



Adaptive Normalization for Non-stationary Time Series Forecasting: A Temporal Slice Perspective

Zhiding Liu^{1,2}, Mingyue Cheng^{1,2}, Zhi Li³, Zhenya Huang^{1,2}, Qi Liu^{1,2},
Yanhu Xie⁴, Enhong Chen^{1,2*}

¹Anhui Province Key Laboratory of Big Data Analysis and Application,
University of Science and Technology of China

²State Key Laboratory of Cognitive Intelligence

³Shenzhen International Graduate School

⁴The First Affiliated Hospital of University of Science and Technology of China

Presenter: Zhiding Liu

- What is time series?
 - *Any signals* collected in chronological order.
 - Ubiquitous/Noisy & Chaotic/Extremely long/Multivariate
- What is forecasting?
 - Given the observation of past S time steps, predict the values of future T steps.
 - Temporal/Channel dependence
- Why forecasting?
 - Weather report
 - Healthcare analysis
 - Decision making
 - ...

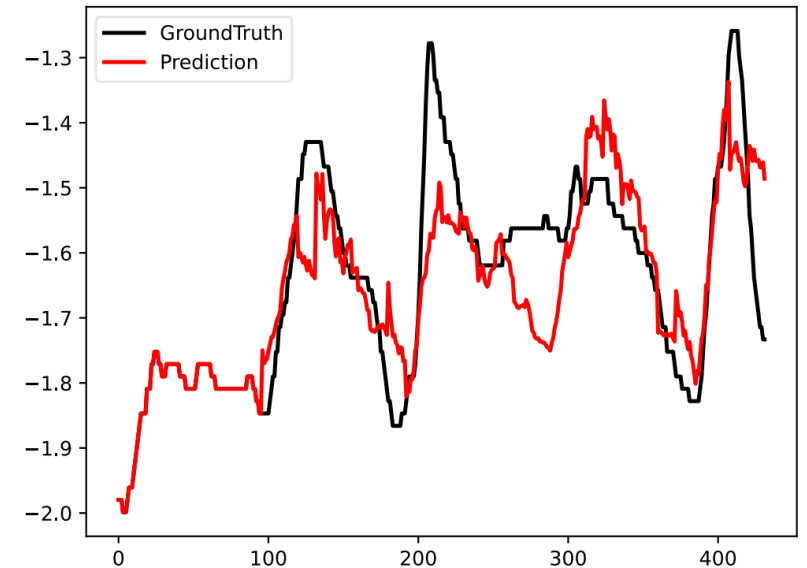


Figure 1: Illustration of forecasting.

- **Tremendous efforts** have been devoted in **designing powerful networks** and therefore greatly advanced the accuracy of forecasting performance.
- The intrinsic **non-stationary property** hinders the generalizability of deep-learning-based models.
 - The distribution shift in time series, i.e., $\forall_{i,j} p(x^i) \neq p(x^j)$
 - Existing forecasting methods barely rely on the non-linear capacity to tackle the challenge.
- To **explicitly alleviate the impact of non-stationarity**, adaptive normalization on input series is the feasible and mainstream solution.
 - Ensure the inputs are ***I.I.D.*** through normalization.

□ Motivation

- Existing normalization methods are based on an assumption that *the input series of an instance follows the same distribution*.
 - In the real-world scenarios, time series points rapidly change over time. For any given time slices of instance k , $x_i^k, x_j^k, p(x_i^k) \neq p(x_j^k)$.
- Previous works either *ignore* the restoration of non-stationary information or simply *adopt the statistical properties of input series to denormalize the output results*.
 - Lead to a prediction shift of the final forecasting results due to a bad estimation of future statistics.

□ Our thoughts

- Split series into non-overlap equally-sized slices and model the **local-region non-stationarity** under them.
- Employ a **statistics prediction module** learning to estimate the distribution of future slices precisely.

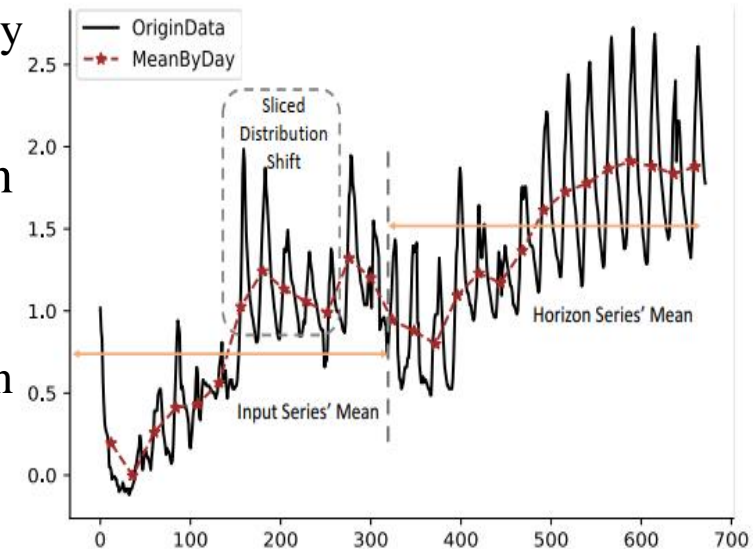


Figure 2: A forecasting instance with non-stationarity.

□ Sliced normalization

- Removing the local non-stationarity for each time slice according to their statistics.

$$\mu_j^i = \frac{1}{T} \sum_{t=1}^T \mathbf{x}_{j,t}^i, (\sigma_j^i)^2 = \frac{1}{T} \sum_{t=1}^T (\mathbf{x}_{j,t}^i - \mu_j^i)^2, \quad \bar{\mathbf{x}}_j^i = \frac{1}{\sigma_j^i + \epsilon} \cdot (\mathbf{x}_j^i - \mu_j^i).$$

□ Statistics prediction

- Under our assumption, a natural challenge is that how to **estimate the evolving distributions** for each future slice.
- We adopt a two-layer perceptron network responsible for this task for simplicity and efficiency.

$$\hat{\mu}^i = \boxed{\mathbf{W}_1} * MLP(\boxed{\mu^i - \rho^i}, \boxed{\bar{\mathbf{x}}^i - \rho^i}) + \boxed{\mathbf{W}_2} * \rho^i, \quad \boxed{} : \text{residual learning}$$
$$\hat{\sigma}^i = MLP(\sigma^i, \bar{\mathbf{x}}^i). \quad \boxed{} : \text{individual preference}$$

- The overall mean of the input sequence is a *maximum likelihood estimation* of the target sequence's mean → **residual learning**.
- Different variables may exhibit distinct patterns in scale changes → **individual preference**.

□ Sliced de-normalization

- The non-stationarity information is vital for forecasting.
- Restore them into predicted results in a slice perspective.

$$\hat{y}_j^i = \bar{y}_j^i * (\hat{\sigma}_j^i + \epsilon) + \hat{\mu}_j^i.$$

□ Slicing Adaptive Normalization (SAN)

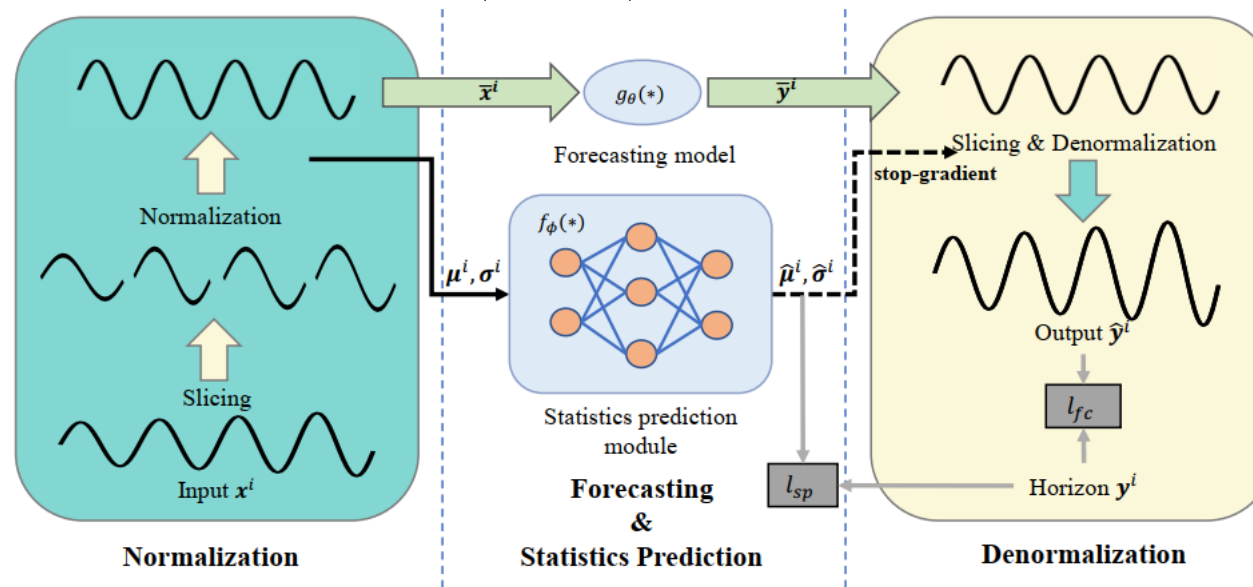


Figure 3: Illustration of the proposed SAN framework.

□ Two-stage training schema

- It forms to a bi-level optimization problem when joint training SAN and backbone model, as the target is to ensure the similarity between distributions of **denormalized output and ground truth**.

$$\begin{aligned} & \arg \min_{\theta} \sum_{(\mathbf{x}^i, \mathbf{y}^i)} l_{fc}(\theta, \phi^*, (\mathbf{x}^i, \mathbf{y}^i)), \\ & \text{s.t. } \phi^* = \arg \min_{\phi} \sum_{(\mathbf{x}^i, \mathbf{y}^i)} l_{sp}(\theta, \phi, (\mathbf{x}^i, \mathbf{y}^i)). \end{aligned}$$

- Relax the optimization objective of statistic prediction module to **estimating the future distribution**.
 - The original non-stationary forecasting task is divided into **decoupled** statistic prediction task and stationary forecasting task.
- Qualities:
 - **Simplifies** the task of non-stationary forecasting through **divide-and-conquer**.
 - Estimate **more accurately** on future distributions.

□ Setup

- The widely used benchmark with 9 datasets:

| Dataset | Variables | Sampling Frequency | Length | Slicing Length | ADF* |
|-------------|-----------|--------------------|--------|----------------|--------------|
| Electricity | 321 | 1 Hour | 26,304 | 24 | -8.44 |
| Exchange | 8 | 1 Day | 7,588 | 6 | -1.90 |
| Traffic | 862 | 1 Hour | 17,544 | 24 | -15.02 |
| Weather | 21 | 10 Minutes | 52,696 | 12 | -26.68 |
| ILI | 7 | 1 Week | 966 | 6 | -5.33 |
| ETTh1&ETTh2 | 7 | 1 Hour | 17,420 | 24 | -5.91&-4.13 |
| ETTh1&ETTh2 | 7 | 15 Minutes | 69,680 | 12 | -14.98&-5.66 |

*A smaller ADF test result indicates a more stationary time series data

□ Baseline models:

- RevIN (ICLR'22), NST(NIPS'22), Dish-TS(AAAI'23)

□ Backbone models:

- Autoformer(NIPS'21),FEDformer(ICML'22),SCINet(NIPS'22),DLinear(AAAI'23)
- Slice-based models: PatchTST(ICLR'23),Crossformer(ICLR'23)

Results

Overall forecasting performance on SAN-enhanced backbone models.

| Methods Metric | Dlinear | | + SAN | | FEDformer | | + SAN | | Autoformer | | + SAN | | SCINet | | + SAN | | |
|-------------------|---------|--------------|--------------|--------------|--------------|--------------|-------|--------------|--------------|--------------|--------------|--------------|--------------|-------|-------|--------------|--------------|
| | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | |
| Electricity | 96 | 0.140 | 0.237 | 0.137 | 0.234 | 0.185 | 0.300 | 0.164 | 0.272 | 0.195 | 0.309 | 0.172 | 0.281 | 0.213 | 0.316 | 0.152 | 0.256 |
| | 192 | 0.153 | 0.250 | 0.151 | 0.247 | 0.196 | 0.310 | 0.179 | 0.286 | 0.215 | 0.325 | 0.195 | 0.300 | 0.224 | 0.329 | 0.163 | 0.266 |
| | 336 | 0.168 | 0.267 | 0.166 | 0.264 | 0.215 | 0.330 | 0.191 | 0.299 | 0.237 | 0.344 | 0.211 | 0.316 | 0.230 | 0.334 | 0.178 | 0.283 |
| | 720 | 0.203 | 0.301 | 0.201 | 0.295 | 0.244 | 0.352 | 0.230 | 0.334 | 0.292 | 0.375 | 0.236 | 0.335 | 0.260 | 0.356 | 0.206 | 0.307 |
| Exchange | 96 | 0.086 | 0.213 | 0.085 | 0.214 | 0.152 | 0.281 | 0.079 | 0.205 | 0.152 | 0.283 | 0.082 | 0.208 | 0.126 | 0.269 | 0.082 | 0.200 |
| | 192 | 0.161 | 0.297 | 0.177 | 0.317 | 0.273 | 0.380 | 0.156 | 0.295 | 0.369 | 0.437 | 0.157 | 0.296 | 0.266 | 0.392 | 0.169 | 0.293 |
| | 336 | 0.338 | 0.437 | 0.294 | 0.407 | 0.452 | 0.498 | 0.260 | 0.384 | 0.534 | 0.544 | 0.262 | 0.385 | 0.574 | 0.541 | 0.320 | 0.409 |
| | 720 | 0.999 | 0.755 | 0.726 | 0.649 | 1.151 | 0.830 | 0.697 | 0.633 | 1.222 | 0.848 | 0.689 | 0.629 | 1.136 | 0.818 | 0.892 | 0.712 |
| Traffic | 96 | 0.411 | 0.283 | 0.412 | 0.288 | 0.579 | 0.363 | 0.536 | 0.330 | 0.654 | 0.403 | 0.569 | 0.350 | 0.626 | 0.393 | 0.542 | 0.344 |
| | 192 | 0.423 | 0.289 | 0.429 | 0.297 | 0.608 | 0.376 | 0.565 | 0.345 | 0.654 | 0.410 | 0.594 | 0.364 | 0.613 | 0.396 | 0.545 | 0.358 |
| | 336 | 0.437 | 0.297 | 0.445 | 0.306 | 0.620 | 0.385 | 0.580 | 0.354 | 0.629 | 0.391 | 0.591 | 0.363 | 0.625 | 0.398 | 0.563 | 0.369 |
| | 720 | 0.467 | 0.316 | 0.474 | 0.319 | 0.630 | 0.387 | 0.607 | 0.367 | 0.657 | 0.402 | 0.623 | 0.380 | 0.639 | 0.409 | 0.607 | 0.381 |
| Weather | 96 | 0.175 | 0.237 | 0.152 | 0.210 | 0.246 | 0.328 | 0.179 | 0.239 | 0.247 | 0.320 | 0.194 | 0.256 | 0.181 | 0.260 | 0.169 | 0.232 |
| | 192 | 0.217 | 0.275 | 0.196 | 0.254 | 0.281 | 0.341 | 0.234 | 0.296 | 0.302 | 0.361 | 0.258 | 0.316 | 0.239 | 0.311 | 0.215 | 0.275 |
| | 336 | 0.263 | 0.314 | 0.246 | 0.294 | 0.337 | 0.376 | 0.304 | 0.348 | 0.362 | 0.394 | 0.329 | 0.367 | 0.293 | 0.348 | 0.267 | 0.314 |
| | 720 | 0.325 | 0.366 | 0.315 | 0.346 | 0.414 | 0.426 | 0.400 | 0.404 | 0.427 | 0.433 | 0.440 | 0.438 | 0.345 | 0.380 | 0.338 | 0.365 |
| ILI | 24 | 2.297 | 1.055 | 2.122 | 1.001 | 3.205 | 1.255 | 2.614 | 1.119 | 3.309 | 1.270 | 2.777 | 1.157 | 7.467 | 2.039 | 2.776 | 1.163 |
| | 36 | 2.323 | 1.070 | 2.029 | 0.978 | 3.148 | 1.288 | 2.537 | 1.079 | 3.207 | 1.216 | 2.649 | 1.104 | 7.035 | 1.948 | 2.411 | 1.026 |
| | 48 | 2.262 | 1.065 | 2.041 | 0.971 | 2.913 | 1.168 | 2.416 | 1.032 | 3.166 | 1.198 | 2.420 | 1.029 | 7.225 | 1.955 | 2.295 | 1.004 |
| | 60 | 2.443 | 1.124 | 2.089 | 0.973 | 2.853 | 1.161 | 2.299 | 1.003 | 2.947 | 1.159 | 2.401 | 1.021 | 7.335 | 1.957 | 2.487 | 1.063 |
| ETTh2 | 96 | 0.292 | 0.356 | 0.277 | 0.338 | 0.341 | 0.382 | 0.300 | 0.355 | 0.384 | 0.420 | 0.316 | 0.366 | 0.690 | 0.625 | 0.294 | 0.347 |
| | 192 | 0.383 | 0.418 | 0.340 | 0.378 | 0.426 | 0.436 | 0.392 | 0.413 | 0.457 | 0.454 | 0.413 | 0.426 | 0.991 | 0.742 | 0.374 | 0.398 |
| | 336 | 0.473 | 0.477 | 0.356 | 0.398 | 0.481 | 0.479 | 0.459 | 0.462 | 0.468 | 0.473 | 0.446 | 0.457 | 1.028 | 0.759 | 0.412 | 0.430 |
| | 720 | 0.708 | 0.599 | 0.396 | 0.435 | 0.458 | 0.477 | 0.462 | 0.472 | 0.473 | 0.485 | 0.471 | 0.474 | 1.363 | 0.885 | 0.437 | 0.461 |

| Methods Metric | PatchTST | | + SAN | | Crossformer | | + SAN | | |
|-------------------|----------|--------------|--------------|--------------|--------------|--------------|-------|--------------|--------------|
| | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | |
| Electricity | 96 | 0.138 | 0.233 | 0.136 | 0.234 | 0.150 | 0.258 | 0.143 | 0.246 |
| | 192 | 0.153 | 0.247 | 0.150 | 0.247 | 0.175 | 0.284 | 0.162 | 0.265 |
| | 336 | 0.170 | 0.263 | 0.165 | 0.264 | 0.218 | 0.325 | 0.177 | 0.280 |
| | 720 | 0.206 | 0.296 | 0.200 | 0.296 | 0.226 | 0.324 | 0.221 | 0.318 |
| Exchange | 96 | 0.094 | 0.216 | 0.087 | 0.218 | 0.283 | 0.393 | 0.087 | 0.219 |
| | 192 | 0.191 | 0.311 | 0.181 | 0.323 | 1.087 | 0.804 | 0.171 | 0.313 |
| | 336 | 0.343 | 0.427 | 0.305 | 0.418 | 1.367 | 0.905 | 0.286 | 0.401 |
| | 720 | 0.888 | 0.706 | 0.659 | 0.620 | 1.546 | 0.987 | 0.749 | 0.653 |
| Weather | 96 | 0.147 | 0.197 | 0.150 | 0.205 | 0.148 | 0.214 | 0.151 | 0.210 |
| | 192 | 0.191 | 0.240 | 0.194 | 0.252 | 0.201 | 0.270 | 0.198 | 0.253 |
| | 336 | 0.244 | 0.282 | 0.243 | 0.290 | 0.248 | 0.311 | 0.248 | 0.294 |
| | 720 | 0.320 | 0.334 | 0.311 | 0.343 | 0.366 | 0.395 | 0.322 | 0.350 |
| ETTh1 | 96 | 0.382 | 0.403 | 0.375 | 0.398 | 0.390 | 0.417 | 0.387 | 0.402 |
| | 192 | 0.416 | 0.423 | 0.413 | 0.422 | 0.424 | 0.448 | 0.413 | 0.425 |
| | 336 | 0.441 | 0.440 | 0.428 | 0.434 | 0.486 | 0.492 | 0.436 | 0.431 |
| | 720 | 0.470 | 0.475 | 0.445 | 0.461 | 0.507 | 0.519 | 0.467 | 0.474 |
| ETTh2 | 96 | 0.174 | 0.261 | 0.167 | 0.260 | 0.330 | 0.401 | 0.170 | 0.262 |
| | 192 | 0.238 | 0.307 | 0.222 | 0.298 | 0.623 | 0.543 | 0.224 | 0.301 |
| | 336 | 0.293 | 0.346 | 0.276 | 0.334 | 0.887 | 0.637 | 0.274 | 0.333 |
| | 720 | 0.373 | 0.401 | 0.366 | 0.393 | 0.844 | 0.640 | 0.366 | 0.390 |

□ Results

- Comparison between SAN and existing normalization approaches.

| Methods | FEDformer | | | | | Autoformer | | | | |
|-------------|--------------|--------|--------------|----------|--------|--------------|--------------|--------------|----------|--------|
| | +SAN | +RevIN | +NST | +Dish-TS | IMP(%) | +SAN | +RevIN | +NST | +Dish-TS | IMP(%) |
| Electricity | 0.191 | 0.200 | 0.198 | 0.203 | 3.54 | 0.204 | 0.219 | 0.213 | 0.231 | 4.23 |
| Exchange | 0.298 | 0.474 | 0.480 | 0.704 | 37.13 | 0.297 | 0.495 | 0.494 | 1.008 | 39.88 |
| Traffic | 0.572 | 0.647 | 0.649 | 0.652 | 11.59 | 0.594 | 0.666 | 0.664 | 0.677 | 10.54 |
| Weather | 0.279 | 0.268 | 0.267 | 0.398 | -4.49 | 0.305 | 0.290 | 0.290 | 0.433 | -5.17 |
| ILI | 2.467 | 2.962 | 3.084 | 2.846 | 13.32 | 2.562 | 3.151 | 3.235 | 3.180 | 18.69 |
| ETTh1 | 0.447 | 0.463 | 0.456 | 0.461 | 1.97 | 0.518 | 0.519 | 0.521 | 0.521 | 0.19 |
| ETTh2 | 0.404 | 0.465 | 0.481 | 1.004 | 13.12 | 0.411 | 0.489 | 0.465 | 1.175 | 11.61 |
| ETTh1 | 0.377 | 0.415 | 0.411 | 0.422 | 8.27 | 0.406 | 0.562 | 0.535 | 0.567 | 24.11 |
| ETTh2 | 0.287 | 0.310 | 0.315 | 0.759 | 7.42 | 0.311 | 0.325 | 0.331 | 0.894 | 4.31 |

Results

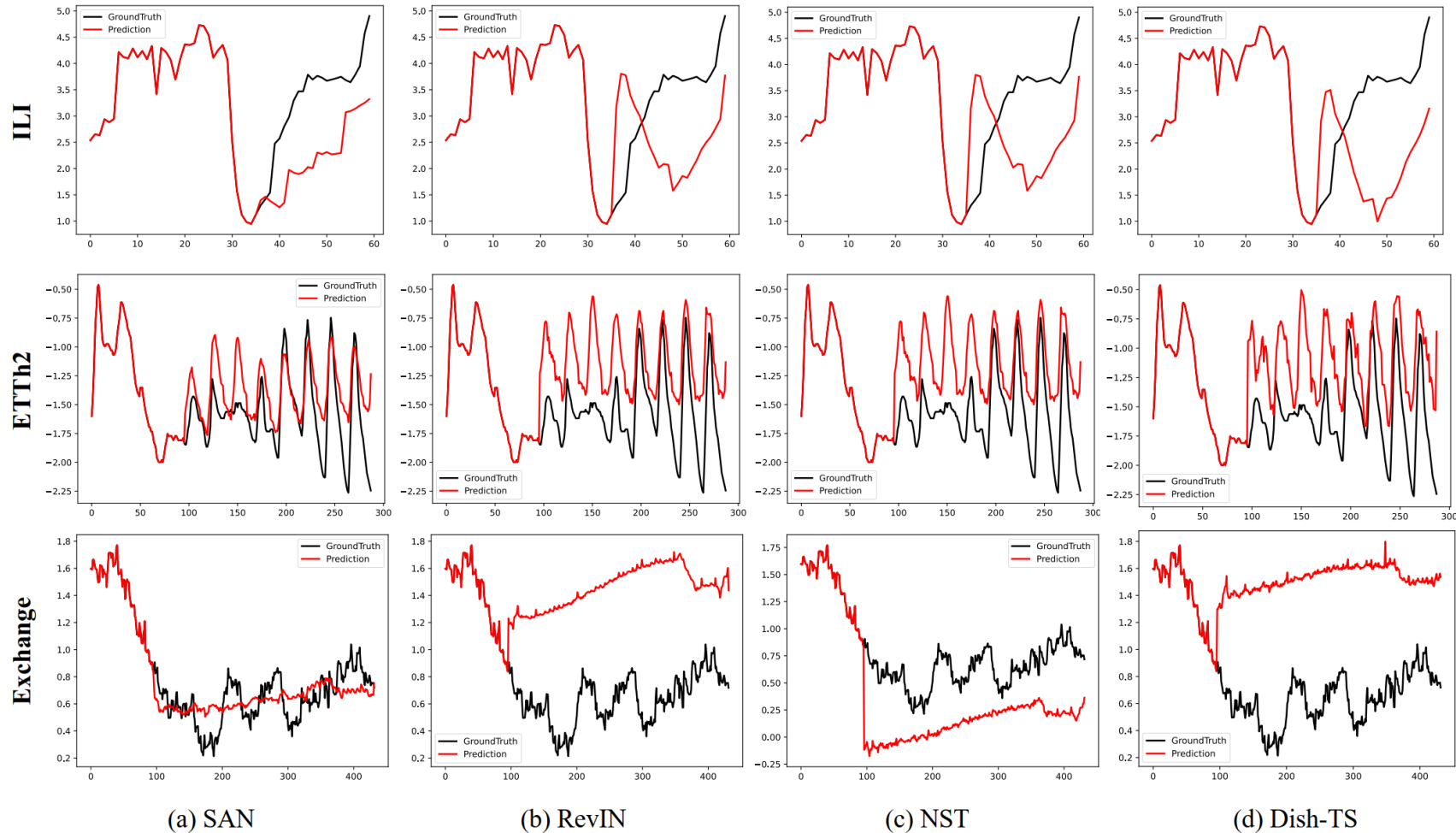


Figure 4: Visualization of forecasting results comparing SAN and baseline models.

- We focused on alleviating the **non-stationary property** of time series data using a novel **slice view** in the forecasting task.
- We proposed **Slicing Adaptive Normalization (SAN)**
 - A **model-agnostic** approach that removes the non-stationary factors the input by normalization and restores them to the output through denormalization **on a per-slice basis**.
 - With a two-stage training schema for the statistics prediction module, SAN **simplifies the non-stationary forecasting task through divide and conquer**.
 - Compared to existing normalization methods, SAN could better alleviate **the local-region non-stationarity** and provides more **accurate estimation on future distributions**.
- Extensive experiments validated the effectiveness of our method.

Thanks



<https://github.com/icantnamemyself/SAN>
zhiding@mail.usc.edu.cn