

# Streaming Factor Trajectory Learning for Temporal Tensor Decomposition

Shikai Fang, Xin Yu, Shibo Li, Zheng Wang, Robert Kirby, Shandian Zhe

Presenter: Shikai Fang

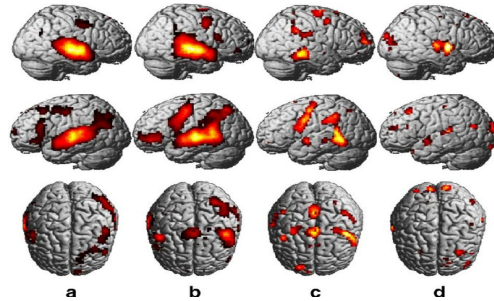
For NeurIPS 2023

Github Repo: <https://github.com/xuangu-fang/Streaming-Factor-Trajectory-Learning>

- Tensor-valued time series
- Represent time-varying and high-order interactions



(region, site, weather)



(subject, voxel, electrode)



(user, user, location, message)

Tensor structure are evolving through time!

Existing methods: **Evolving weights** + **Static Factors**

$$y_{\mathbf{i}}(t) \approx \mathbf{w}(t)^\top \left( \mathbf{u}_{i_1}^1 \circ \dots \circ \mathbf{u}_{i_K}^K \right) \quad \text{Too simple!}$$

**SFTL(ours): Build functional factor trajectories!**

CPD: 
$$y_{\mathbf{i}}(t) \approx \mathbf{w}^\top \left( \mathbf{u}_{i_1}^1(t) \circ \dots \circ \mathbf{u}_{i_K}^K(t) \right)^\top$$

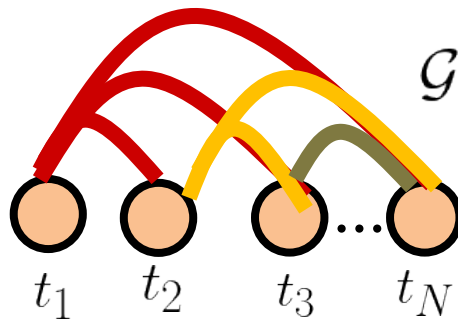
Tucker: 
$$y_{\mathbf{i}}(t) \approx \text{vec}(\mathcal{W})^\top \left( \mathbf{u}_{i_1}^1(t) \otimes \dots \otimes \mathbf{u}_{i_K}^K(t) \right)$$

# State-Space Gaussian Process

## Linear-Cost Functional Prior over Factor Trajectory

**Temporal GPs**

Temporal States:



$$\mathcal{GP}(\mathbf{w}_r \mid \mathbf{0}, \mathbf{K}_r(t, t'))$$

**Space:**  $\mathcal{O}(N^2)$

**Time:**  $\mathcal{O}(N^3)$

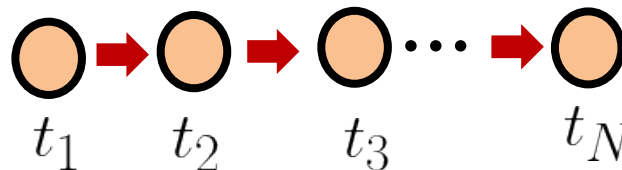
**LTI-SDE**

$$\frac{d\gamma_r(t)}{dt} = \mathbf{F}\gamma_r + \mathbf{L}\xi(t)$$

discrete form

$$p(\gamma_r(t_{n+1}) \mid \gamma_r(t_n)) = \mathcal{N}(\gamma_r(t_{n+1}) \mid \mathbf{A}_n \gamma_r(t_n), \mathbf{Q}_n)$$

**State Space Model**

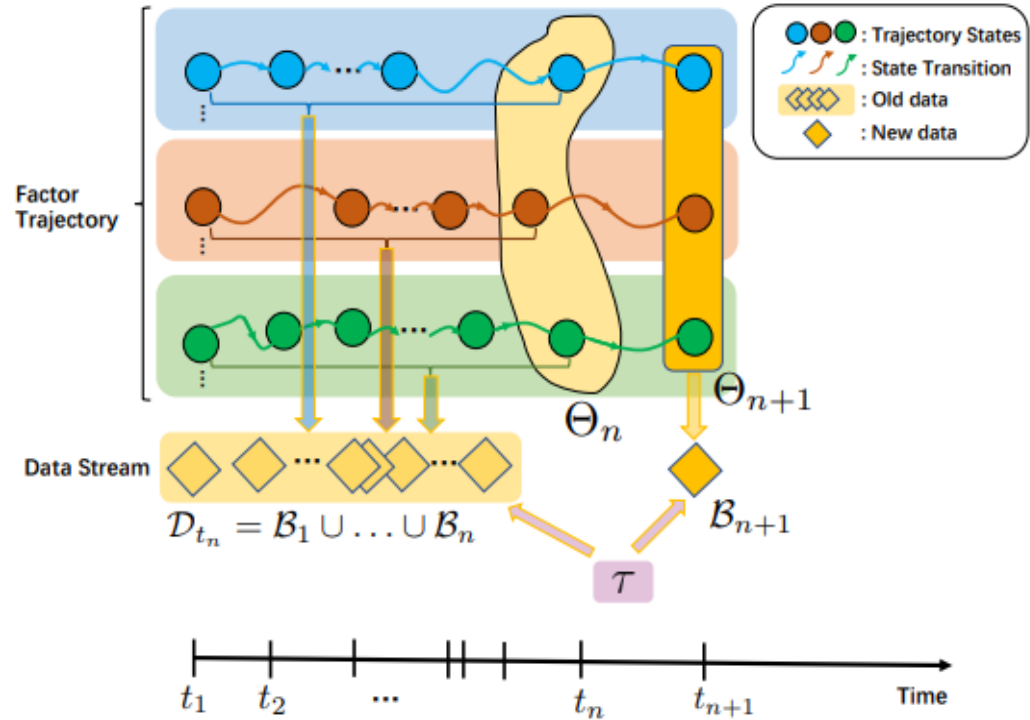


**Space:**  $\mathcal{O}(N)$

**Time:**  $\mathcal{O}(N)$

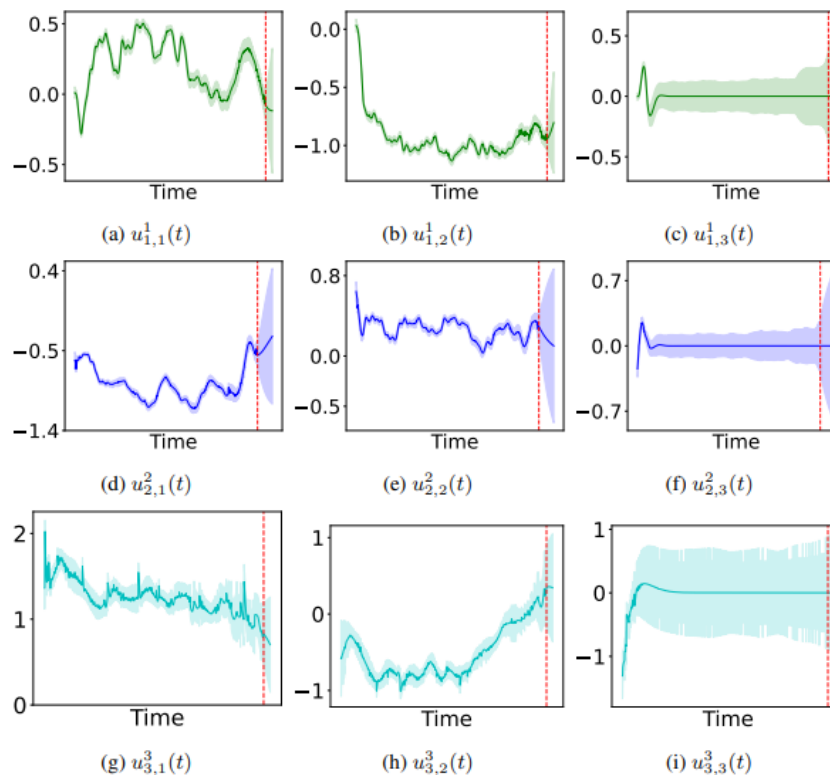
Efficient **Online** algorithm:

- **Closed-form Updates**
- **Conditional Expectation Propagation**
- **Kalman Filter + RTS Smoother**



# Experiment Results

	RMSE	<i>FitRecord</i>
Static	PTucker	$0.656 \pm 0.147$
	Tucker-ALS	$0.846 \pm 0.005$
	CP-ALS	$0.882 \pm 0.017$
	CT-CP	$0.664 \pm 0.007$
	CT-GP	$0.604 \pm 0.004$
	BCTT	$0.518 \pm 0.007$
	NONFAT	$0.503 \pm 0.002$
	THIS-ODE	$0.526 \pm 0.004$
Stream	POST	$0.696 \pm 0.019$
	ADF-CP	$0.648 \pm 0.008$
	BASS-Tucker	$0.076 \pm 0.024$
	SFTL-CP	<b><math>0.424 \pm 0.014</math></b>
	SFTL-Tucker	$0.430 \pm 0.010$
MAE		
Static	PTucker	$0.369 \pm 0.009$
	Tucker-ALS	$0.615 \pm 0.006$
	CP-ALS	$0.642 \pm 0.012$
	CT-CP	$0.46 \pm 0.004$
	CT-GP	$0.414 \pm 0.001$
	BCTT	$0.355 \pm 0.005$
	NONFAT	$0.341 \pm 0.001$
	THIS-ODE	$0.363 \pm 0.004$
Stream	POST	$0.478 \pm 0.014$
	ADF-CP	$0.449 \pm 0.006$
	BASS	$0.772 \pm 0.031$
	SFTL-CP	<b><math>0.242 \pm 0.006</math></b>
	SFTL-Tucker	$0.246 \pm 0.001$



Thanks.