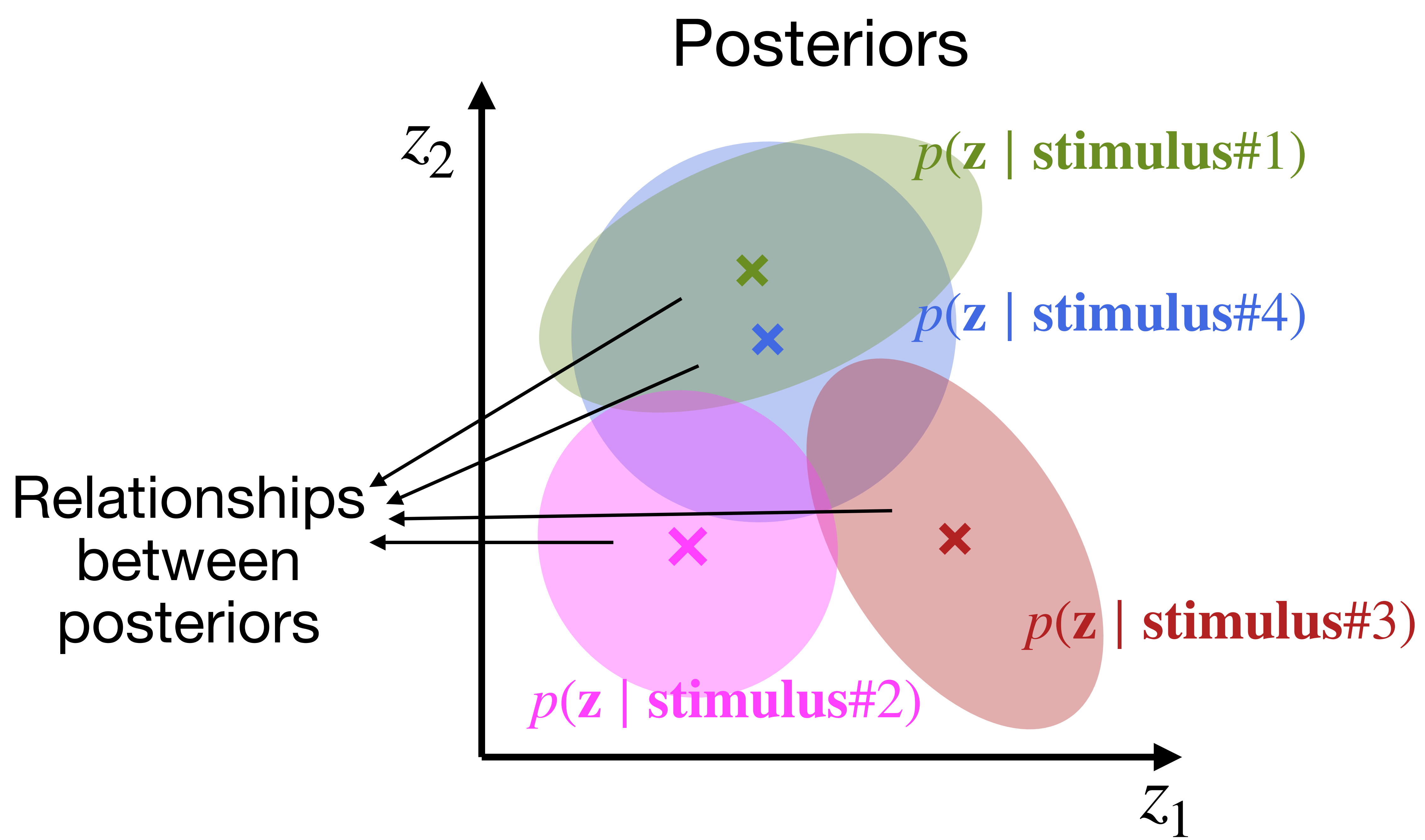

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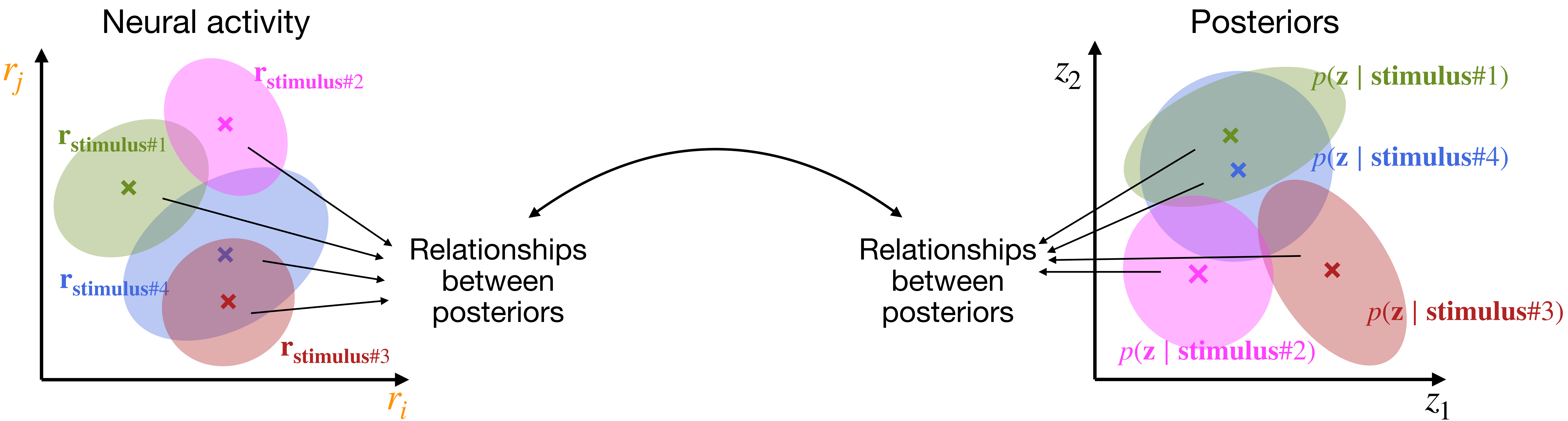

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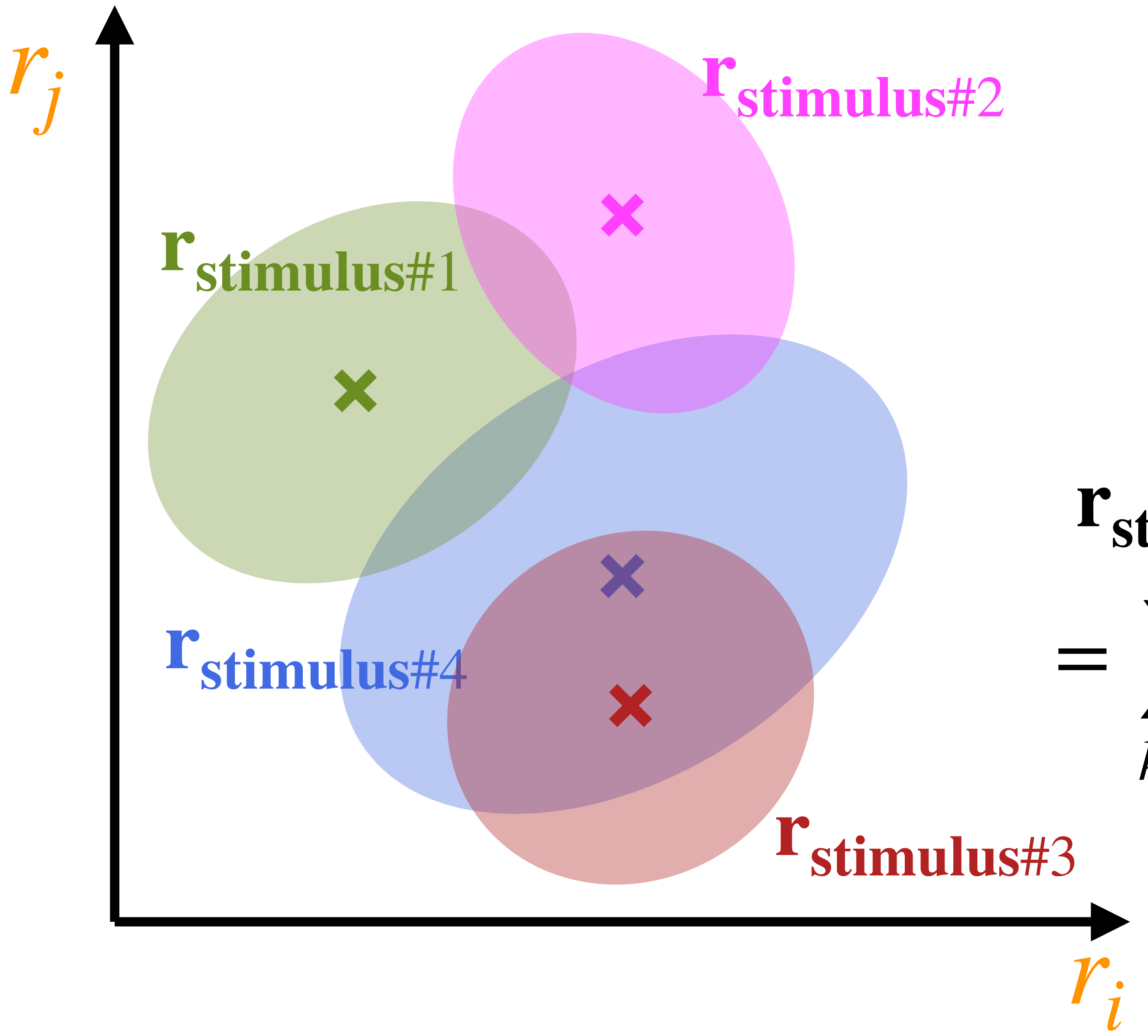
Neural activity

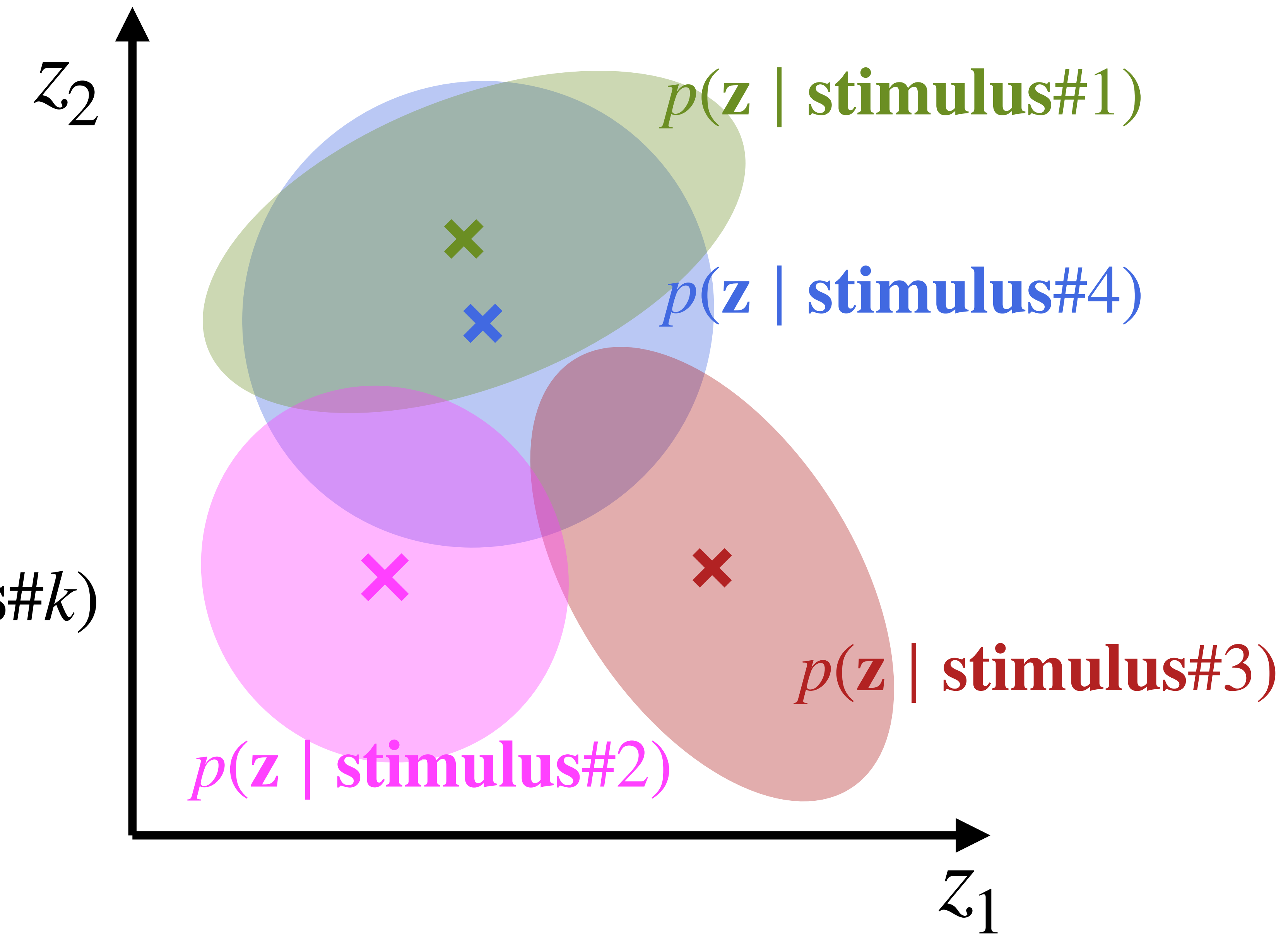


Comparing relationships
by comparing w

$$\mathbf{r}_{\text{stimulus\#}n} = \sum_{k \neq n} w_k \mathbf{r}_{\text{stimulus\#}k}$$

$$p(\mathbf{z} \mid \textbf{stimulus\#}n) = \sum_{k \neq n} w_k p(\mathbf{z} \mid \textbf{stimulus\#}k)$$

Posteriors



Application

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- Generating neural predictions
  - For single neurons in area MT
  - From a Bayesian causal inference model of motion perception fitted to behavior in a center-surround motion experiment (Shivkumar et al. 2023)

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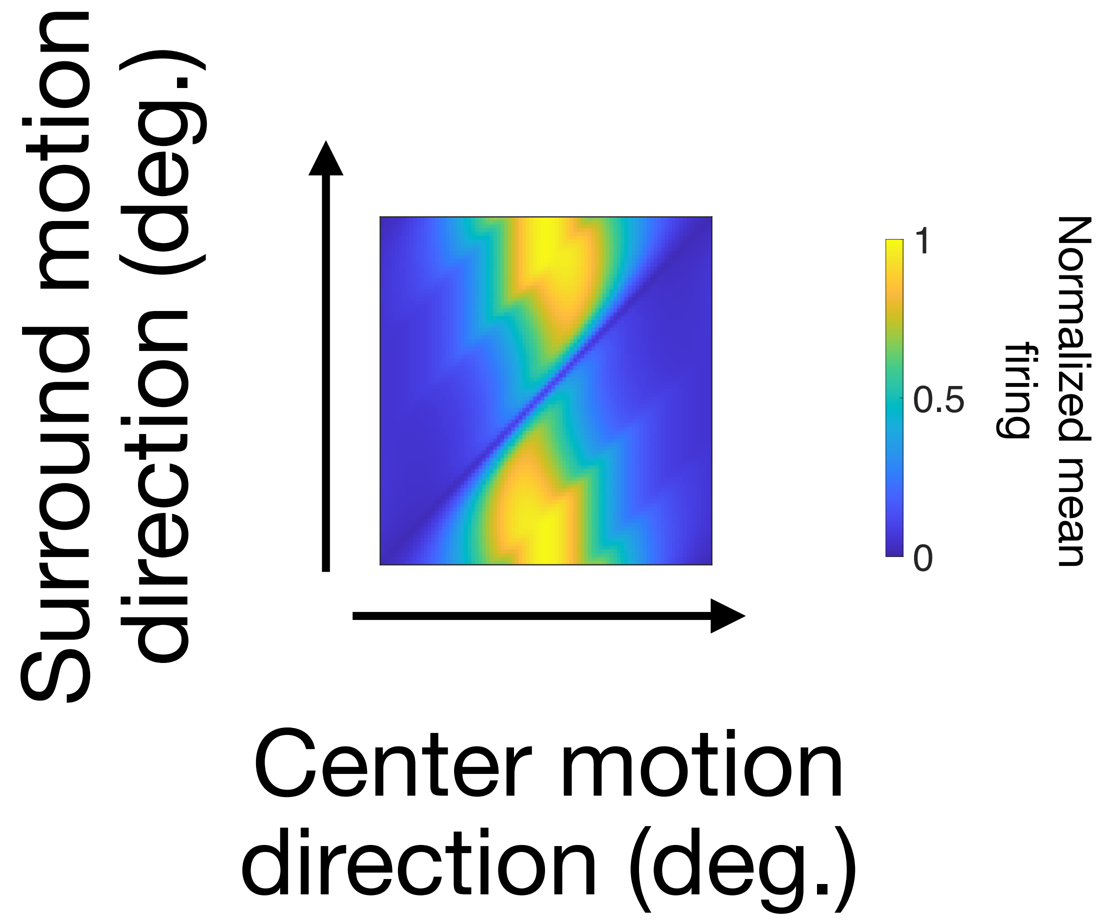

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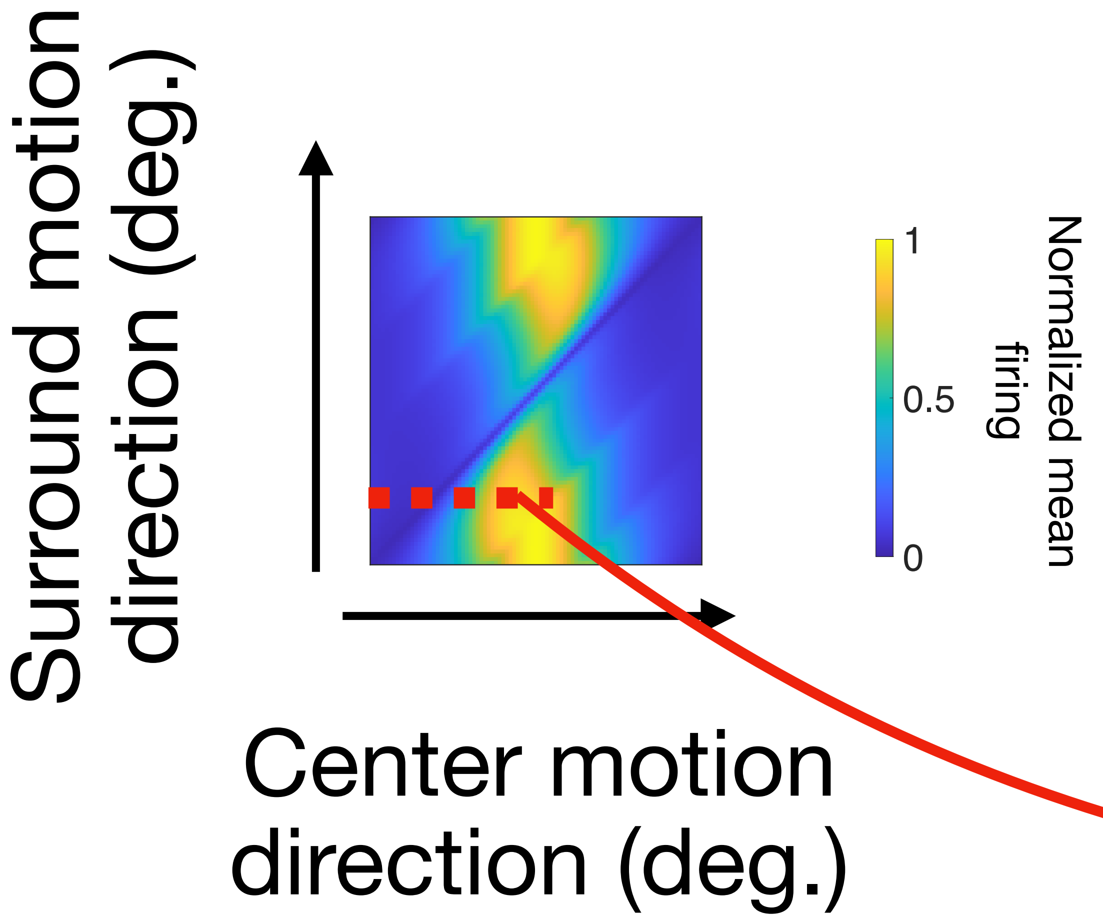
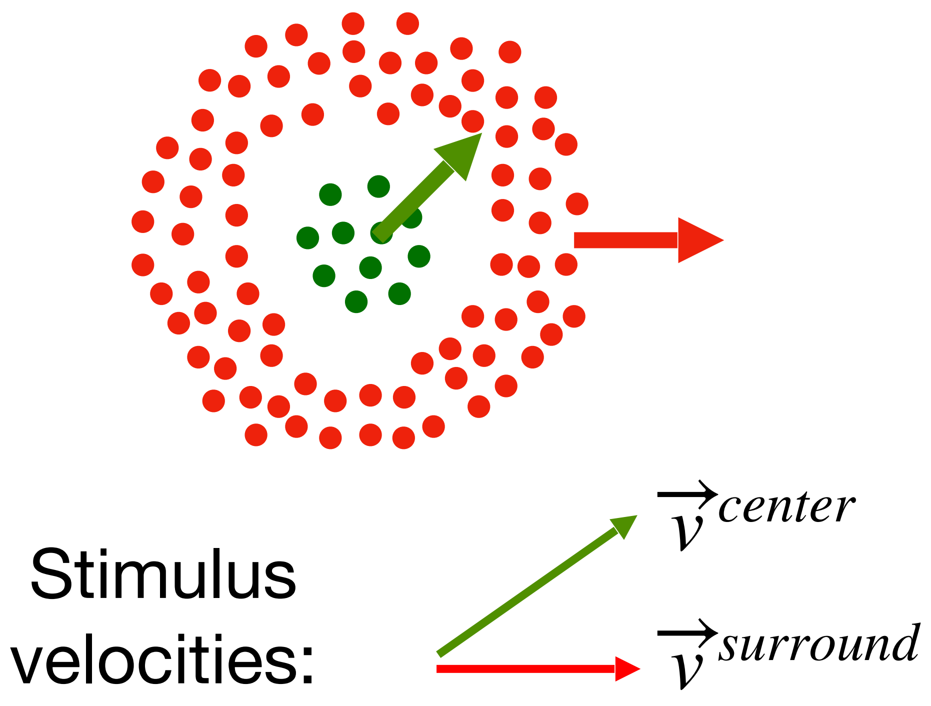


Surround direction = -90 deg.

Recordings from area MT while monkey passively observing center-surround motion stimuli (done by Zhexin Xu & Gregory C. DeAngelis)



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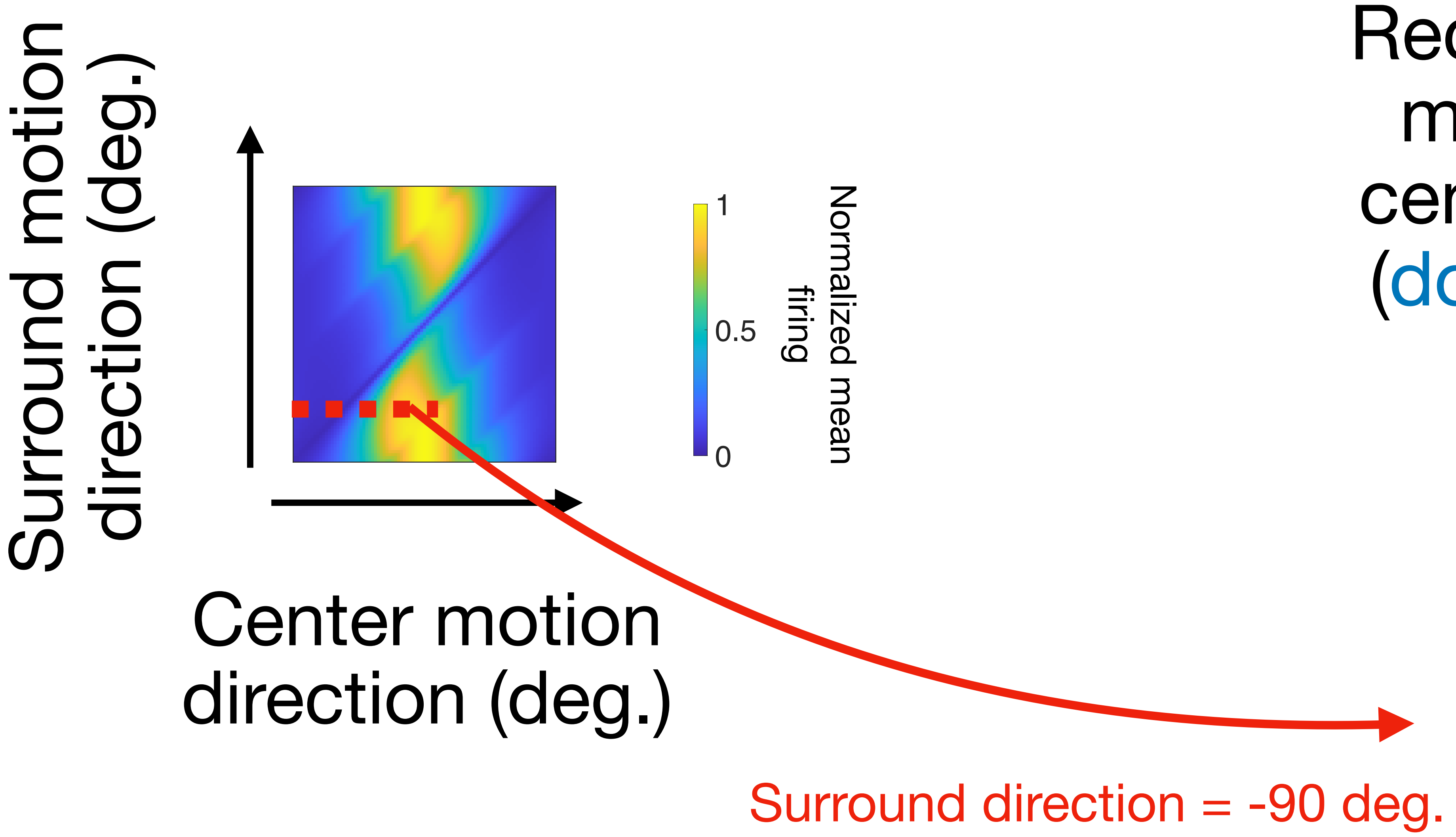

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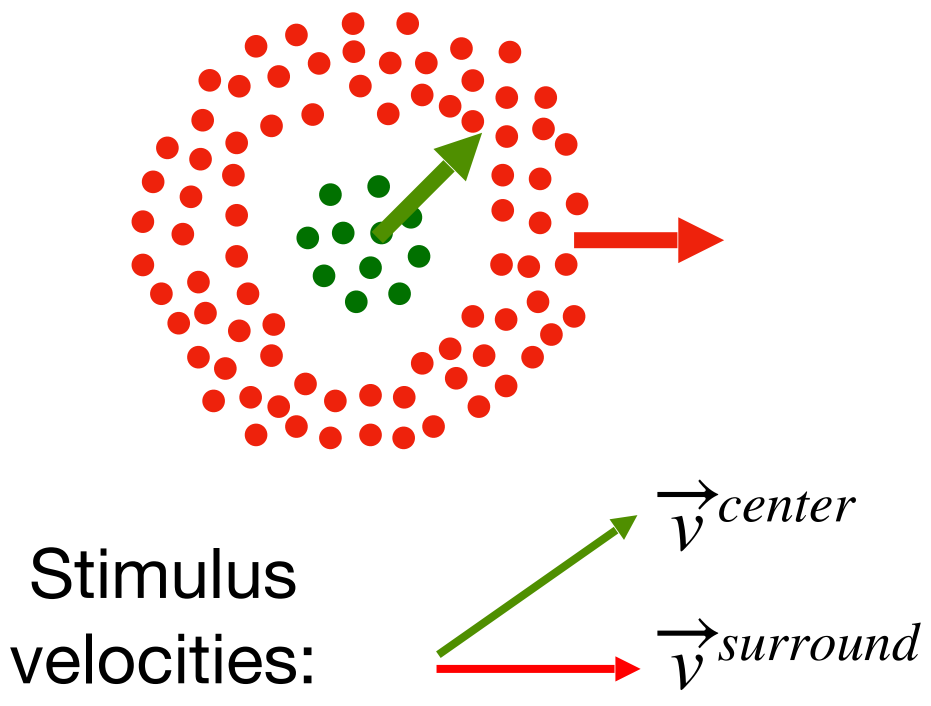

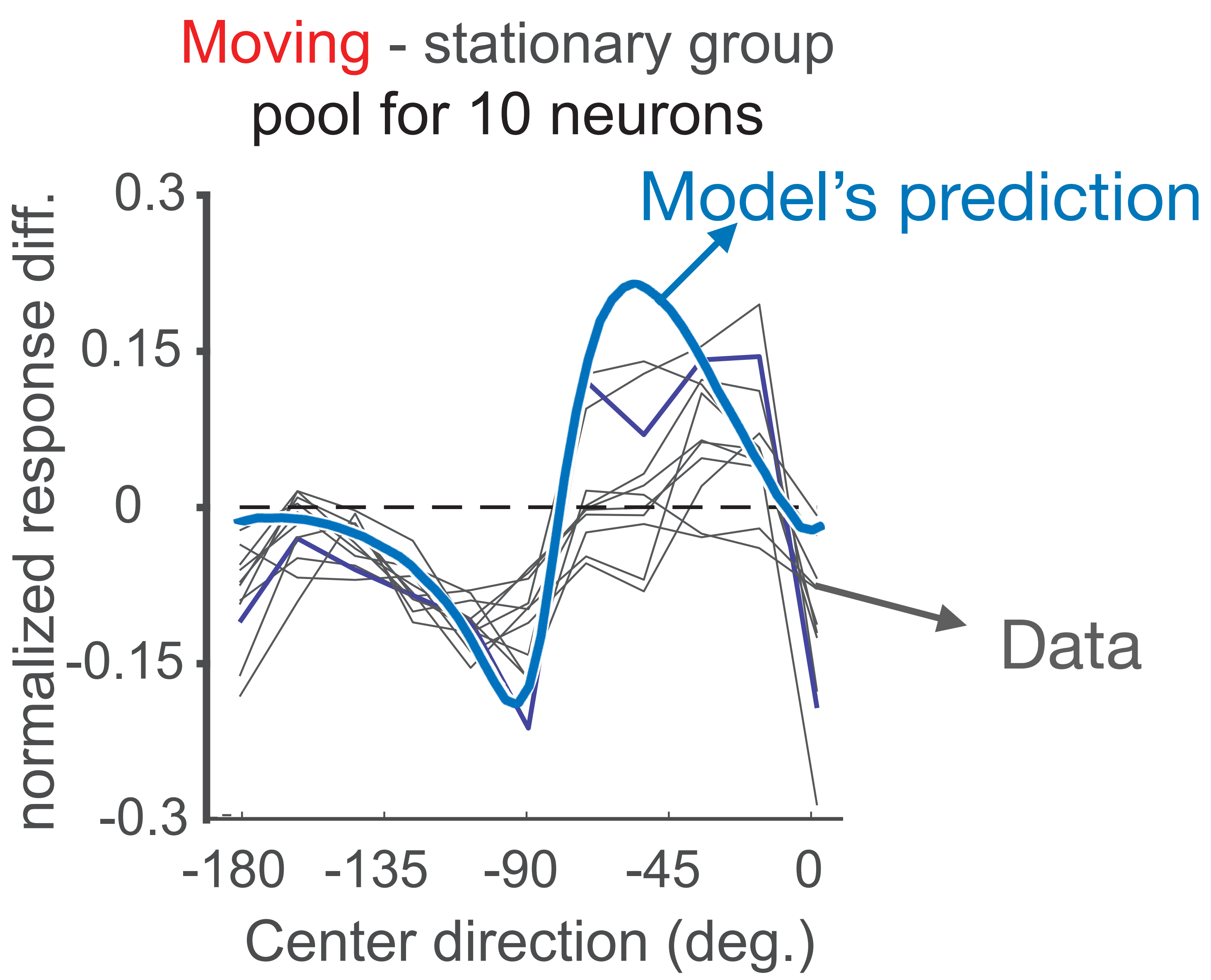

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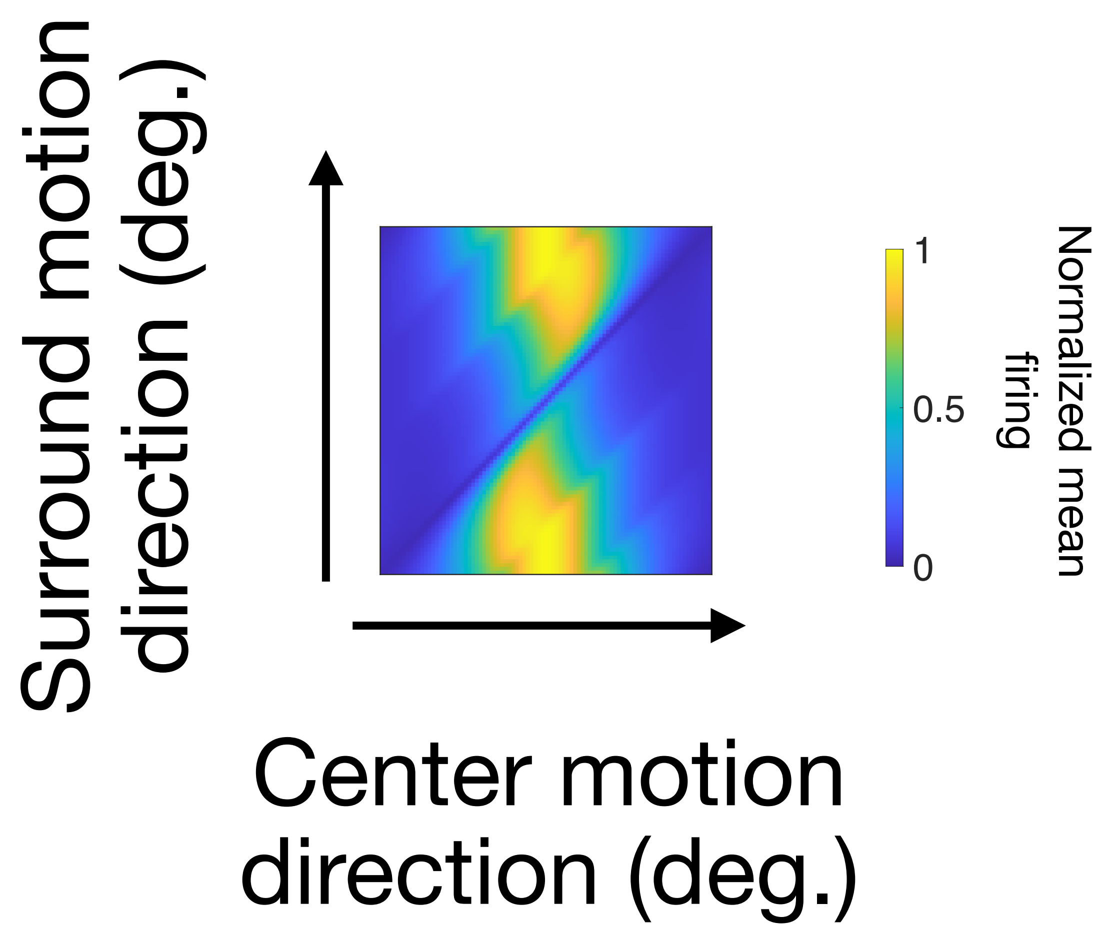

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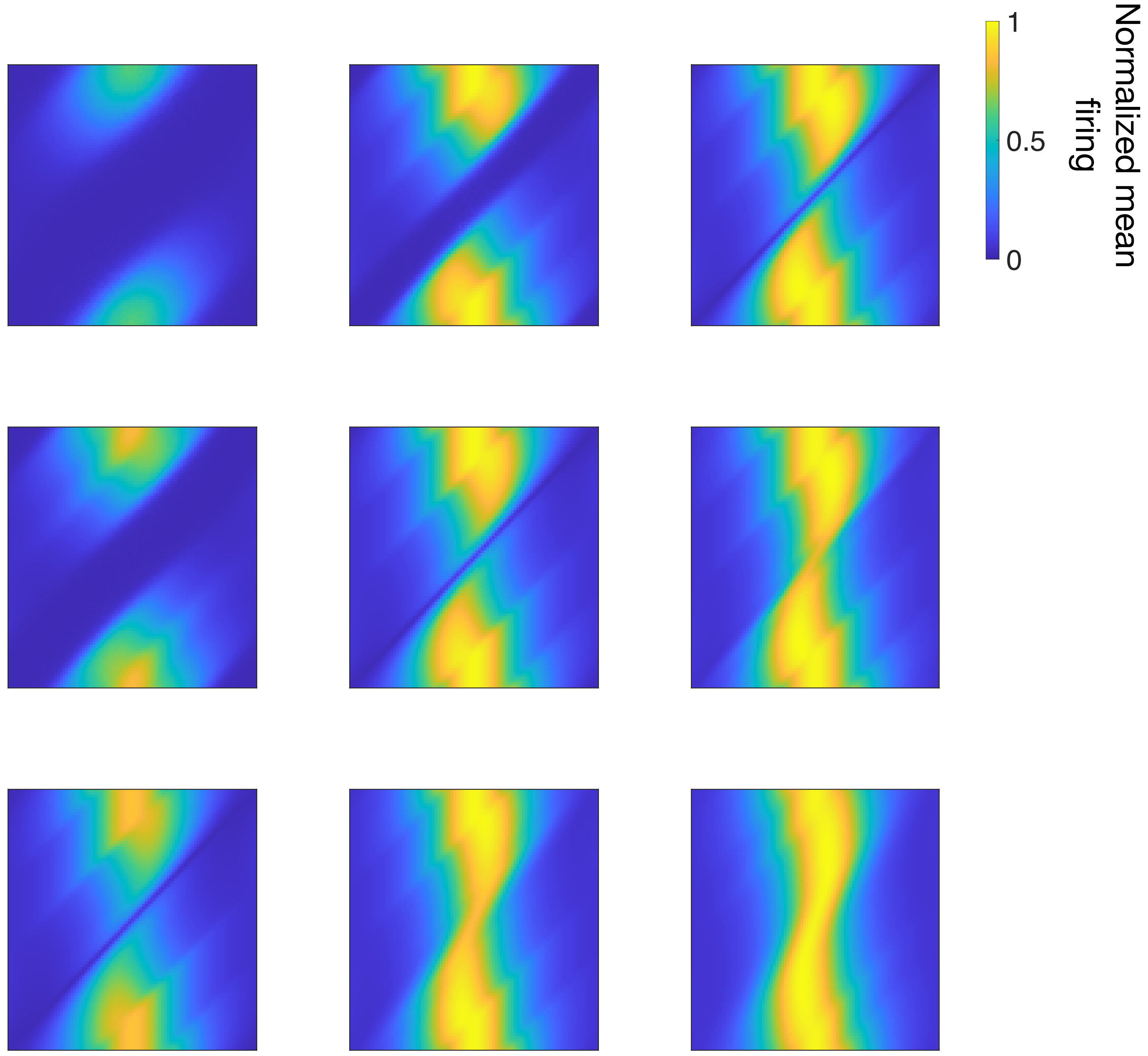

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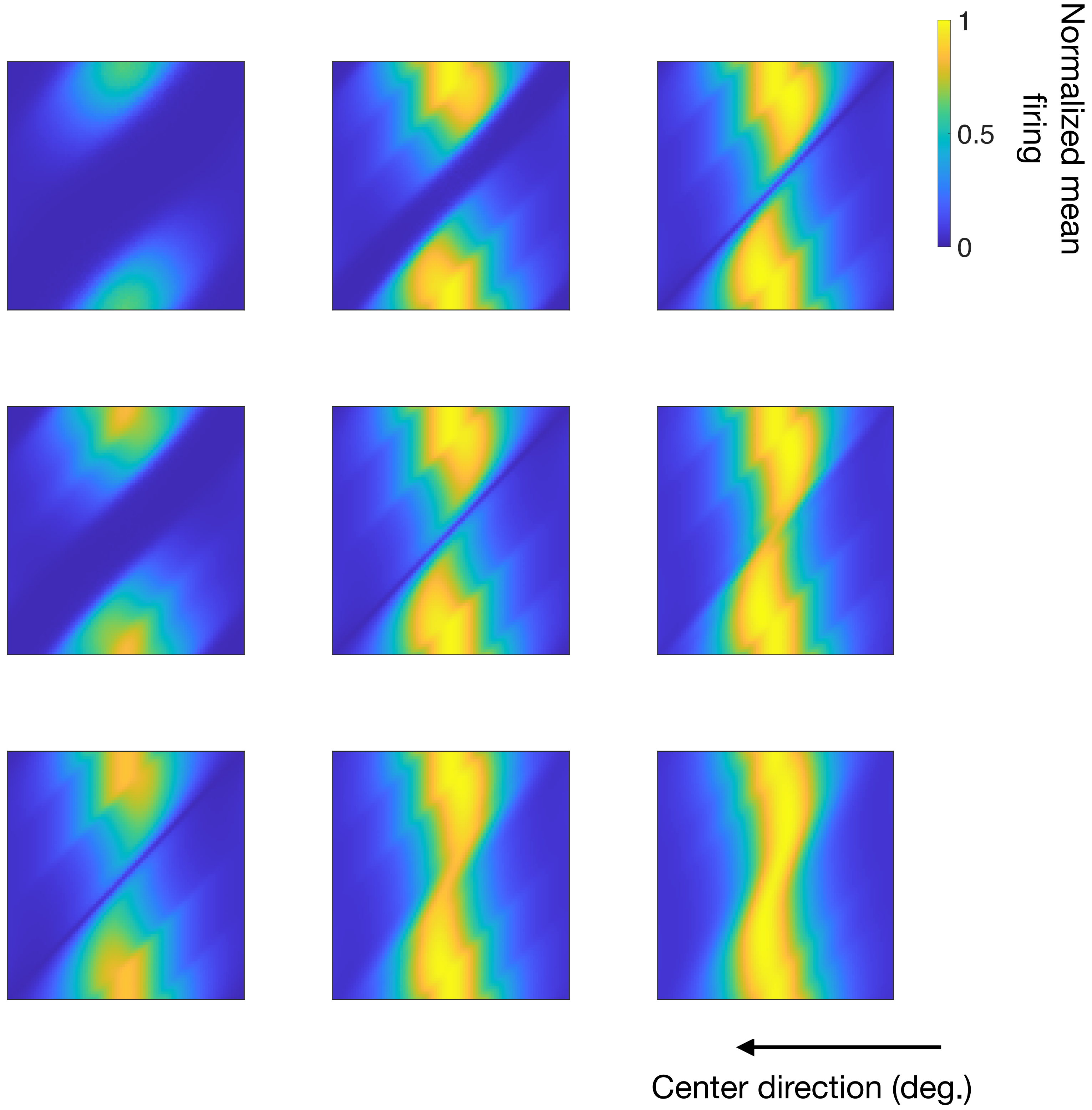

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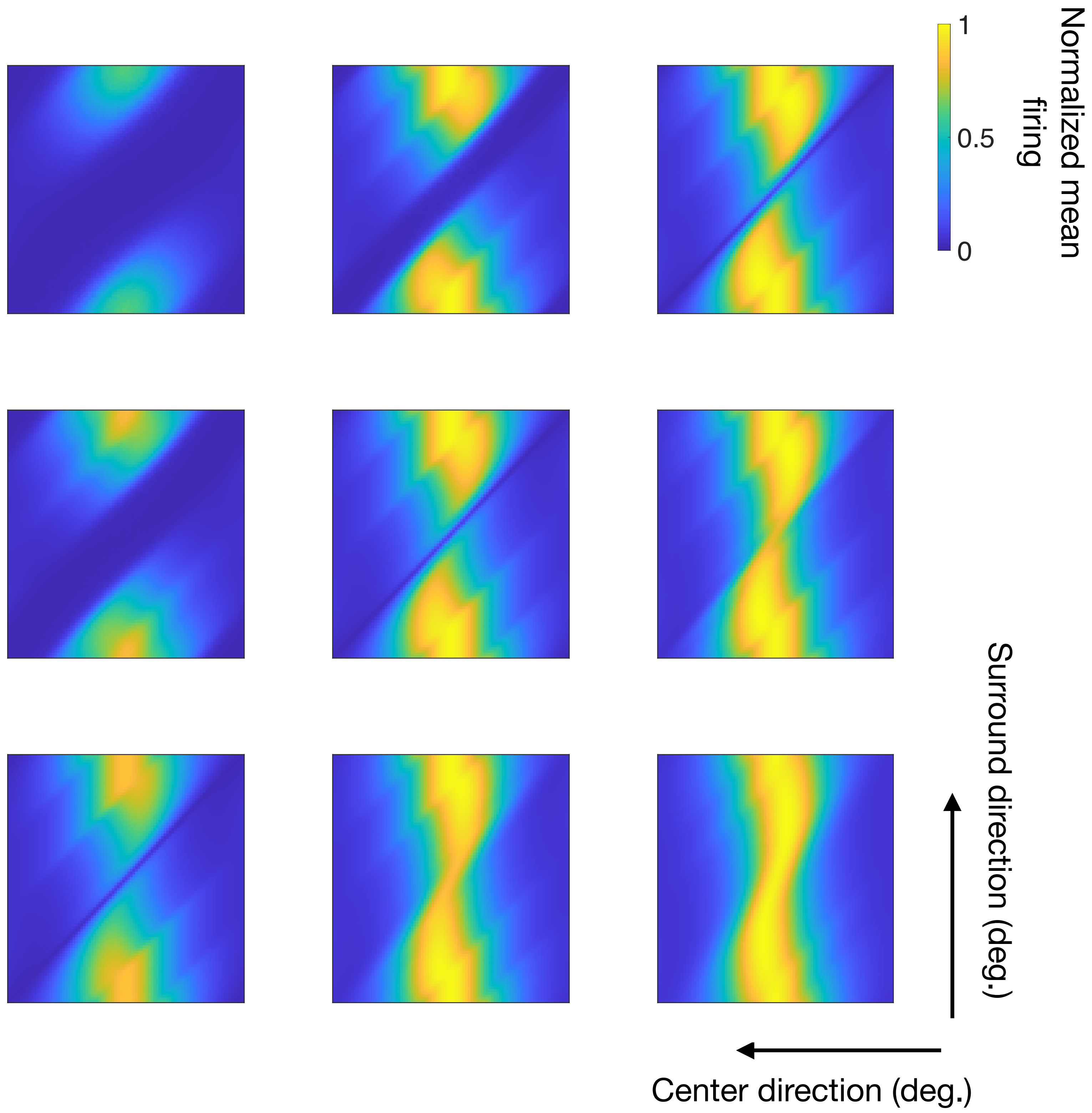

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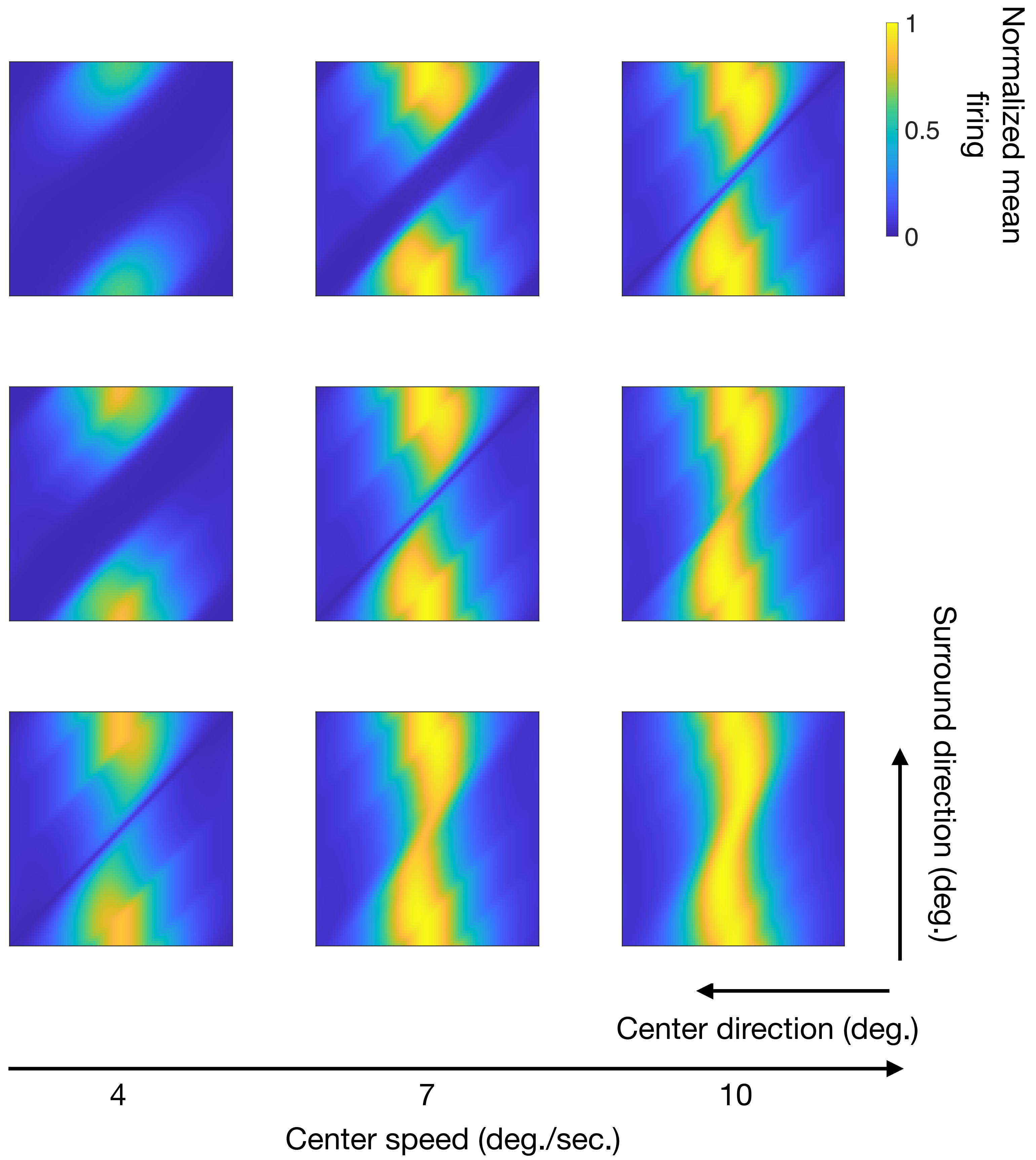

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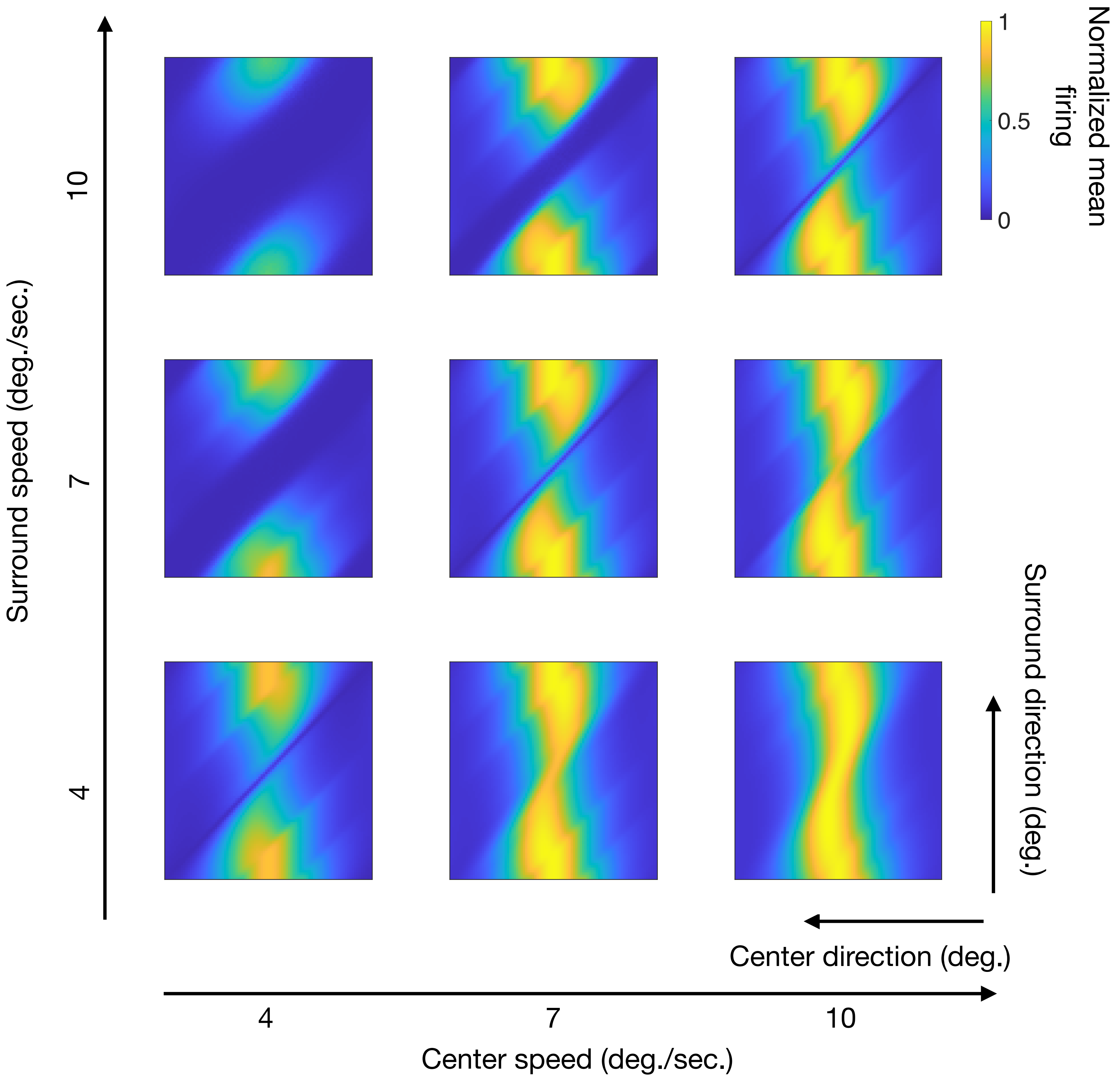

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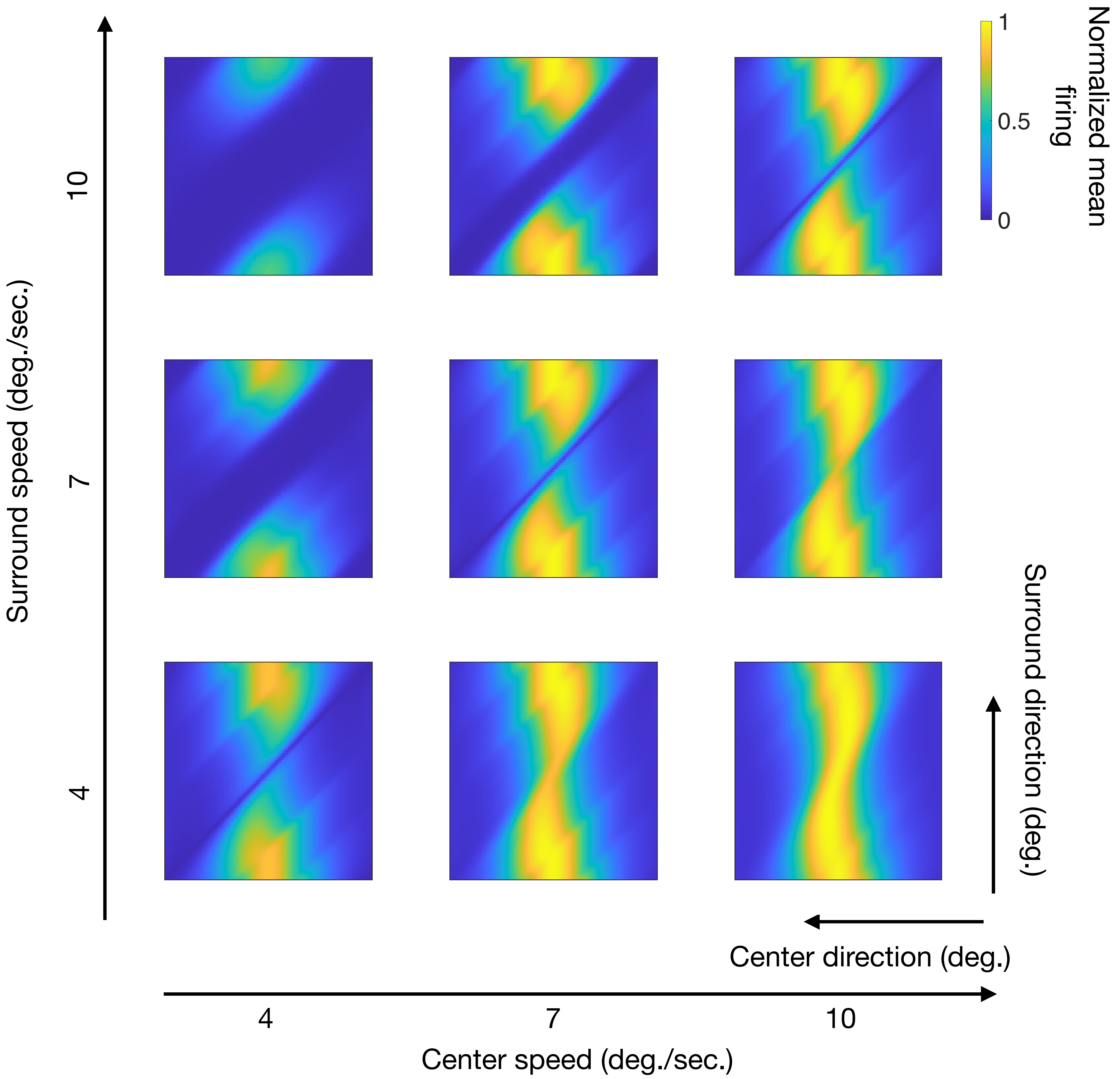

Testable predictions for the whole stimulus space

Conclusions

- We developed a method for testing Bayesian models that is invariant to most encodings
  - It works for all Linear Distributional Codes (including neural sampling and Distributed Distributional Codes)
  - It works for many types of neural data: single neuron responses, population activity, fMRI signal, ANN node activity
- Derivations, assessing validity, and application to simulated data can found in our paper *Lengyel et al. (2023). Proceedings of the I edition of the Workshop on Unifying Representations in Neural Models, in proceedings of Machine Learning Research*
- Applying the model to test causal inference in motion perception (Shivkumar et al. 2023) with data from are MT is in prep.

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