PETAL: Physics Emulation Through Averaged Linearizations for Solving Inverse Problems

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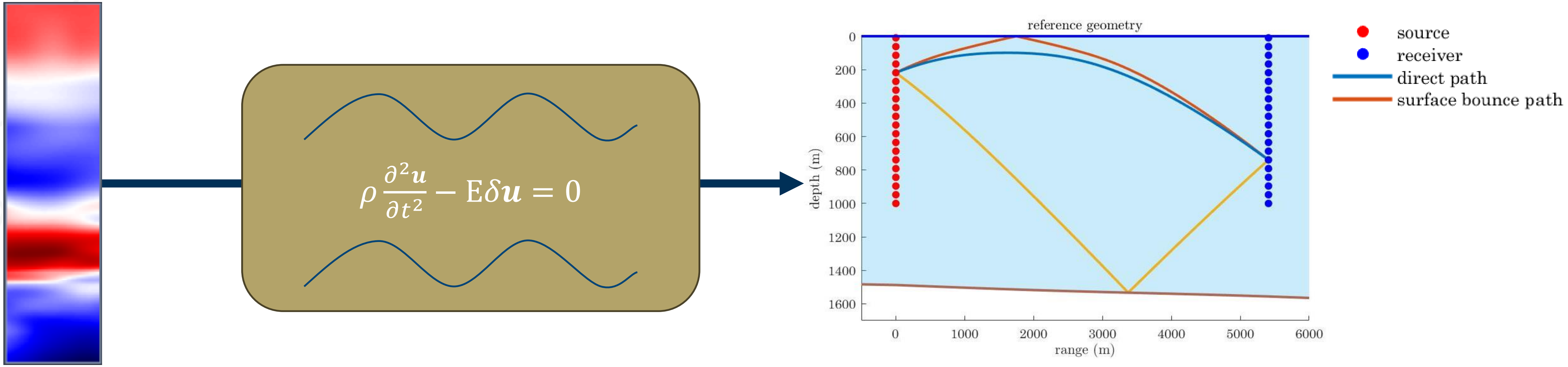
Inverse Problems



x F y

$$y = F(x)$$

- x – underlying signal of interest (Sound Speed Profile)
- y – observables (arrival times observed at receivers)
- F – "Forward" model that maps signal to observables

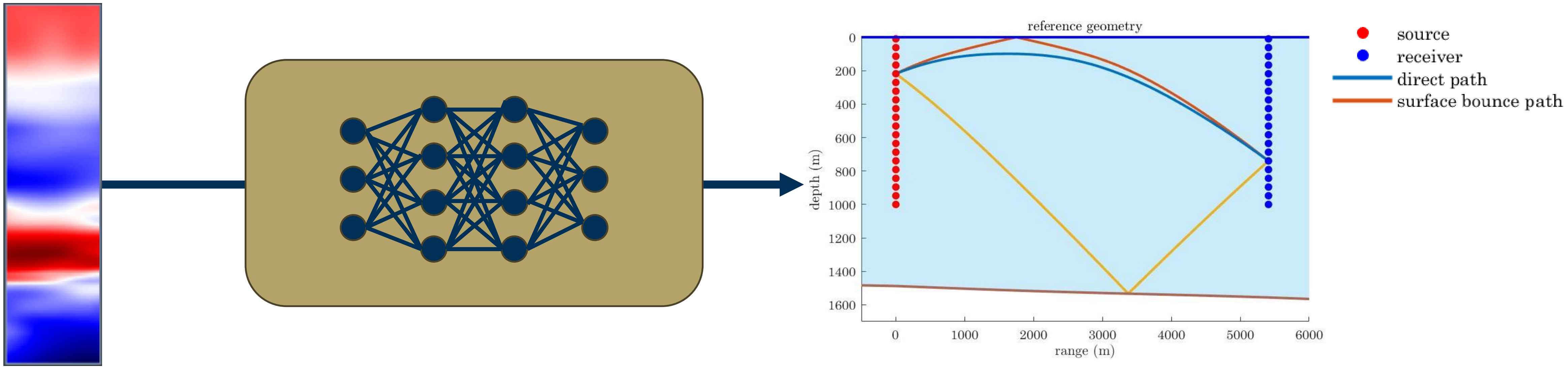
Neural Adjoint: Forward Model Surrogate

- Physics-based Forward Model

$$F: x \rightarrow y$$

- Train a **Neural Net** surrogate model

$$G_\theta: x \rightarrow y$$



x G_θ y

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- Formulate as optimization problem

$$\hat{x} = \arg\min_x \frac{1}{2} |G_\theta(x) - y|^2$$

- Solve for iteratively

$$x^{k+1} = x^k - \lambda d^k$$

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- Descent direction

$$d^k = J_G(x)^\top (G_\theta(x) - y)$$

GPU acceleration (Fast!)

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Autograd/backprop (cheap)

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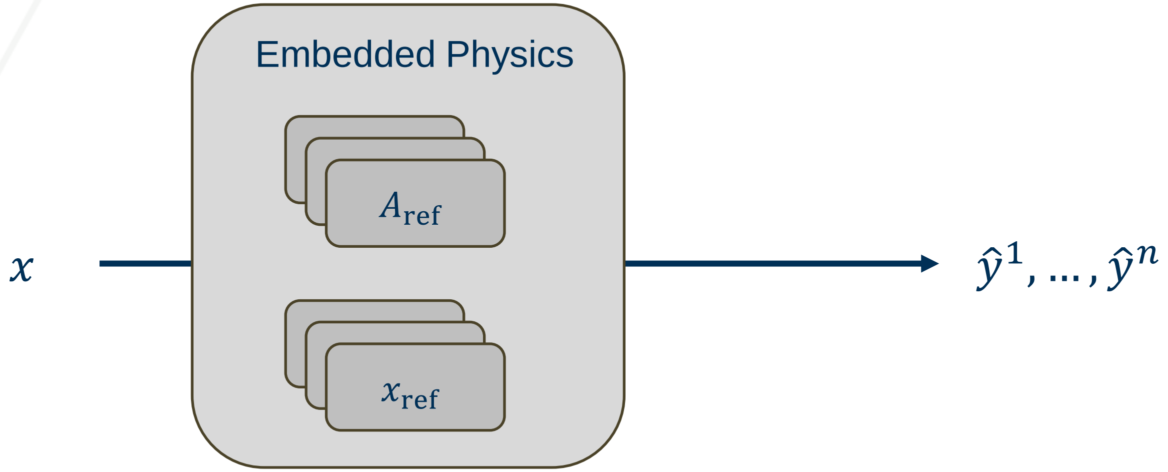
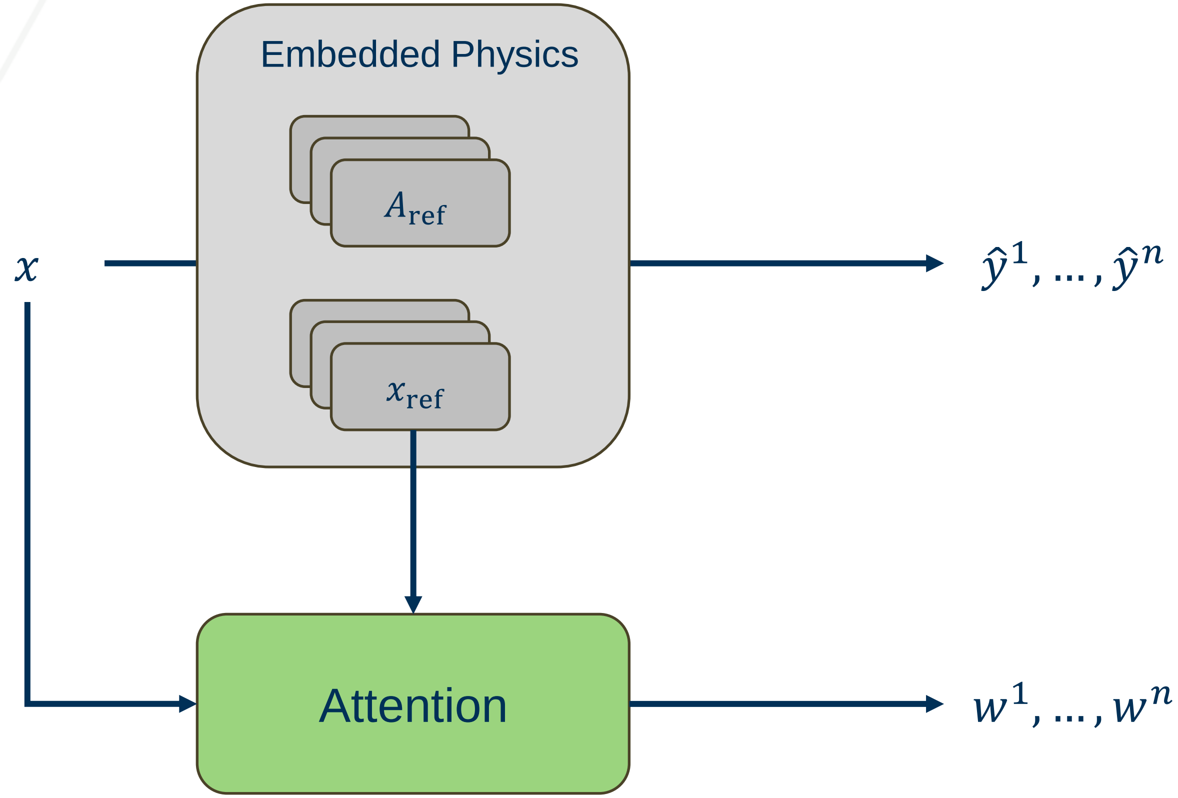
Black box: no physics

PETAL – Embedding Physics



- Ensemble of cheap approximations of physics based forward model
  - $y = J_F(x_{\text{ref}})^\top (x - x_{\text{ref}}) + y_{\text{ref}}$
    $y = A_{\text{ref}}(x - x_{\text{ref}}) + y_{\text{ref}}$

PETAL



- Compute weights via a learned attention module

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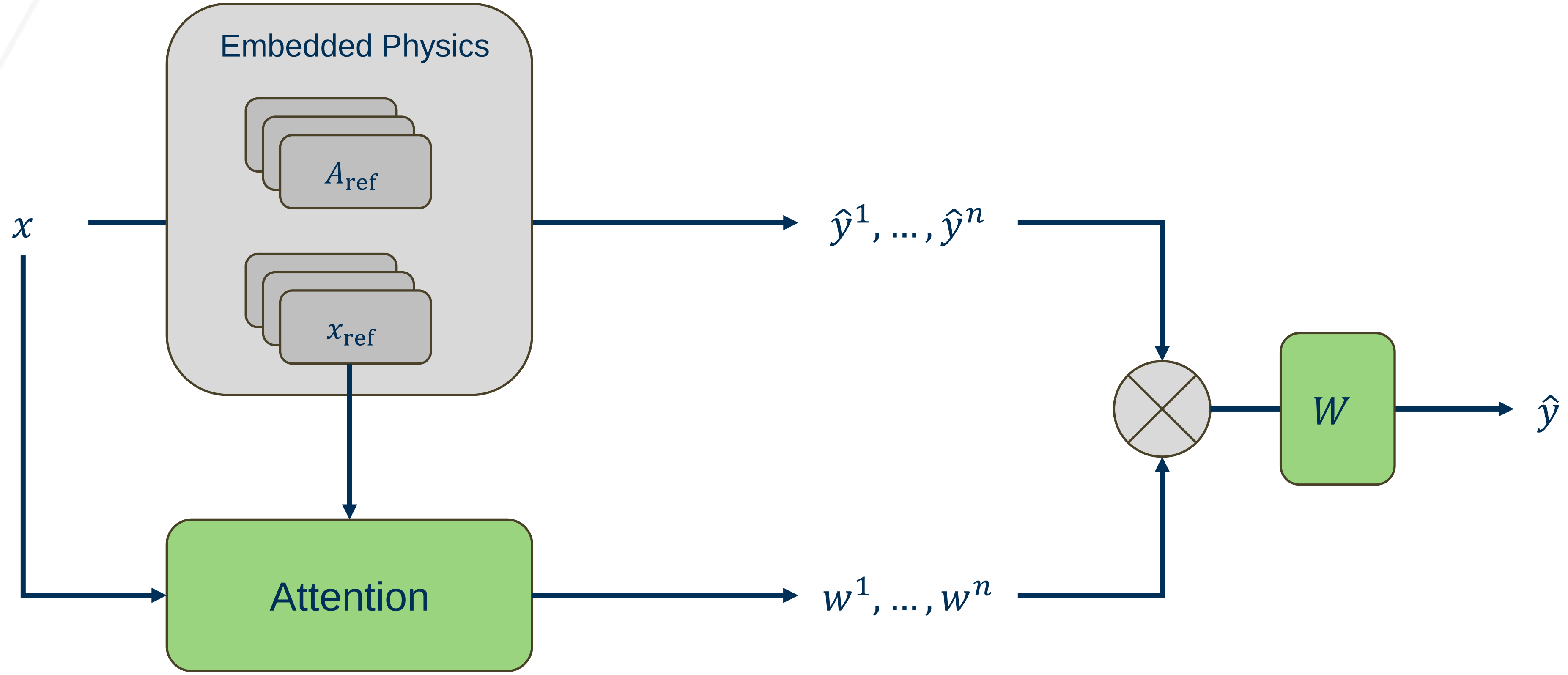

- Take a weighted average over the set of reference models

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- Replace F with G_θ

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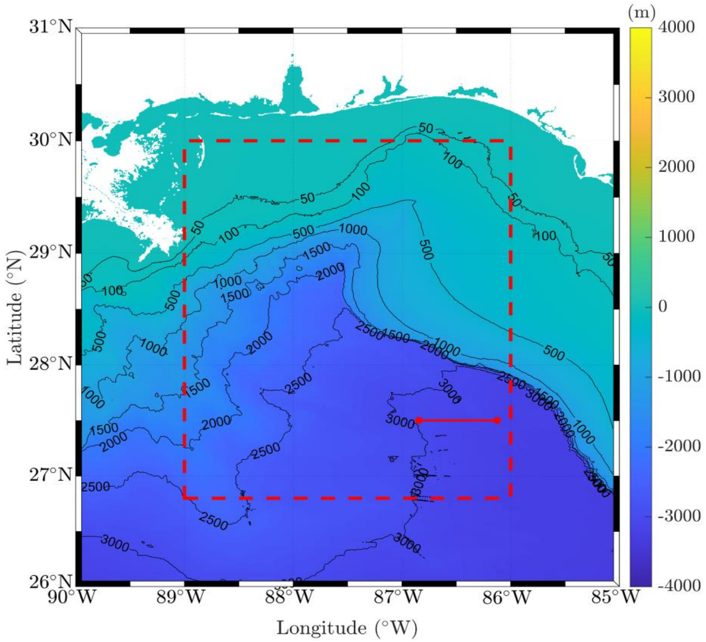
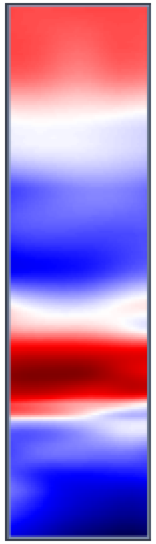
Embedded Physics via Linearizations

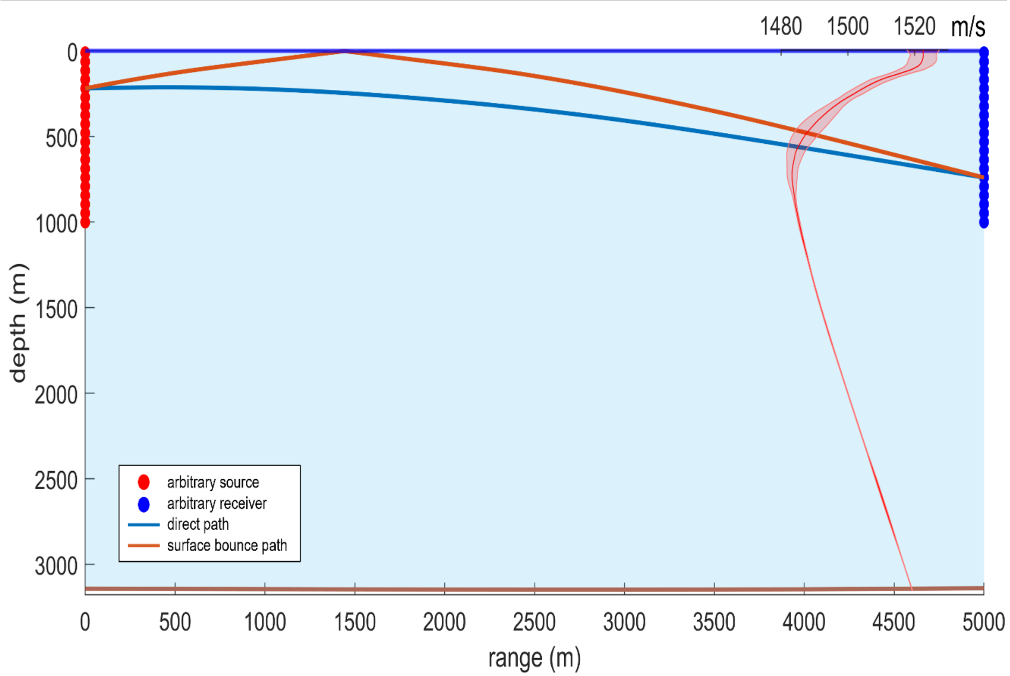
Experimental Set Up



Gulf of Mexico



Data



Forward Model

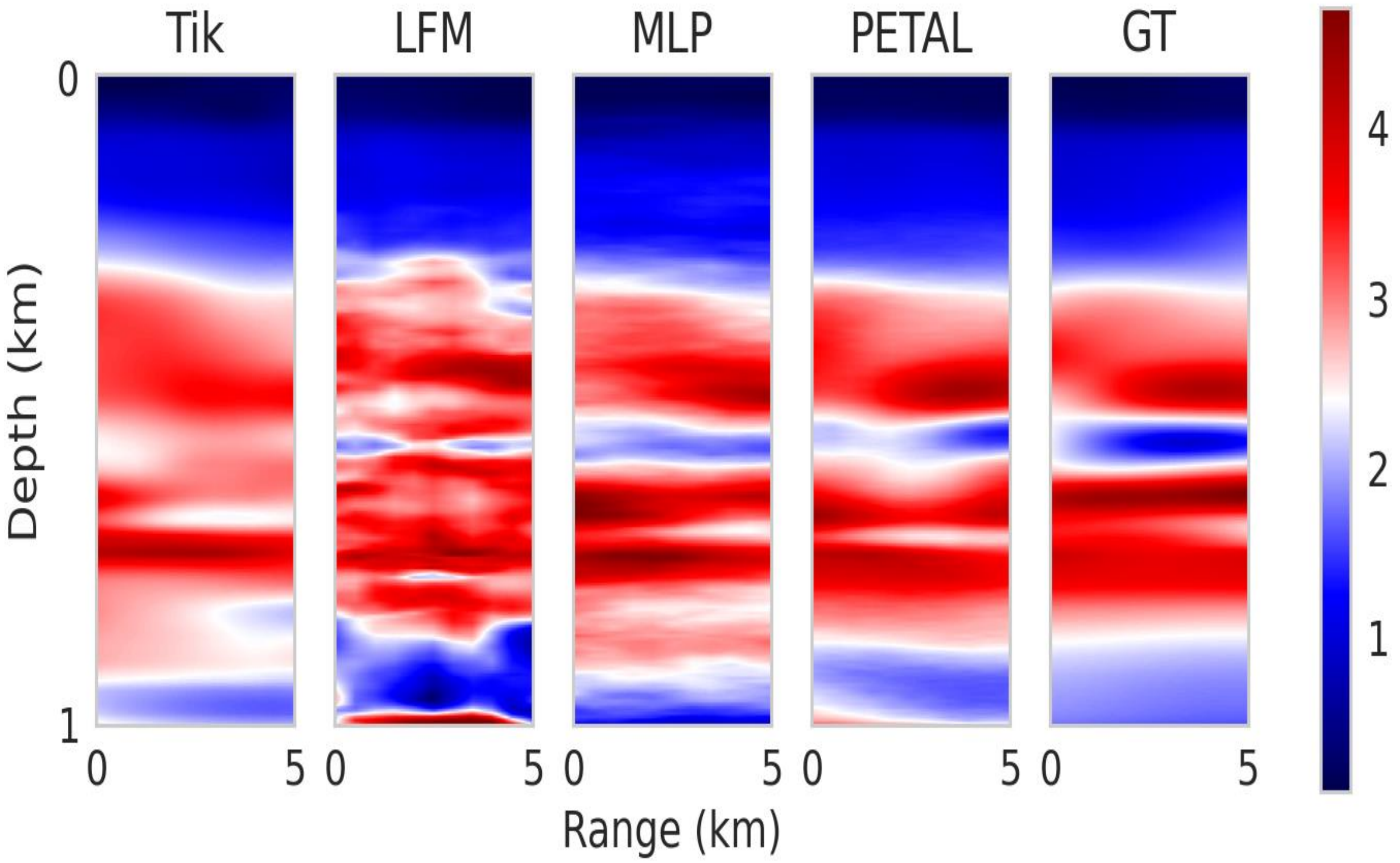
- Data collected from simulations of the Gulf of Mexico
- Forward Model: Direct and surface bounce path between source-receiver pairs
- Data: Sound Speed Profiles
- Observables: Arrival times

Results

	Low Variability		Med Variability		High Variability	
	Avg	Tik	Avg	Tik	Avg	Tik
Tik	0.647	---	0.773	---	0.881	---
LFM	0.620	0.597	0.584	0.580	0.617	0.630
MLP	0.384	0.378	0.406	0.409	0.424	0.428
PETAL (Ours)	**0.365**	**0.339**	**0.360**	**0.346**	**0.361**	**0.374**

- Tik – Classical Inversion with linearized forward model + Tikhonov regularization
- LFM – Optimization framework with linearized forward model
- MLP – Neural adjoint optimization framework with generic learned surrogate
- PETAL – Proposed model in neural adjoint optimization framework

Results



Conclusion

- Introduce the Neural Adjoint method for solving inverse problems
- Introduce a novel architecture that embeds physics into the surrogate
- Demonstrate its efficacy on a ocean acoustic tomography problem