

Energy Transformer

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Energy Transformer combines 3 paradigms

Transformer

Evolves tokens using attention

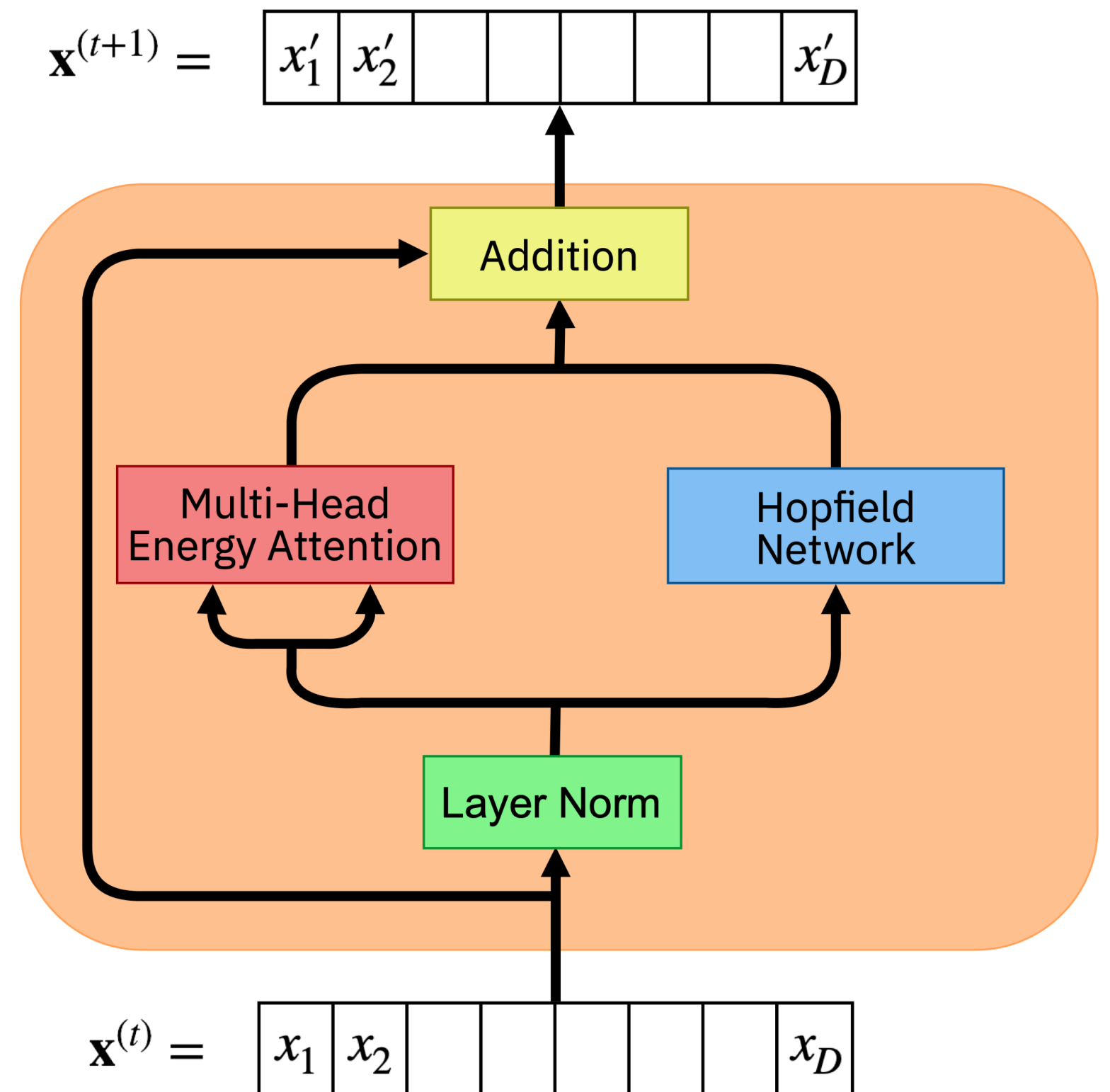
Energy-Based Model

Inference minimizes a computed energy

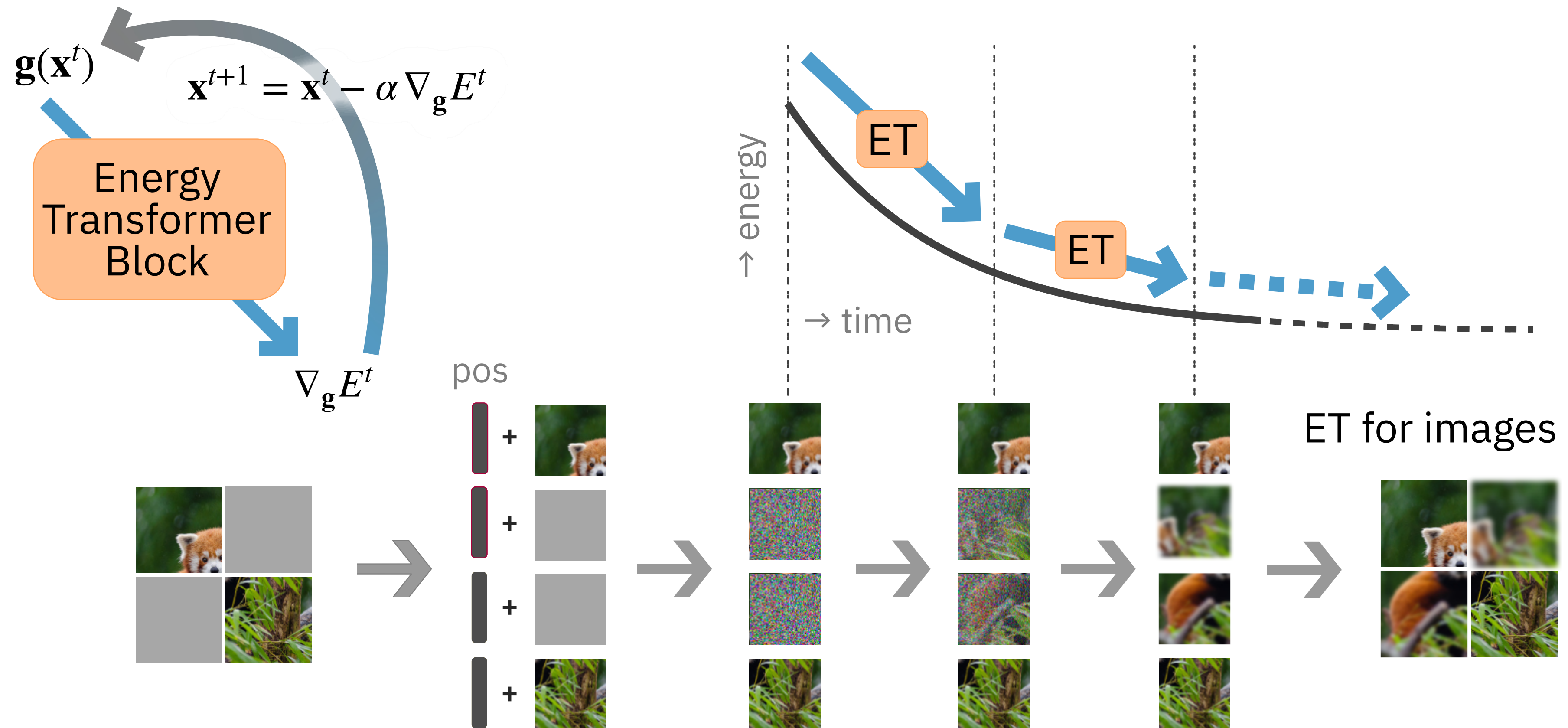
Associative Memory

An dynamical attractor system where inference is pattern completion

✓ Energy transformer block

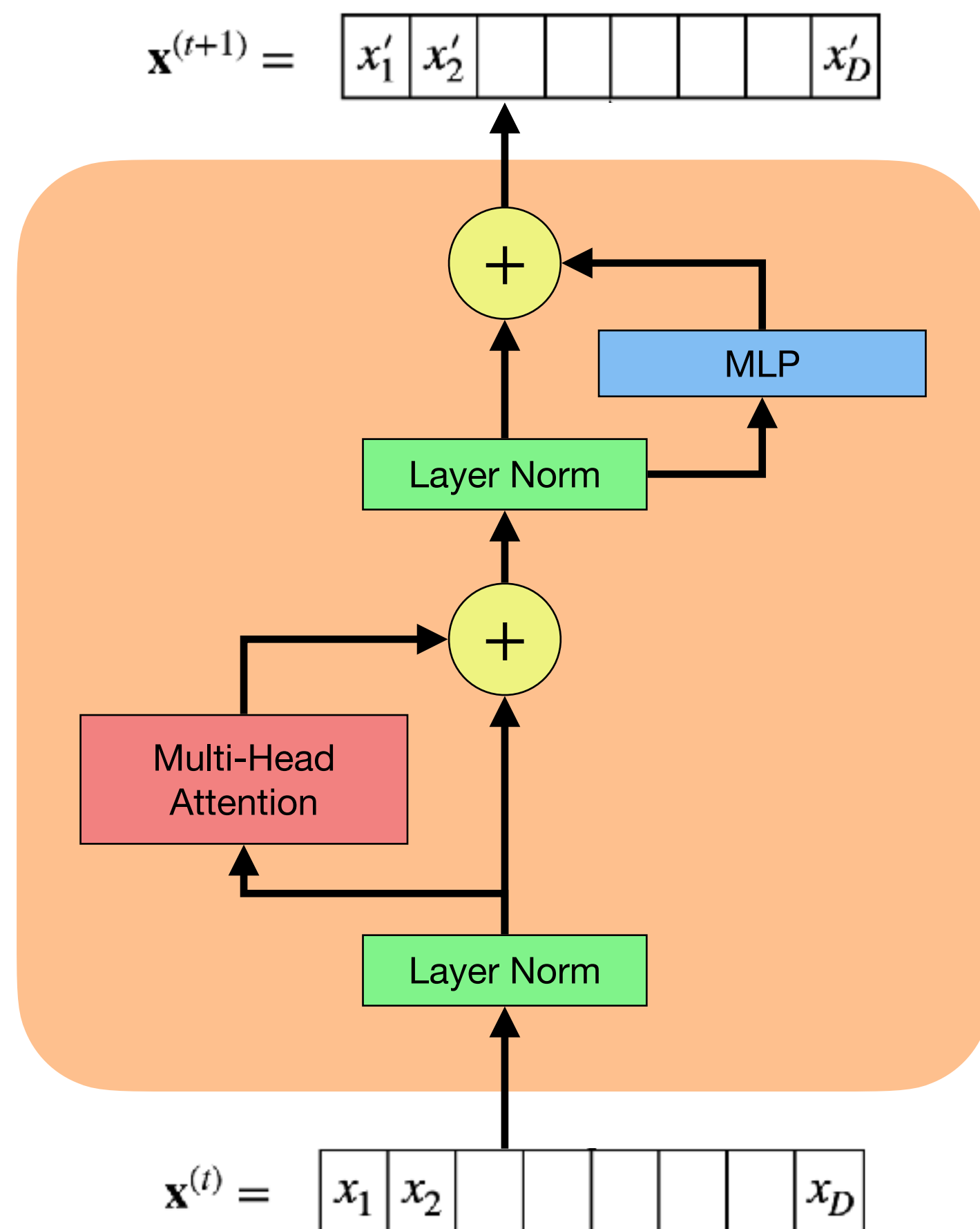


Energy Transformer in practice

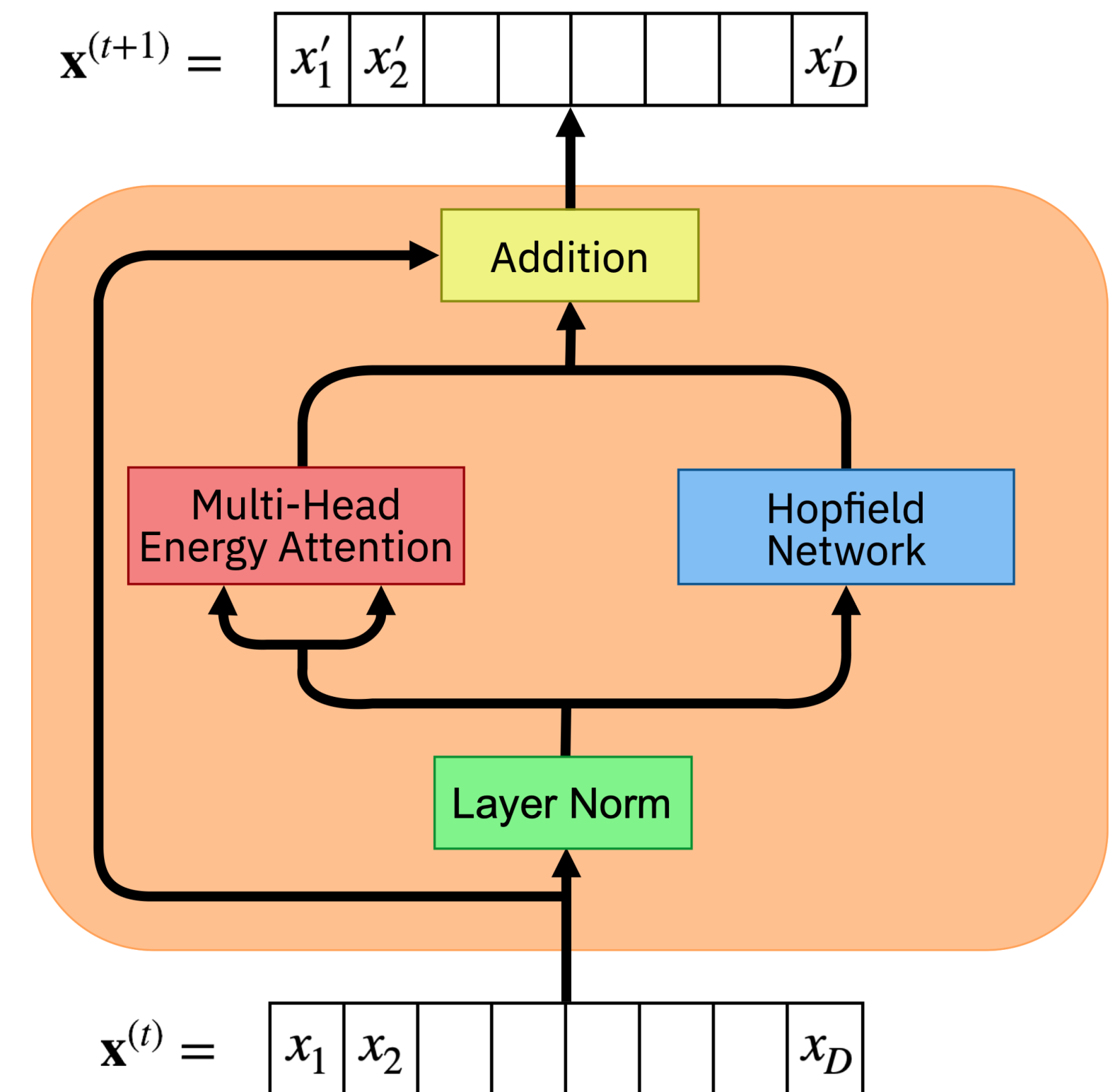


Conventional Transformers vs. Energy Transformer

✗ Conventional transformer block



✓ Energy transformer block



Attention energy is **low when keys are aligned with queries** (and vice versa)

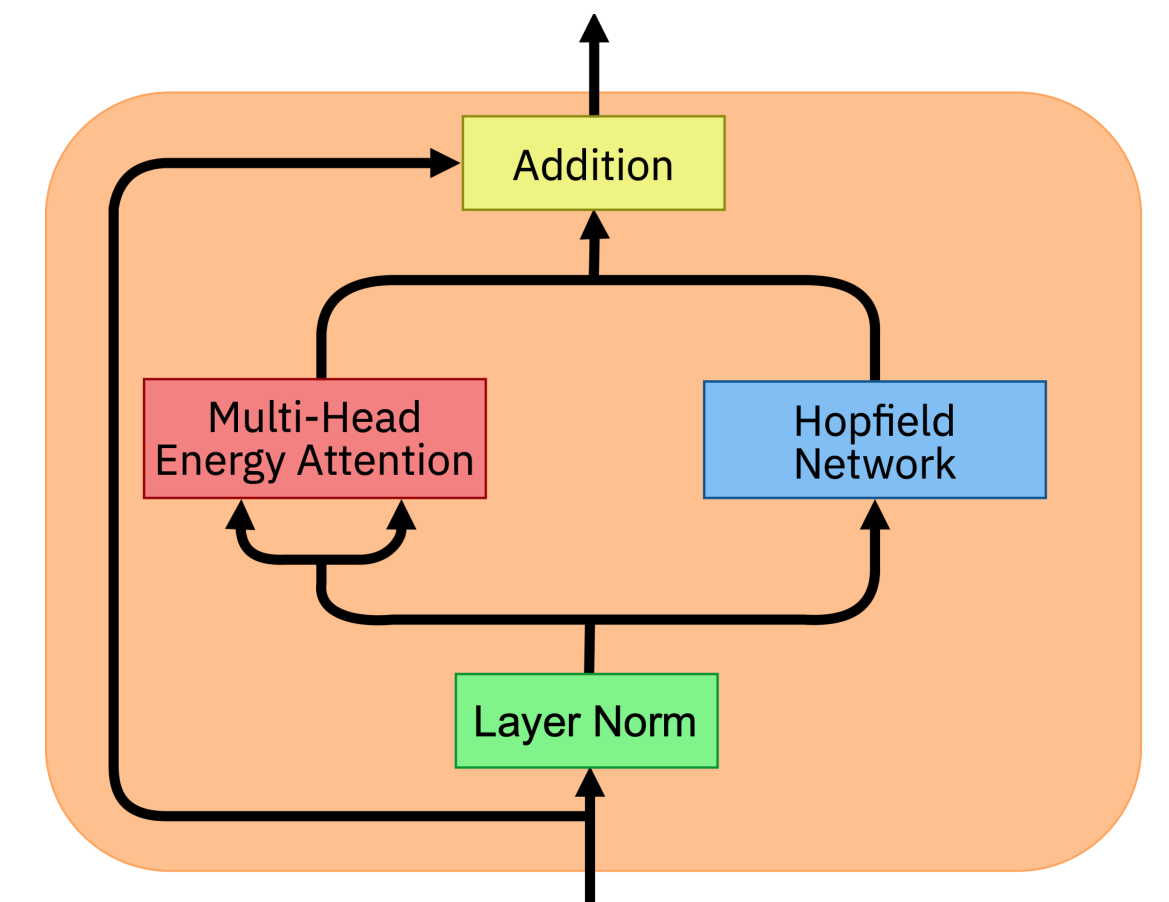
$$E^{\text{ATT}} = -\frac{1}{\beta} \sum_h \sum_C \log \left(\sum_{B \neq C} \exp \left(\beta \sum_{\alpha} K_{\alpha h B} Q_{\alpha h C} \right) \right)$$

Hopfield Network energy is **low when tokens look like memories**

$$E^{\text{HN}} = - \sum_{B=1}^N \sum_{\mu=1}^K G \left(\sum_{j=1}^D \xi_{\mu j} g_{jB} \right)$$

Energy Transformer: **make both sub-blocks happy**

Energy transformer block



$$E = E^{\text{ATT}} + E^{\text{HN}}$$

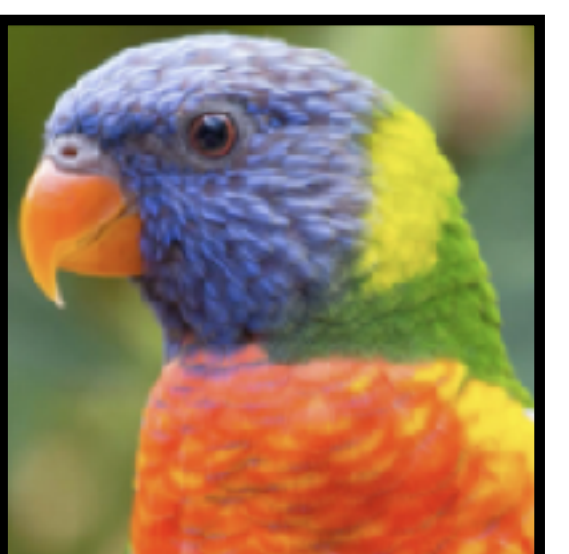
$$\tau \frac{dx_{iA}}{dt} = -\frac{\partial E}{\partial g_{iA}}$$

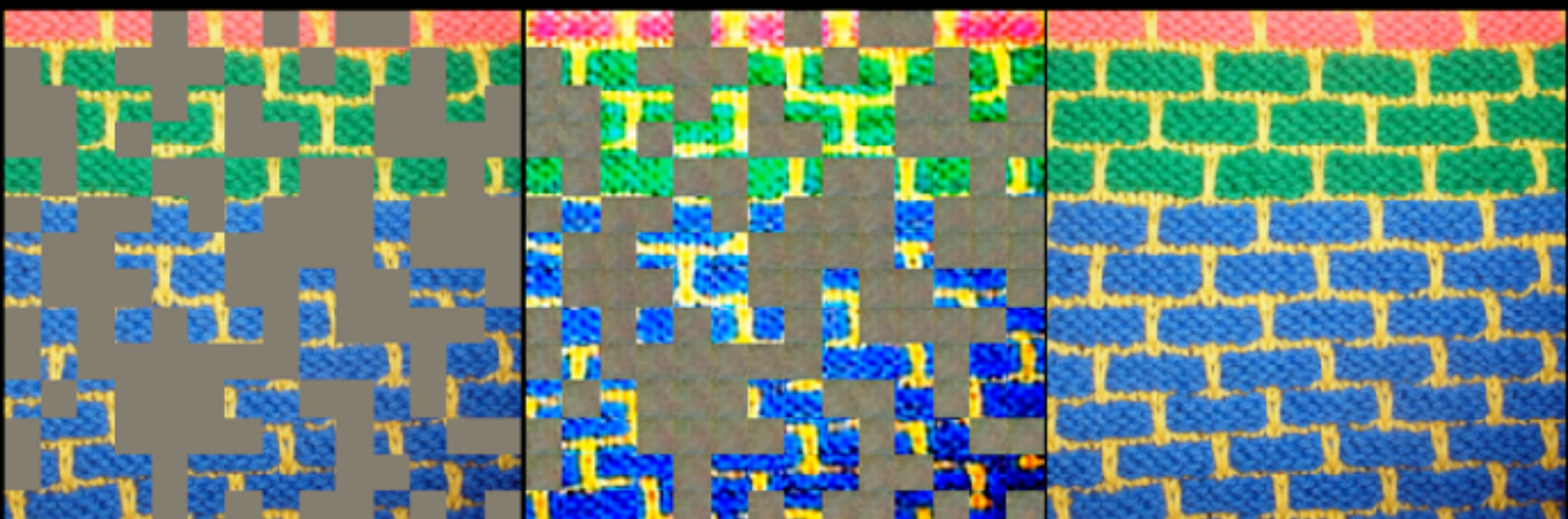
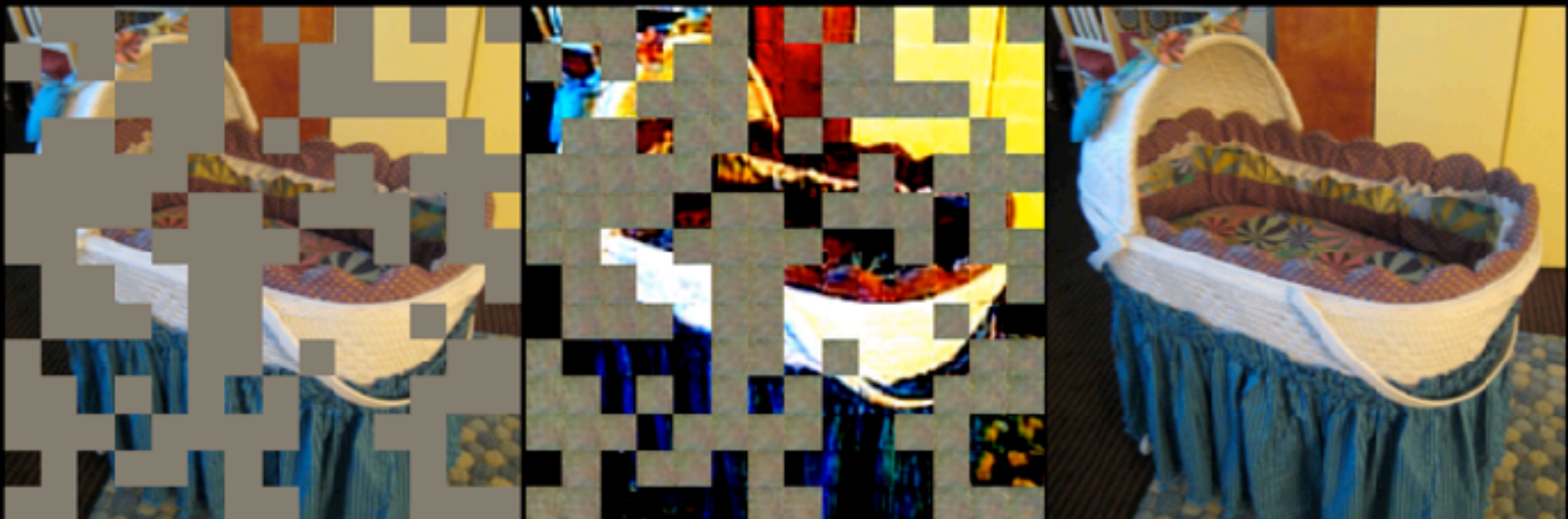
High energy



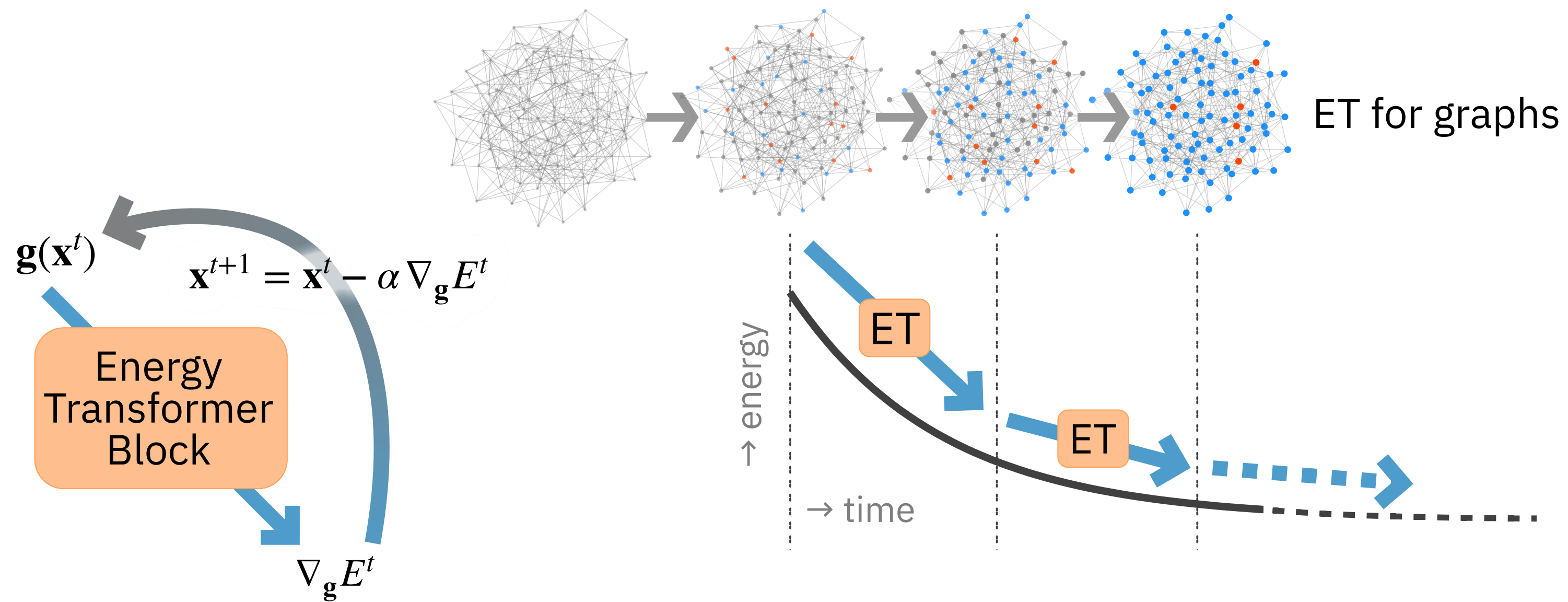
Recurrent ET block

Low energy





Anomaly Detection



| | Datasets | Split | GraphConsis | CAREGNN | PC-GNN | BWGNN | MLP | GT | ET (Ours) |
|----------|-----------|-------|----------------|----------------|--------------------------------|--------------------------------|----------------|--------------------------------|--------------------------------|
| Macro-F1 | Yelp | 1% | 56.8 \pm 2.8 | 62.1 \pm 1.3 | 59.8 \pm 1.4 | 61.1 \pm 0.4 | 53.9 \pm 0.2 | 61.7 \pm 0.4 | 63.0\pm0.6 |
| | | 40% | 58.7 \pm 2.0 | 63.3 \pm 0.9 | 63.0 \pm 2.3 | 71.0 \pm 0.9 | 57.5 \pm 0.8 | 68.7 \pm 0.4 | 71.5\pm0.1 |
| | Amazon | 1% | 68.5 \pm 3.4 | 68.7 \pm 1.6 | 79.8 \pm 5.6 | 90.9\pm0.7 | 74.6 \pm 1.2 | 88.6 \pm 0.5 | 89.3 \pm 0.7 |
| | | 40% | 75.1 \pm 3.2 | 86.3 \pm 1.7 | 89.5 \pm 0.7 | 92.2 \pm 0.4 | 79.1 \pm 1.2 | 91.7 \pm 0.8 | 92.8\pm0.3 |
| | T-Finance | 1% | 71.7 | 73.3 | 62.0 | 84.8 | 61.0 | 81.5 | 85.1\pm1.0 |
| | | 40% | 73.4 | 77.5 | 63.1 | 86.8 | 70.5 | 83.6 | 88.2\pm1.0 |
| T-Social | 1% | 52.4 | 55.8 | 51.1 | 75.9 | 50.0 | 64.3 | 79.1\pm0.7 | |
| | 40% | 56.5 | 56.2 | 52.1 | 83.9 | 50.3 | 68.2 | 83.5\pm0.4 | |
| AUC | Yelp | 1% | 66.4 \pm 3.4 | 75.0 \pm 3.8 | 75.4\pm0.9 | 72.0 \pm 0.5 | 59.8 \pm 0.4 | 72.5 \pm 0.6 | 73.2 \pm 0.8 |
| | | 40% | 69.8 \pm 3.0 | 76.1 \pm 2.9 | 79.8 \pm 0.1 | 84.0 \pm 0.9 | 66.5 \pm 1.0 | 81.9 \pm 0.5 | 84.9\pm0.3 |
| | Amazon | 1% | 74.1 \pm 3.5 | 88.6 \pm 3.5 | 90.4 \pm 2.0 | 89.4 \pm 0.3 | 83.6 \pm 1.7 | 89.0 \pm 1.2 | 91.9\pm1.0 |
| | | 40% | 87.4 \pm 3.3 | 90.5 \pm 1.6 | 95.8 \pm 0.1 | 98.0 \pm 0.4 | 89.8 \pm 1.0 | 95.4 \pm 0.6 | 97.3\pm0.4 |
| | T-Finance | 1% | 90.2 | 90.5 | 90.7 | 91.1 | 82.9 | 90.0 | 92.8\pm1.1 |
| | | 40% | 91.4 | 92.1 | 91.2 | 94.3 | 87.1 | 88.2 | 95.0\pm3.0 |
| | T-Social | 1% | 65.2 | 71.2 | 59.8 | 88.0 | 56.3 | 81.4 | 91.9\pm0.6 |
| | | 40% | 71.2 | 71.8 | 68.4 | 95.2 | 56.9 | 82.5 | 93.9\pm0.2 |

Graph Classification

| Method | Dataset | | | | | | | |
|------------------------|--|--|--|--|--|--|---|--|
| | PROTEINS | NCI1 | NCI109 | DD | ENZYMES | MUTAG | MUTAGENICITY | FRANKENSTEIN |
| WKPI (kmeans) | 78.5 \pm 0.4 \blacktriangledown (6.4) | 87.5 \pm 0.5 | 85.9 \pm 0.4 \blacktriangledown (1.5) | 82.0 \pm 0.5 \blacktriangledown (13.7) | - | 85.8 \pm 2.5 \blacktriangledown (14.2) | - | - |
| WKPI (kcenters) | 75.2 \pm 0.4 \blacktriangledown (9.7) | 84.5 \pm 0.5 \blacktriangledown (3.0) | 87.4 \pm 0.3 | 80.3 \pm 0.4 \blacktriangledown (15.4) | - | 88.3 \pm 2.6 \blacktriangledown (11.7) | - | - |
| Spec-GN | - | 84.8 \pm 1.6 \blacktriangledown (2.7) | 83.6 \pm 0.8 \blacktriangledown (3.8) | - | 72.5 \pm 5.8 \blacktriangledown (5.9) | - | - | - |
| Norm-GN | - | 84.9 \pm 1.7 \blacktriangledown (2.6) | 83.5 \pm 1.3 \blacktriangledown (3.9) | - | 73.3 \pm 8.0 \blacktriangledown (5.1) | - | - | - |
| GWL-WL | 75.8 \pm 0.6 \blacktriangledown (9.1) | - | - | - | 71.3 \pm 1.1 \blacktriangledown (7.1) | - | - | 78.9 \pm 0.3 |
| HGP-SL | 84.9 \pm 1.6 | 78.5 \pm 0.8 \blacktriangledown (9.1) | 80.7 \pm 1.2 \blacktriangledown (6.7) | 81.0 \pm 1.3 \blacktriangledown (14.7) | 68.8 \pm 2.1 \blacktriangledown (9.6) | - | 82.2 \pm 0.6 | - |
| DSGCN | 77.3 \pm 0.4 \blacktriangledown (7.6) | - | - | - | 78.4 \pm 0.6 | - | - | - |
| U2GNN | 80.0 \pm 3.2 \blacktriangledown (4.9) | - | - | 95.7 \pm 1.9 | - | 88.5 \pm 7.1 \blacktriangledown (11.5) | - | - |
| NDP | 73.4 \pm 3.1 \blacktriangledown (11.5) | 74.2 \pm 1.7 \blacktriangledown (13.3) | - | 72.8 \pm 5.4 \blacktriangledown (22.9) | 44.5 \pm 7.4 \blacktriangledown (34.9) | 87.9 \pm 5.7 \blacktriangledown (12.1) | 77.9 \pm 1.4 \blacktriangledown (4.3) | - |
| ASAP | 74.2 \pm 0.8 \blacktriangledown (10.7) | 71.5 \pm 0.4 \blacktriangledown (16.0) | 70.1 \pm 0.6 \blacktriangledown (17.3) | 76.9 \pm 0.7 \blacktriangledown (18.8) | - | - | - | 66.3 \pm 0.5 \blacktriangledown (12.6) |
| EvoG | - | - | - | - | 55.7 \blacktriangledown (22.7) | 100.0 | - | - |
| ET (Ours) | 90.3 \pm 0.7 \blacktriangle (5.4) | 90.1 \pm 0.1 \blacktriangle (2.6) | 90.5 \pm 0.1 \blacktriangle (3.1) | 95.9 \pm 0.8 \blacktriangle (0.2) | 99.8 \blacktriangle (21.4) | 96.6 \pm 0.2 \blacktriangledown (3.4) | 98.7 \pm 0.1 \blacktriangle (16.5) | 99.8 \pm 0.1 \blacktriangle (20.9) |