

# DspGNN

Bringing **Spectral Design** to Discrete Time  
**Dynamic Graph Neural Networks** for  
**Edge Regression**

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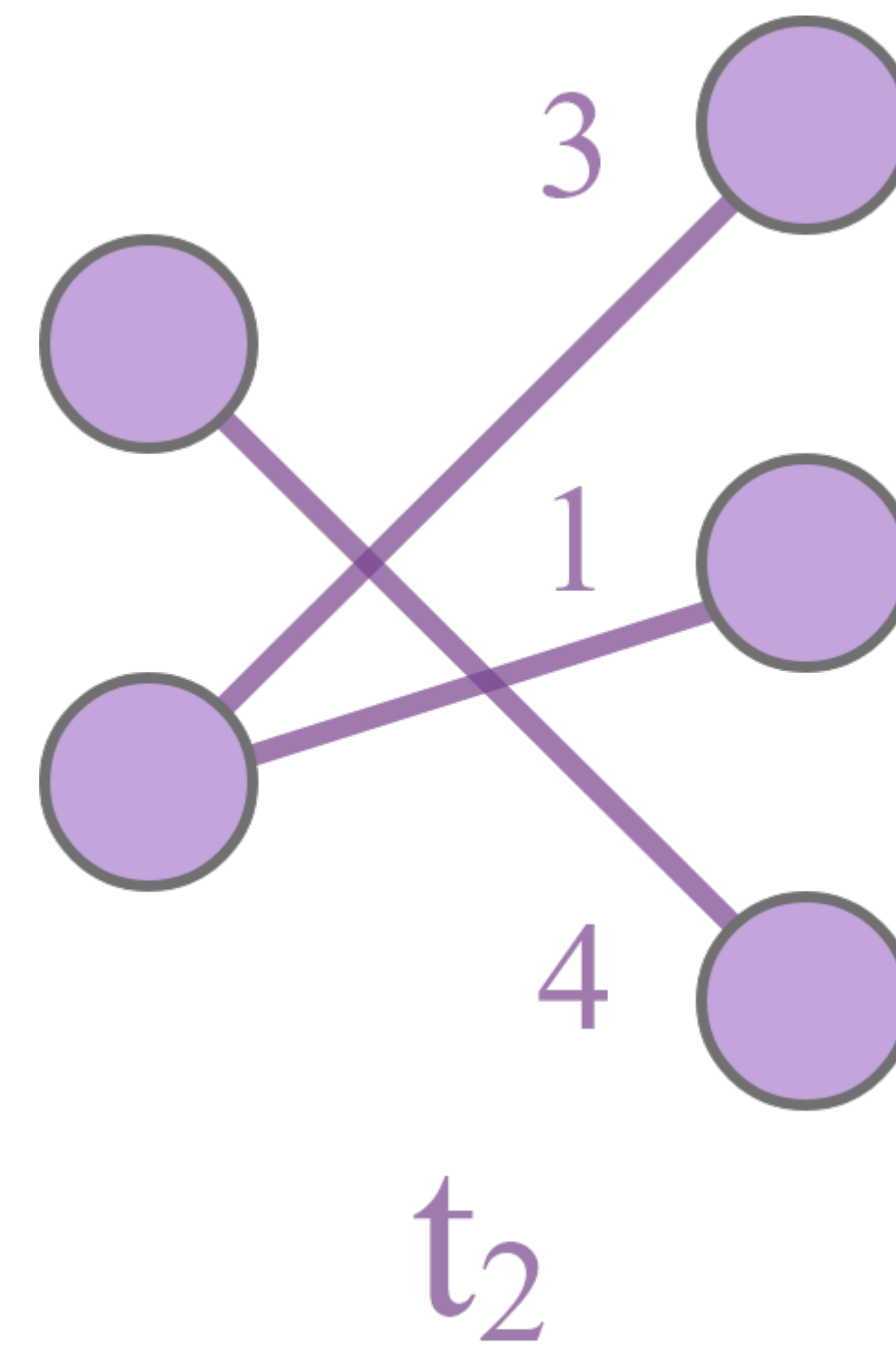
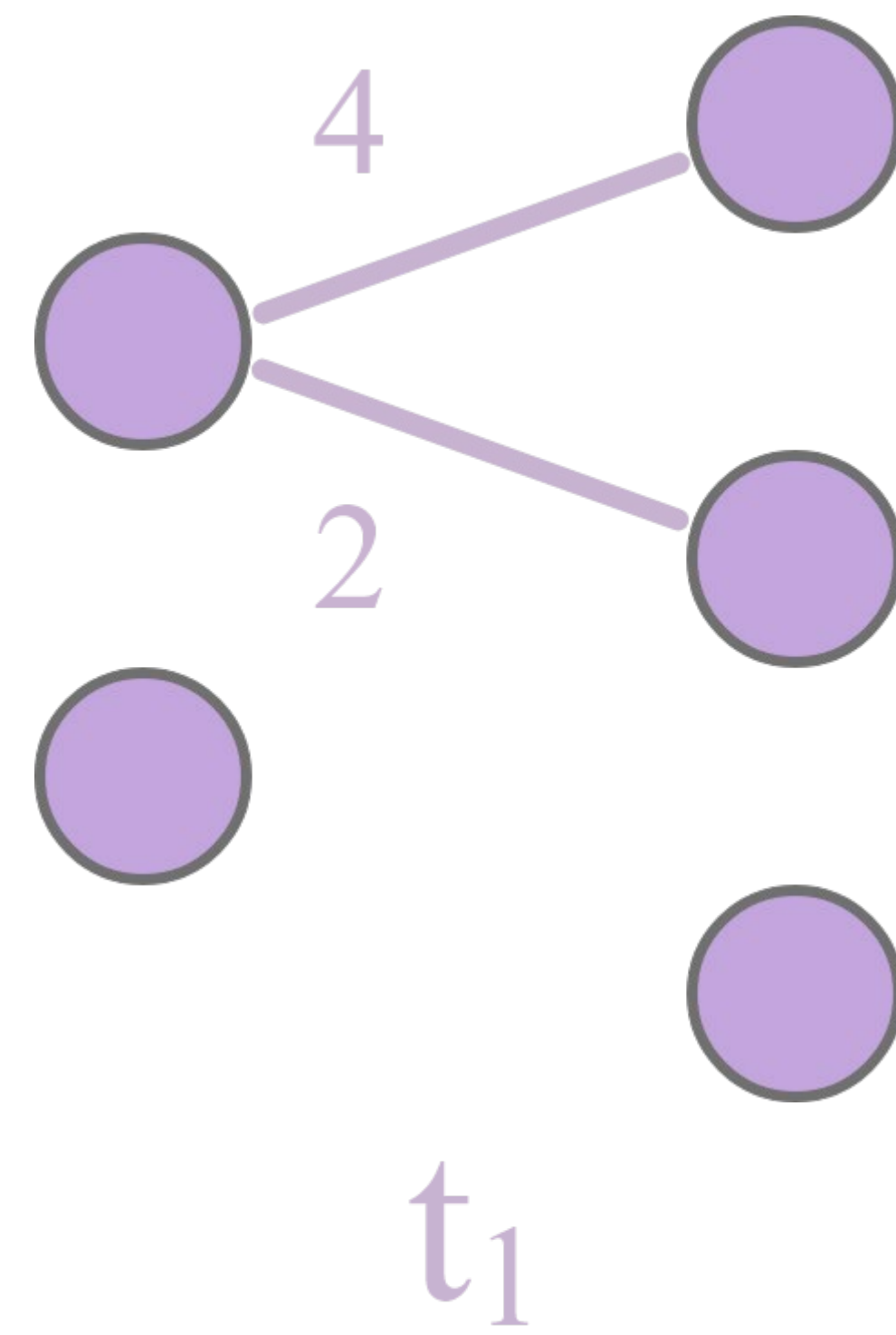
†

§

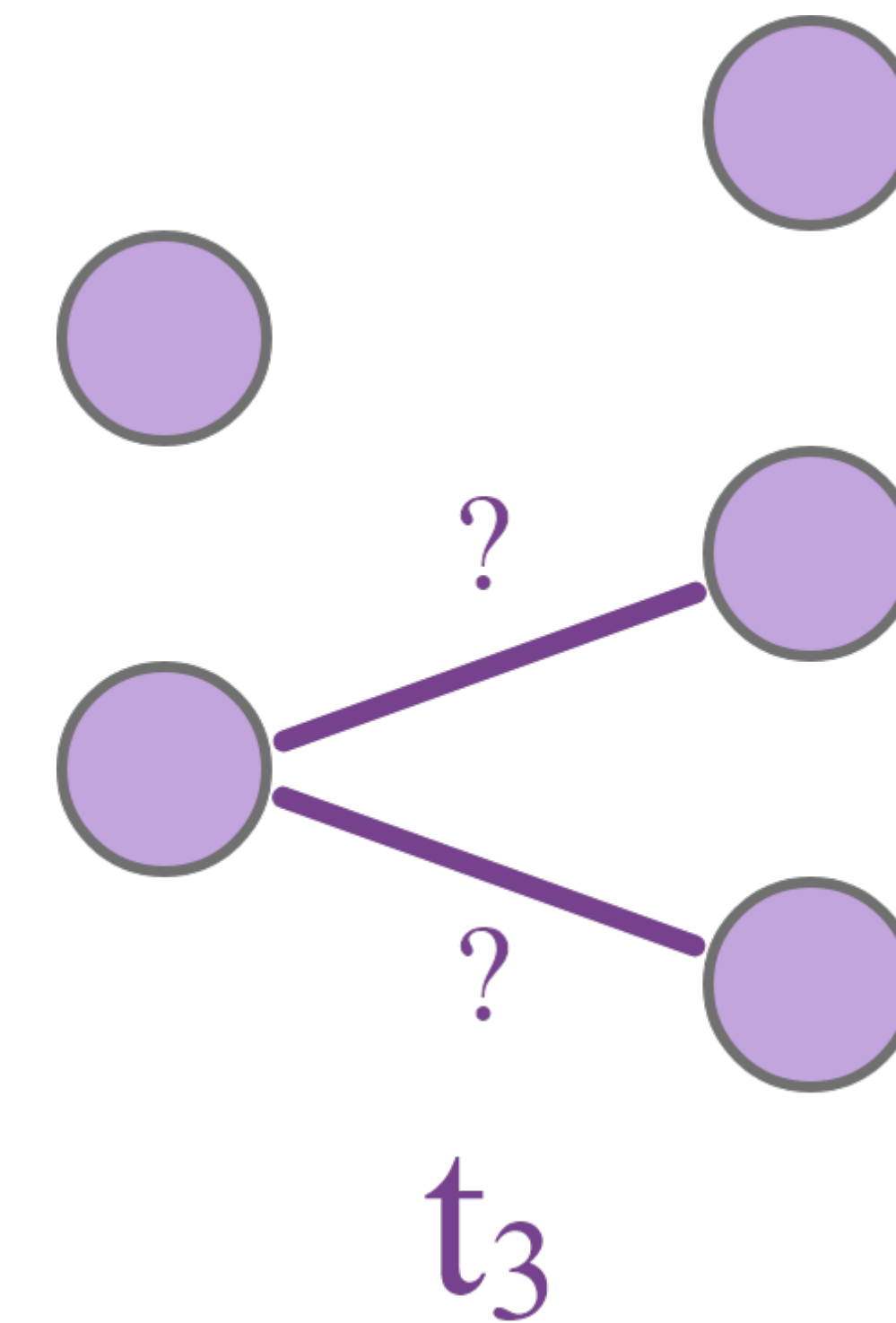
‡

#

# Discrete Time Dynamic Graph Edge Regression

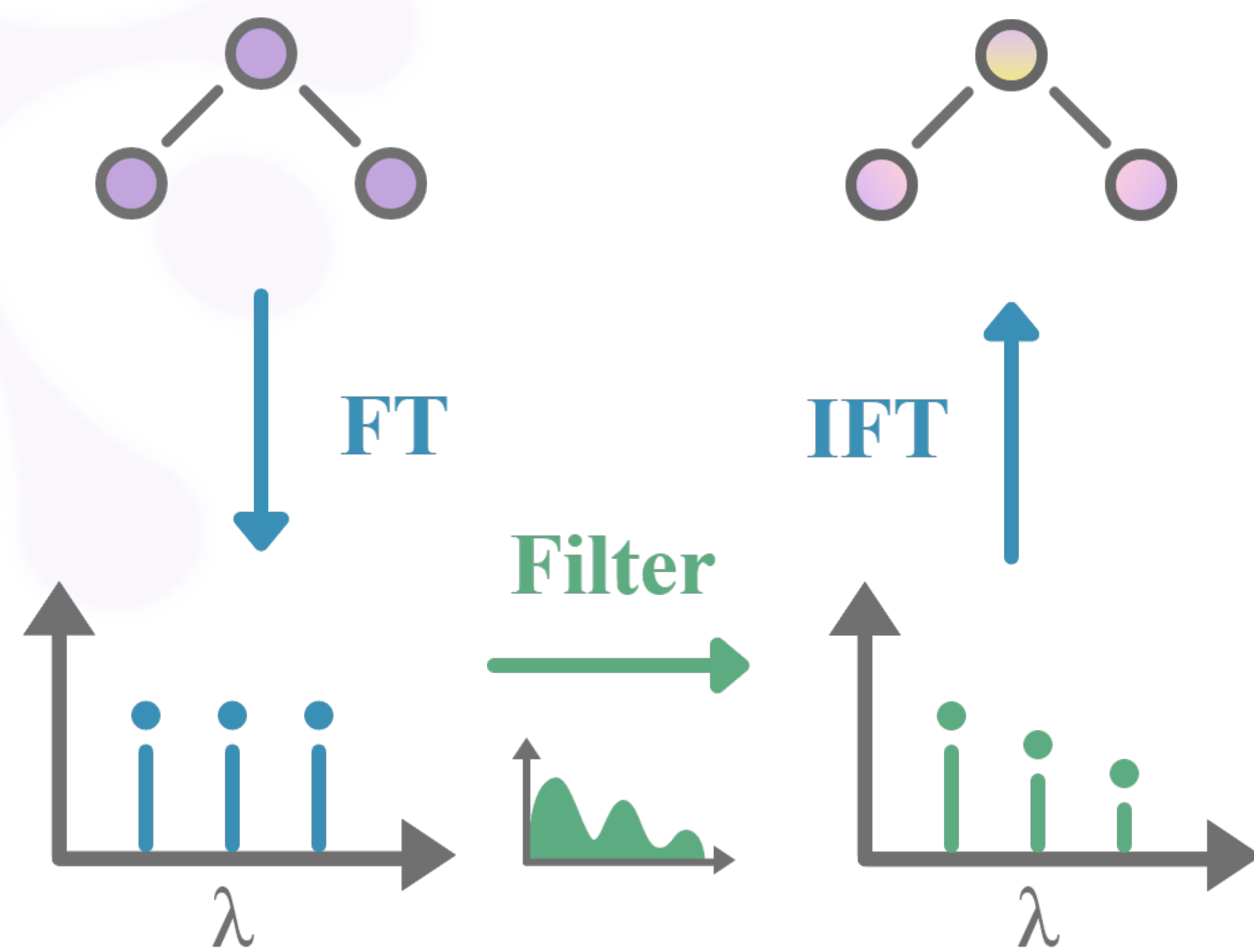


Given



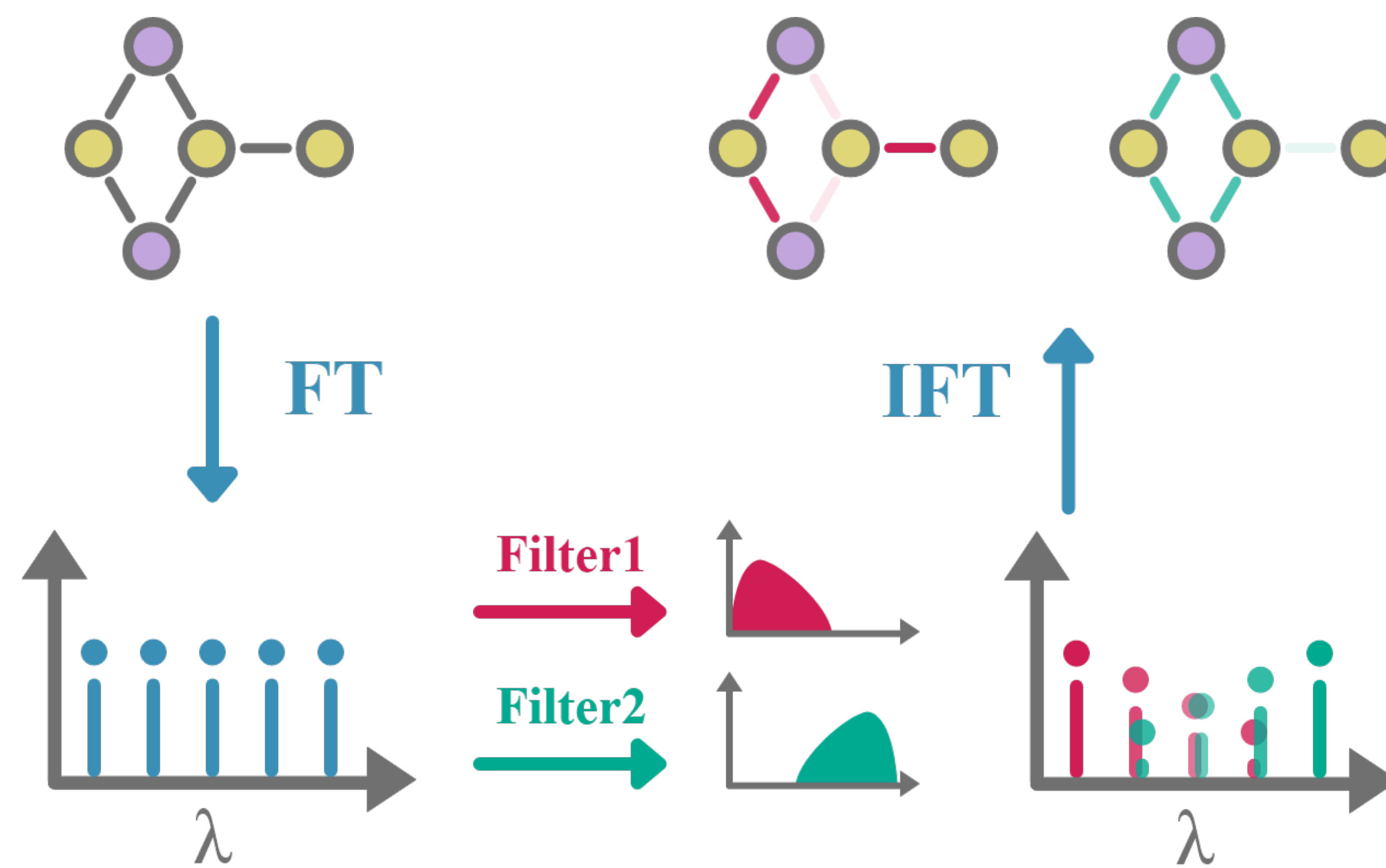
Predict

# Spectral-Designed Graph Convolution



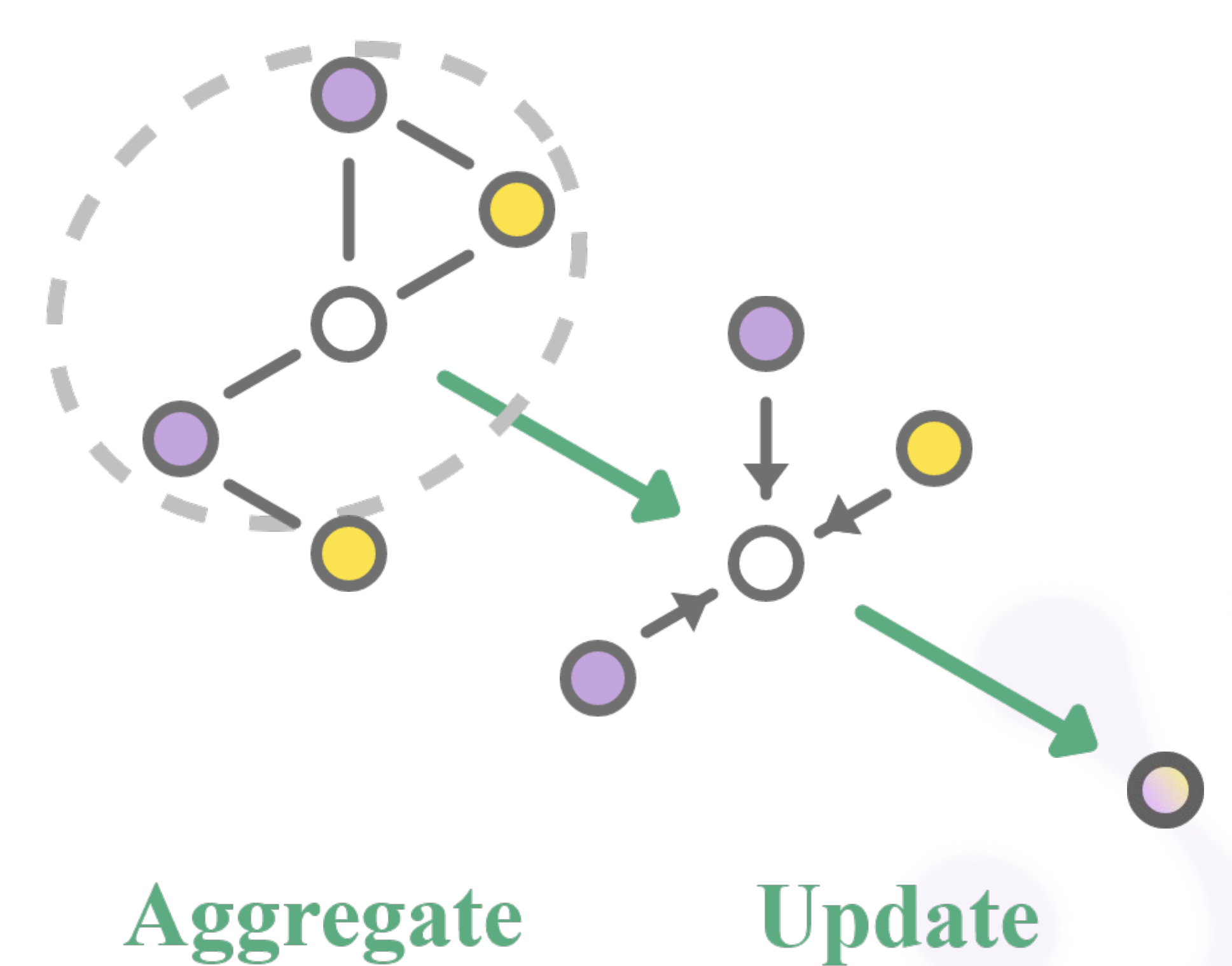
Spectral Convolution

$$\mathbf{H}' = \mathbf{U}g(\Lambda)\mathbf{U}^T\mathbf{H}$$



Spectral-Designed Convolution

$$\mathbf{H}' = \sigma \left( \sum_s \mathbf{C}^{(s)} \mathbf{H} \mathbf{W}^{(s)} \right), \quad \mathbf{C}^s = \mathbf{U} \Phi^s(\Lambda) \mathbf{U}^T$$



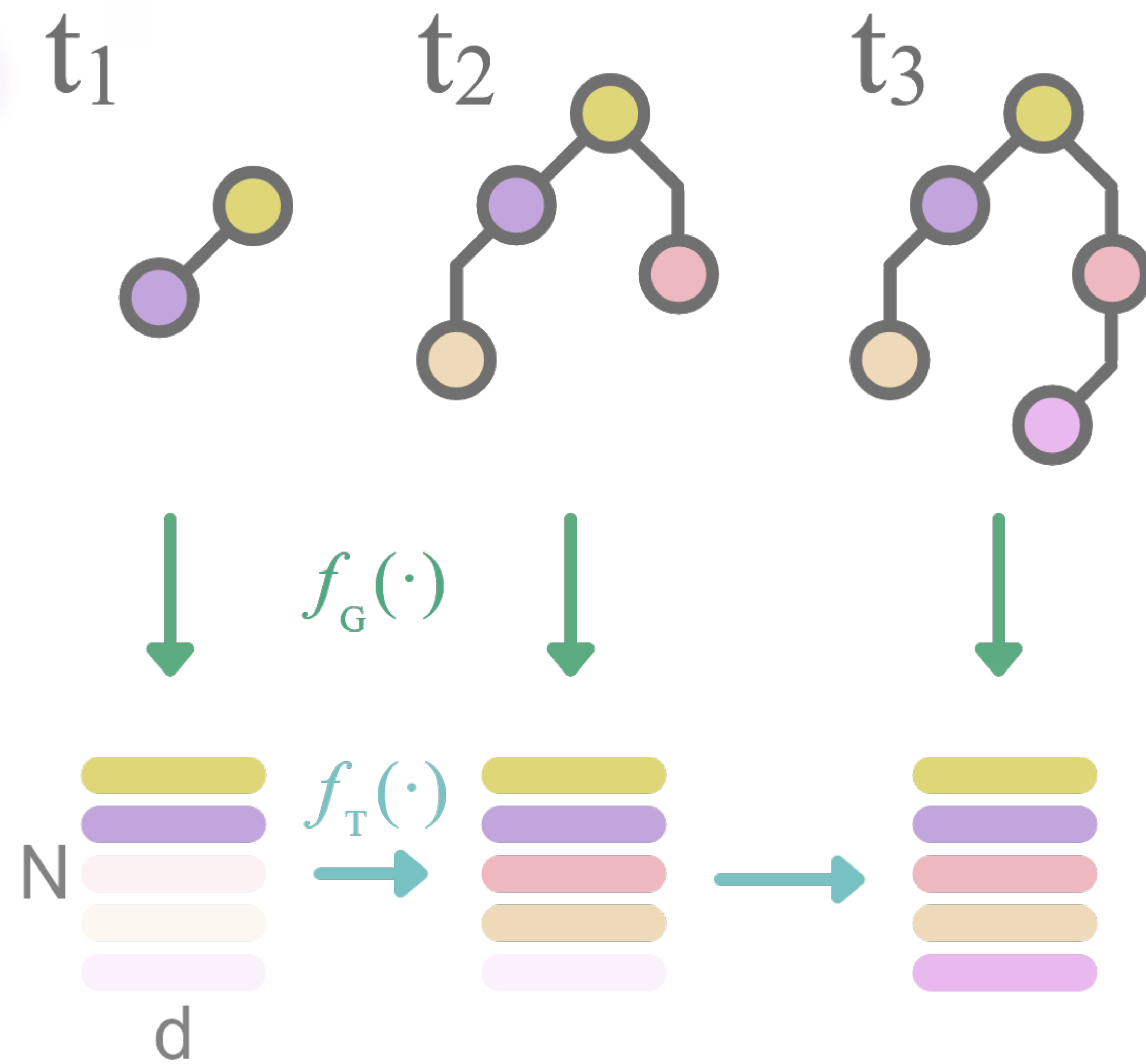
Aggregate

Update

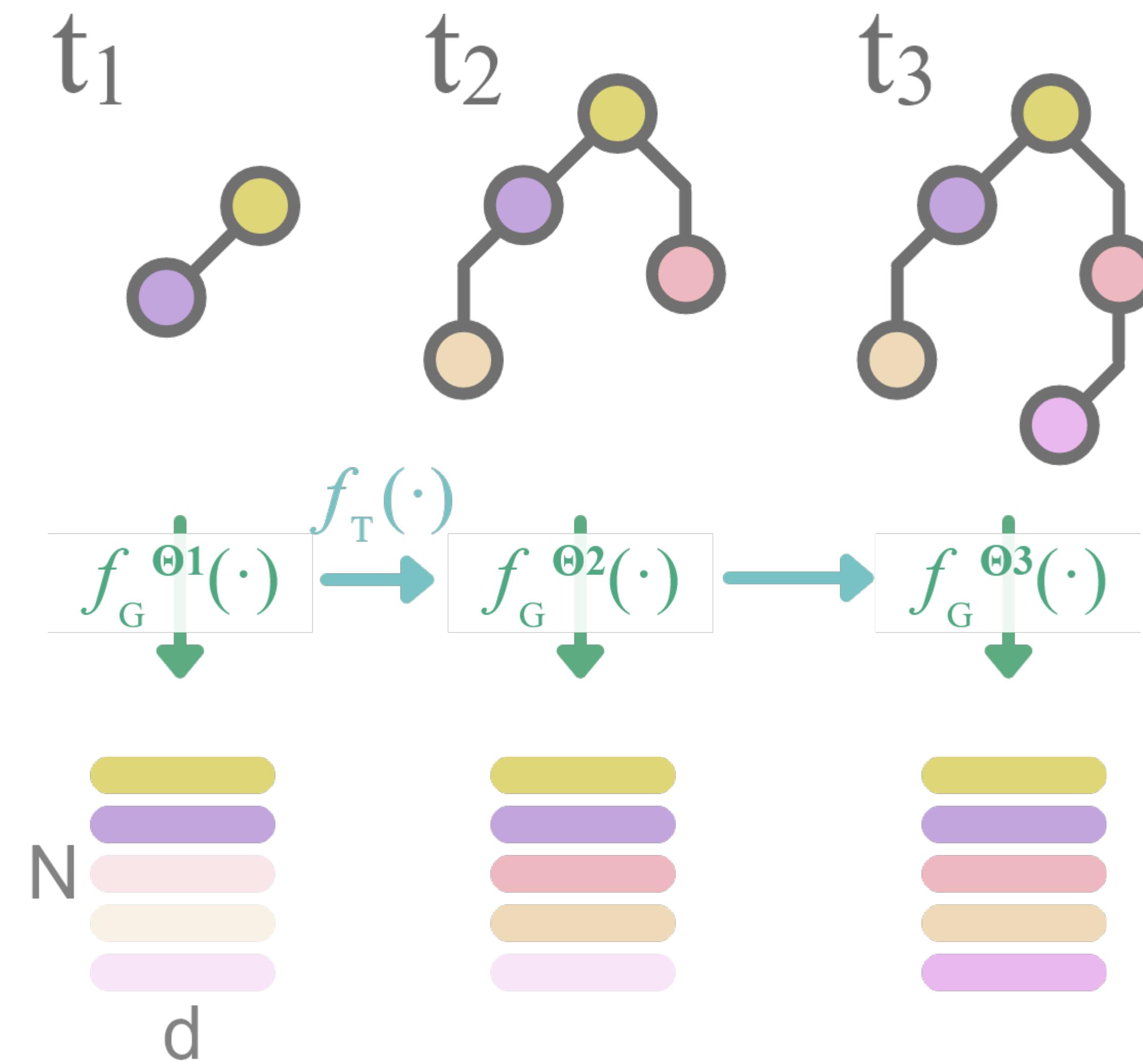
Spatial Convolution

$$\mathbf{H}' = \sigma \left( \sum_s \mathbf{C}^{(s)} \mathbf{H} \mathbf{W}^{(s)} \right)$$

# Discrete Time Dynamic Graph Neural Networks

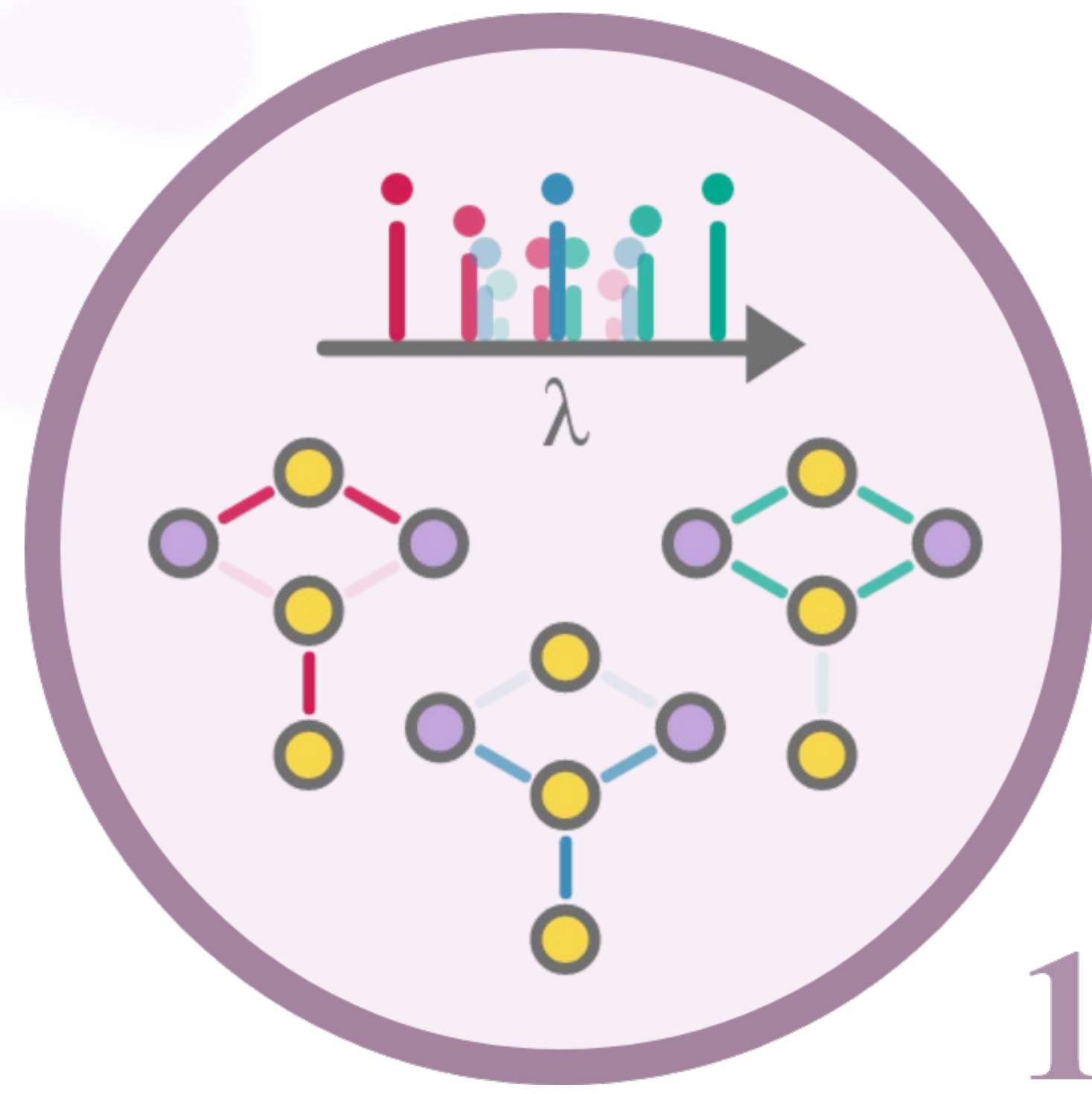


Sequentially Enc(H)



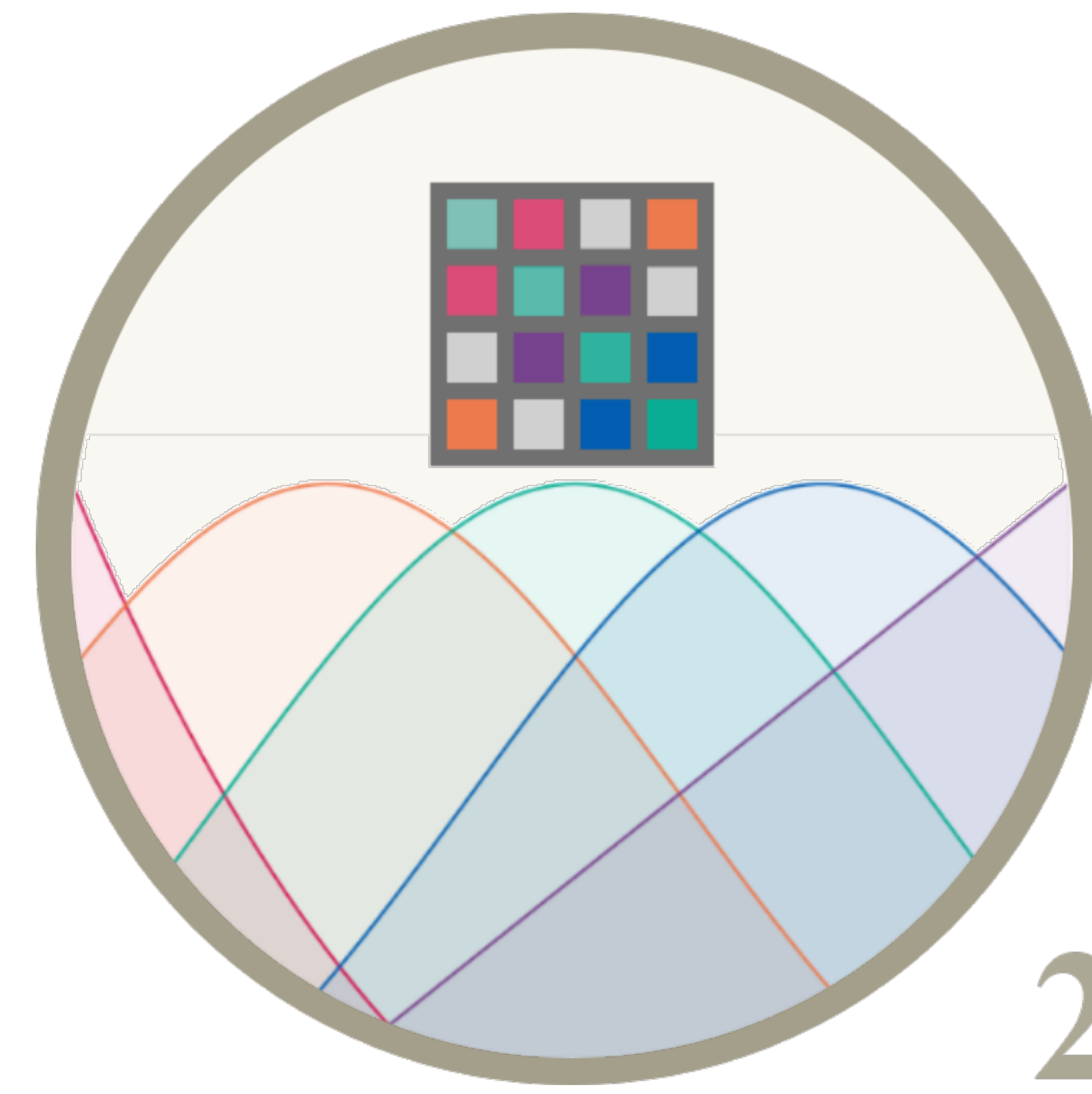
Sequentially Enc( $\Theta$ )

# Motivation



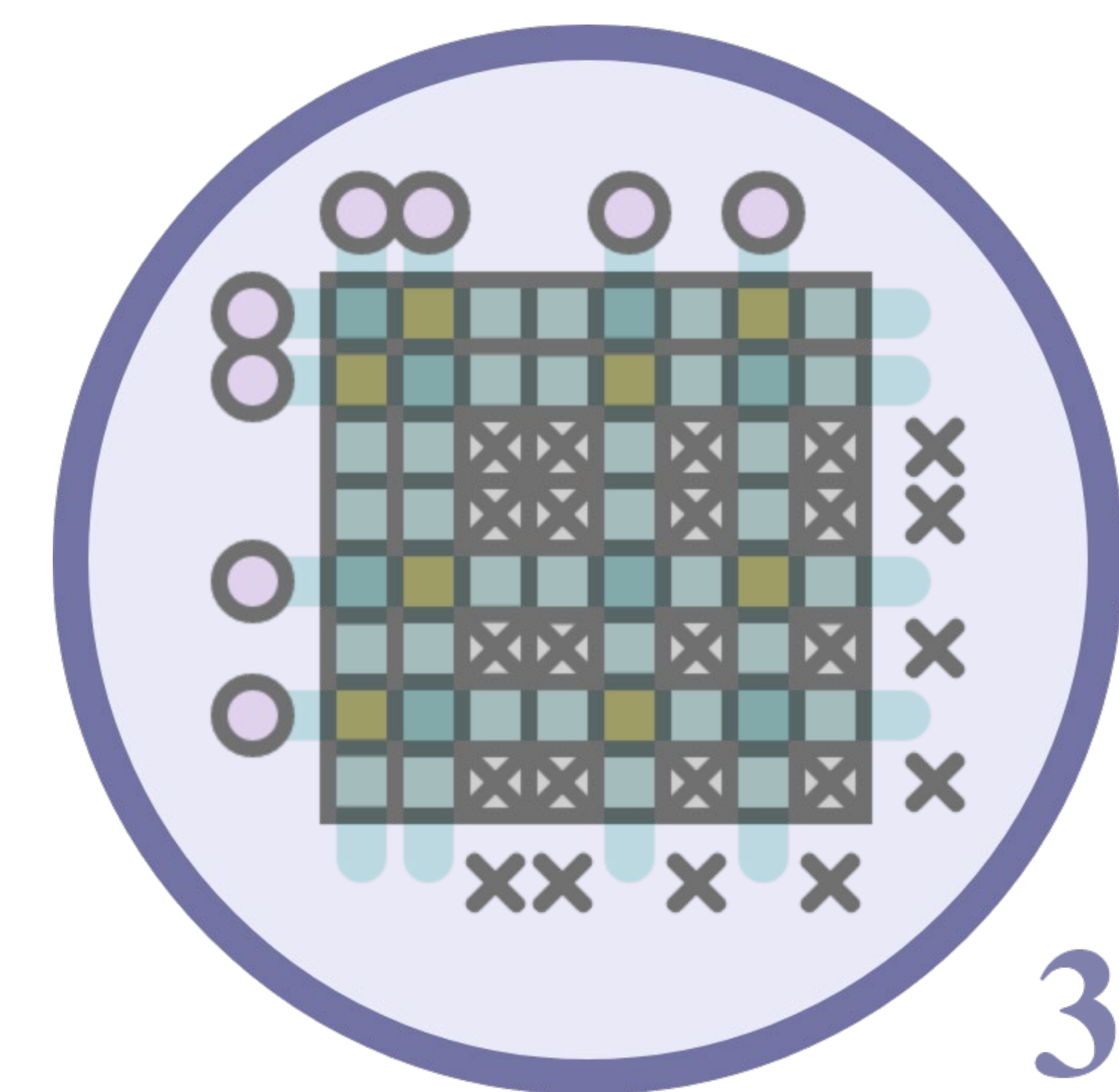
1

DSGCN in DTDGNNs



2

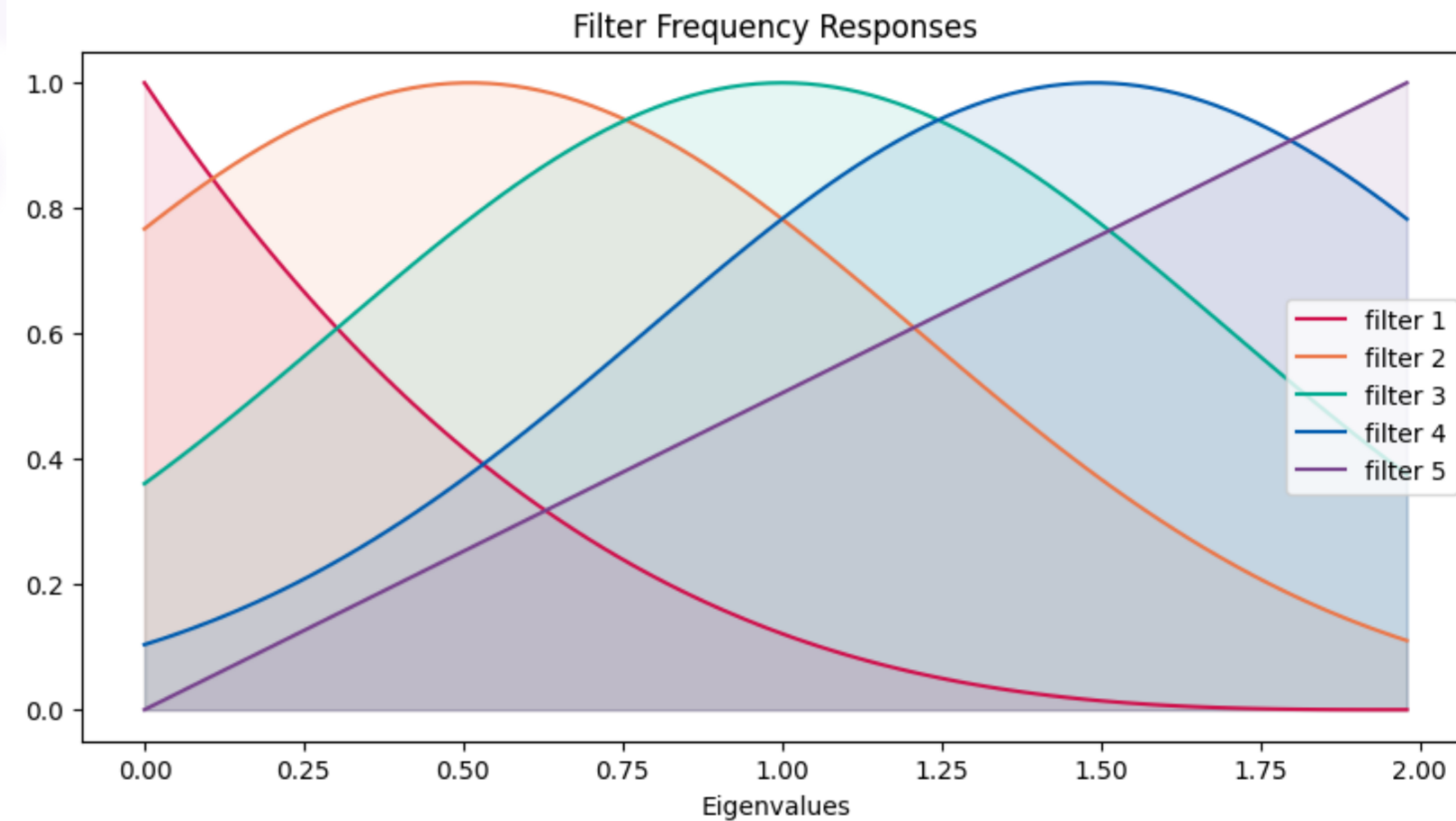
Combine filter outputs



3

Faster eigendecomposition

# Spectral-Designed Filters



$$F_1(\lambda) = \left(1 - \frac{\lambda}{\lambda_{\max}}\right)^3, \quad (\text{Low-pass})$$

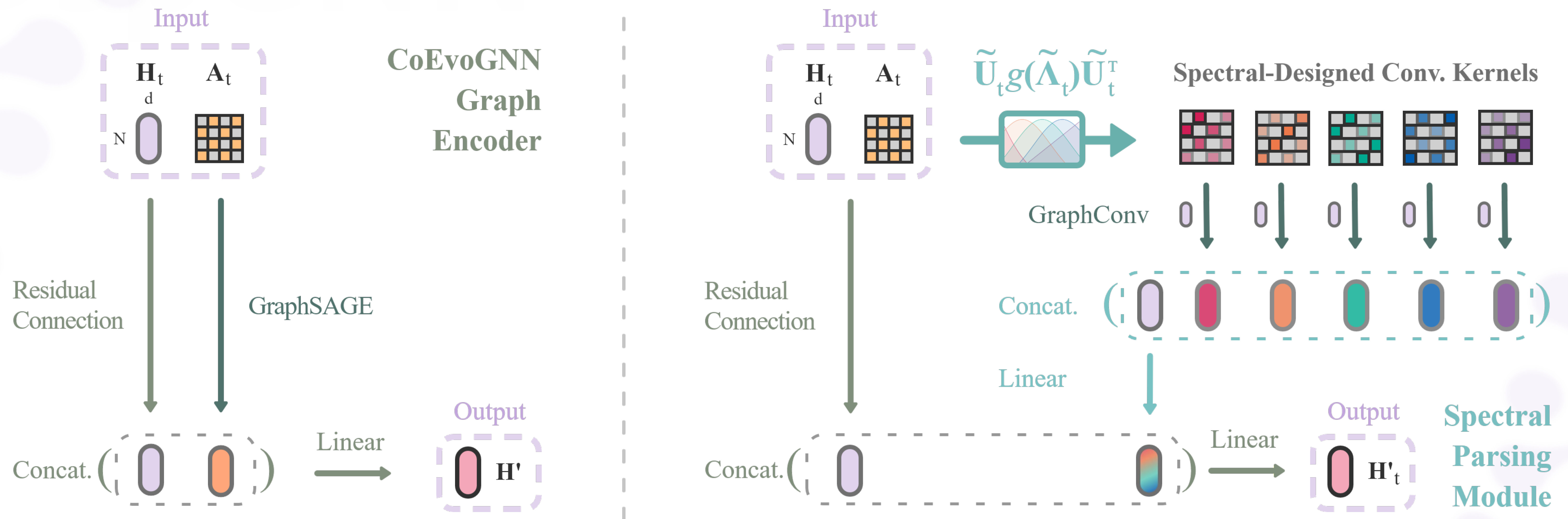
$$F_2(\lambda) = e^{-((\lambda - \lambda_{c1})^2) \cdot \gamma}, \quad (\text{Band-pass 1})$$

$$F_3(\lambda) = e^{-((\lambda - \lambda_{c2})^2) \cdot \gamma}, \quad (\text{Band-pass 2})$$

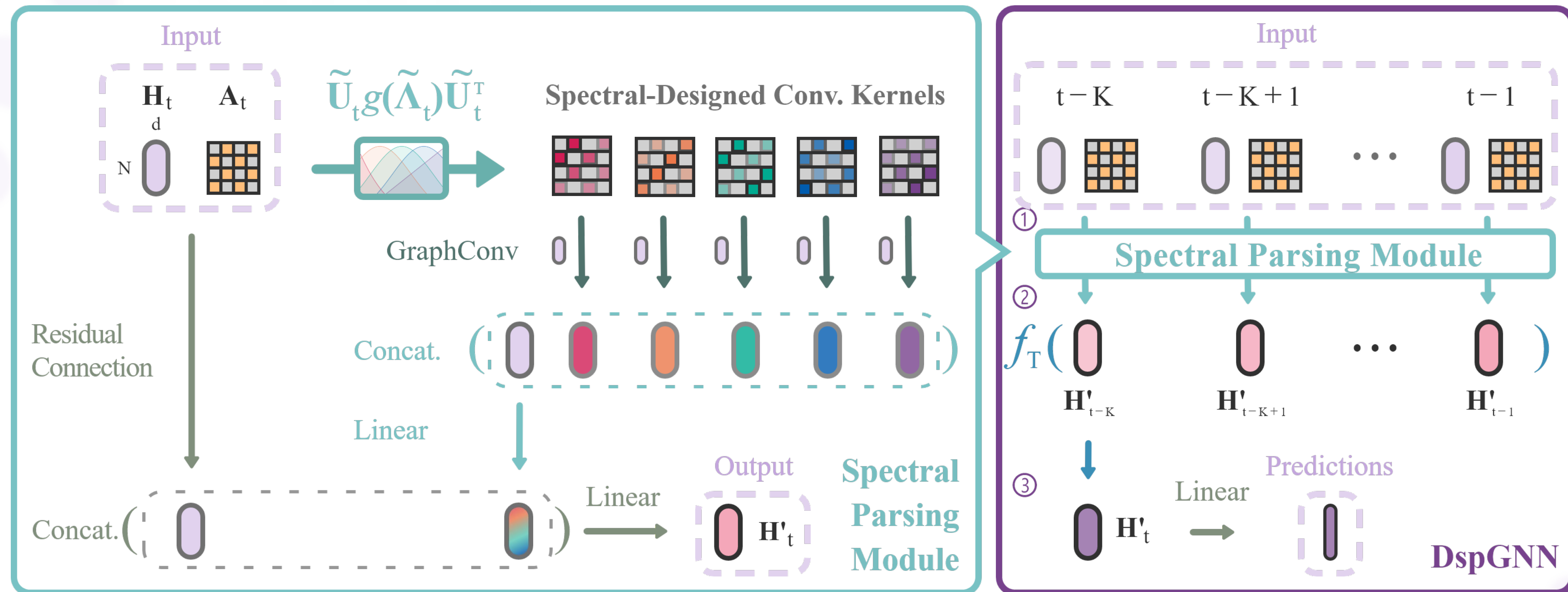
$$F_4(\lambda) = e^{-((\lambda - \lambda_{c3})^2) \cdot \gamma}, \quad (\text{Band-pass 3})$$

$$F_5(\lambda) = \frac{\lambda}{\lambda_{\max}}, \quad (\text{High-pass})$$

# DspGNN (Spectral Parsing Module)



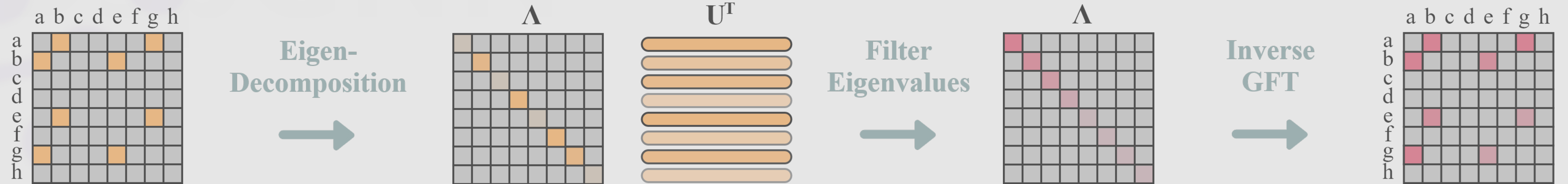
# DspGNN (Overall Architecture)



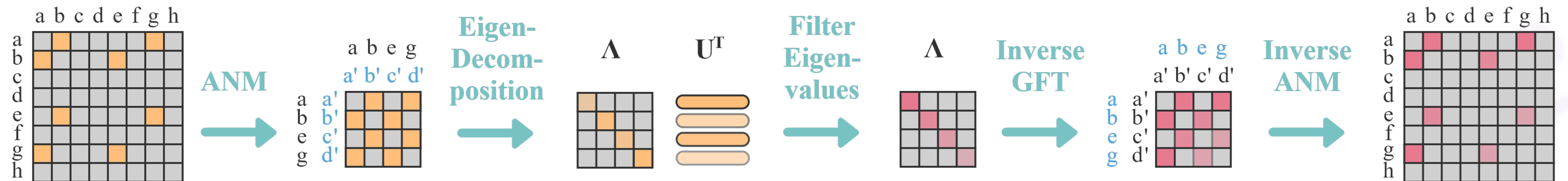
- ① Encode snapshots SP Module    ② Encode with a temporal encoder    ③ Compute the predictions



# Active Node Mapping



Without Active Node Mapping  $O(N^3)$



With Active Node Mapping  $O(N_t^3)$

# Experiments

| Name           | # Nodes | # Average active nodes<br>per snapshot | # Edges | # Time steps | Time step split<br>Train / Valid / Test |
|----------------|---------|--|---------|--------------|---|
| Bitcoin-Alpha  | 3,783   | 106                                    | 24,173  | 137          | 95 / 14 / 28                            |
| Bitcoin-OTC    | 5,881   | 148                                    | 35,588  | 136          | 95 / 13 / 28                            |
| MovieLens-100K | 9,811   | 740                                    | 100,836 | 90           | 63 / 9 / 18                             |

Bitcoin: level of trust, -10~+10    MovieLens-100K: movie rating, 0.5~5.0

|              | Bitcoin-Alpha          |                        | Bitcoin-OTC            |                        | MovieLens-100K         |                        |
|--------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
|              | RMSE                   | MAE                    | RMSE                   | MAE                    | RMSE                   | MAE                    |
| CoEvoGNN*    | <i>0.1171</i> (0.0097) | <i>0.0879</i> (0.0100) | 0.1638 (0.0066)        | 0.1145 (0.0106)        | 0.2381 (0.0058)        | <i>0.1834</i> (0.0048) |
| CoEvoGNN     | 0.1231 (0.0033)        | 0.0928 (0.0030)        | 0.1671 (0.0073)        | 0.1176 (0.0064)        | <i>0.2372</i> (0.0059) | 0.1894 (0.0067)        |
| EvolveGCN    | 0.1173 (0.0187)        | 0.0903 (0.0216)        | <i>0.1556</i> (0.0049) | <i>0.1025</i> (0.0067) | 0.2404 (0.0134)        | 0.1944 (0.0160)        |
| DspGNN(Ours) | <b>0.0968</b> (0.0002) | <b>0.0637</b> (0.0006) | <b>0.1471</b> (0.0009) | <b>0.0831</b> (0.0019) | <b>0.2293</b> (0.0000) | <b>0.1812</b> (0.0000) |

Edge regression performance: MEAN and STD of the normalized scores for the five seeds

# Computing cost comparison

|                              |                | W/O Active Node Mapping | With Active Node Mapping |       |
|------------------------------|----------------|-------------------------|--------------------------|-------|
| Max space complexity (Theo.) | Bitcoin-Alpha  | $1 \times 10^7$         | $3 \times 10^5$          |       |
|                              | Bitcoin-OTC    | $3 \times 10^7$         | $3 \times 10^5$          |       |
|                              | MovieLens-100K | $1 \times 10^8$         | $6 \times 10^6$          |       |
| Time complexity (Theo.)      | Bitcoin-Alpha  | $7 \times 10^{12}$      | $6 \times 10^8$          |       |
|                              | Bitcoin-OTC    | $3 \times 10^{13}$      | $1 \times 10^9$          |       |
|                              | MovieLens-100K | $9 \times 10^{13}$      | $1 \times 10^{11}$       |       |
| Real time consumed (seconds) | Library        | Numpy                   | Scipy                    | Numpy |
|                              | Bitcoin-Alpha  | 1826.31                 | 225.69                   | 0.31  |
|                              | Bitcoin-OTC    | 7203.62                 | 592.23                   | 0.55  |
|                              | MovieLens-100K | 11150.19                | 1159.91                  | 12.83 |

# Acknowledgments



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# Reference

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