NEURAL INFORMATION PROCESSING SYSTEMS

Causal Discovery in Semi-Stationary Time Series

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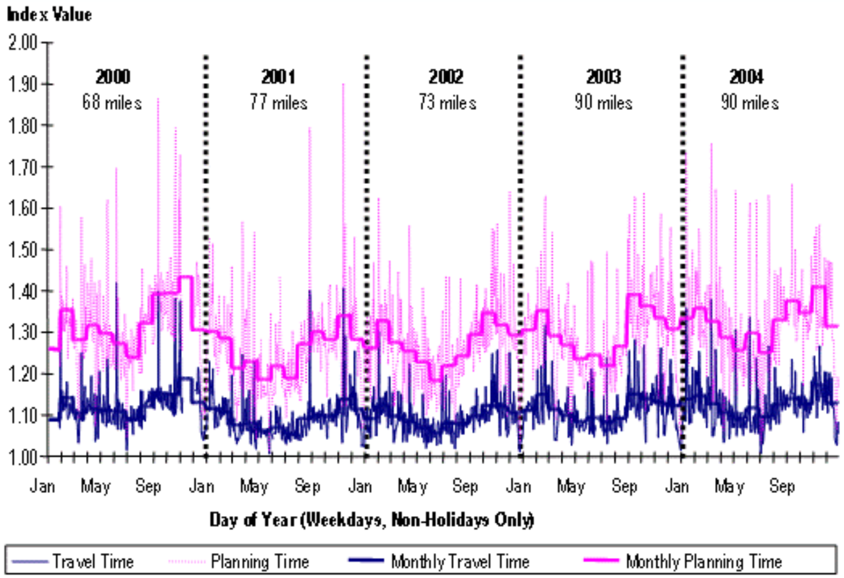
CausaML

Motivations

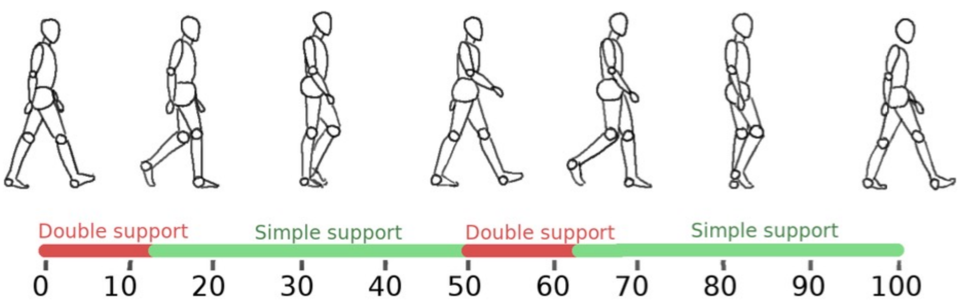
- *Periodic* nature is commonly observed in many real-world time series data.
- *Periodic* changes in the causal relations are expected underlying this type of time series without assuming stationarity.

Daily and Monthly Trends in Congestion
San Antonio, Texas, 2000-2004



Source: Analysis of data from FHWA's *Mobility Monitoring Program*

Human Walking
SS and DS phases duration, measured as percentage of complete cycle.



Source: kalouguine,2020

Introduction

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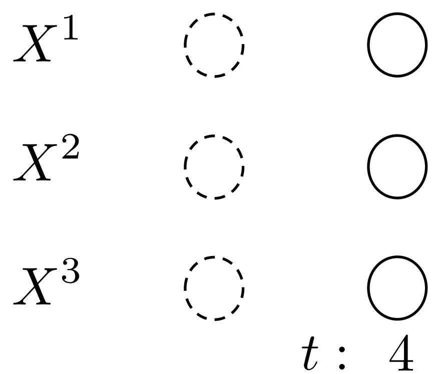

Figure1. Partial causal graph for 3-variate time series $V = \{X^1, X^2, X^3\}$ with a Stationary SCM

$$1.\ X_t^j = f_j(Pa(X_t^j), \epsilon_t^j), j \in [n]$$

$$2.\ Pa(X_{t+\Delta t}^j) = \{X_{s+\Delta t}^i : X_s^i \in Pa(X_t^j), i \in [n]\}, \forall \Delta t \in \mathbf{N}$$

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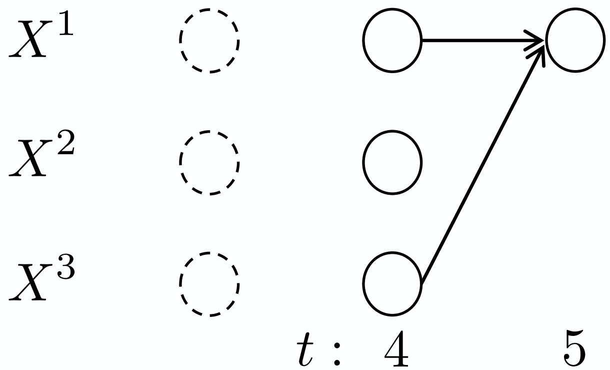

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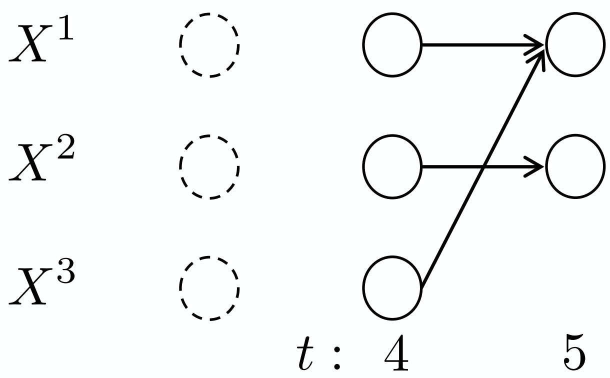

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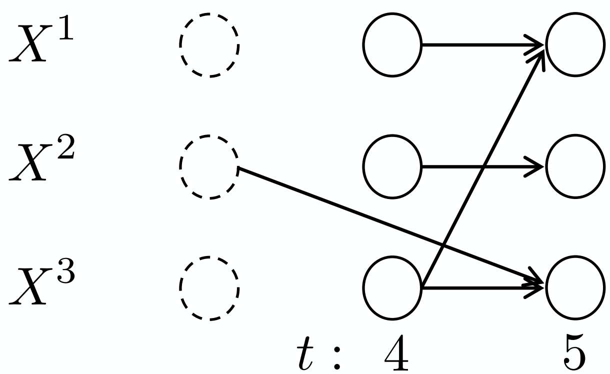

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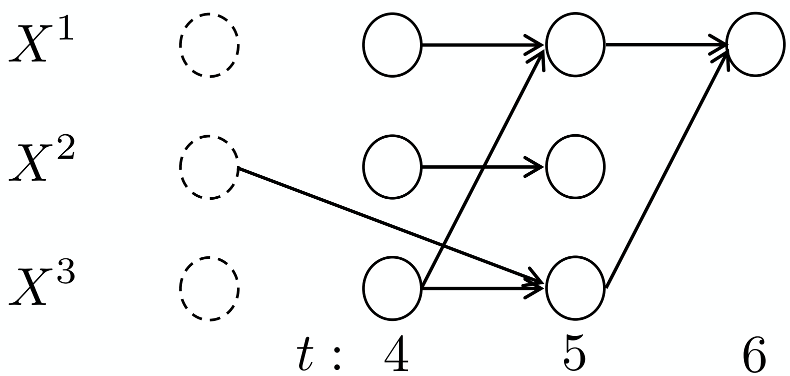

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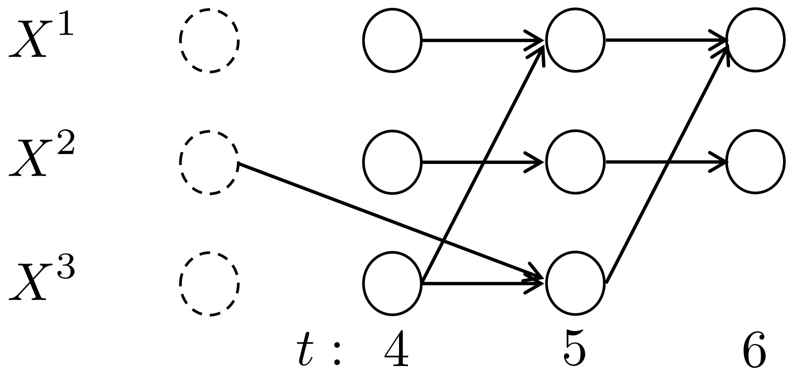

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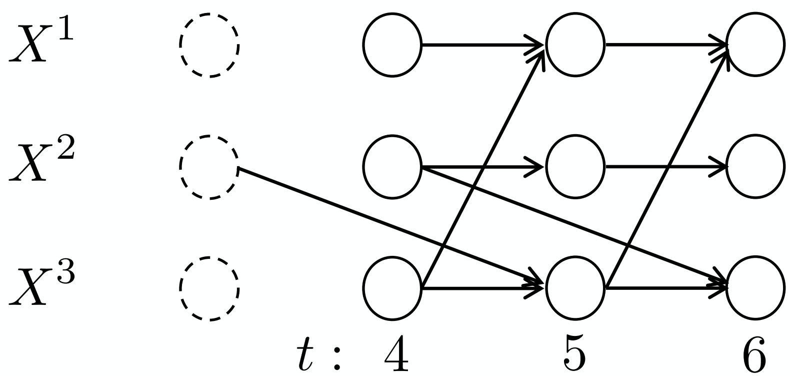

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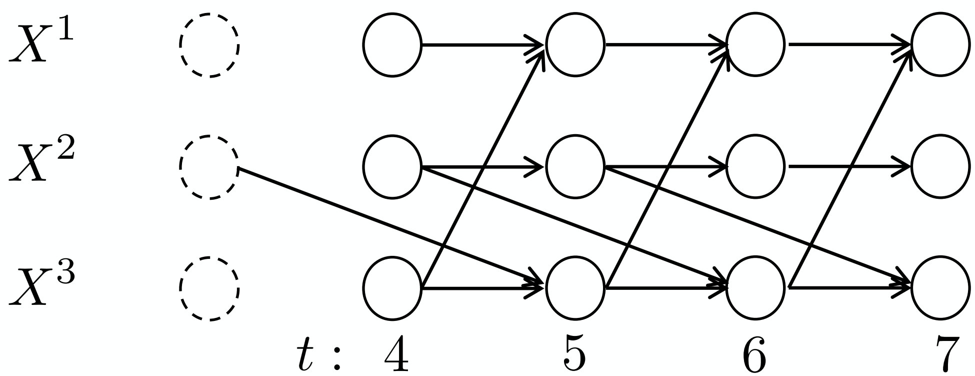

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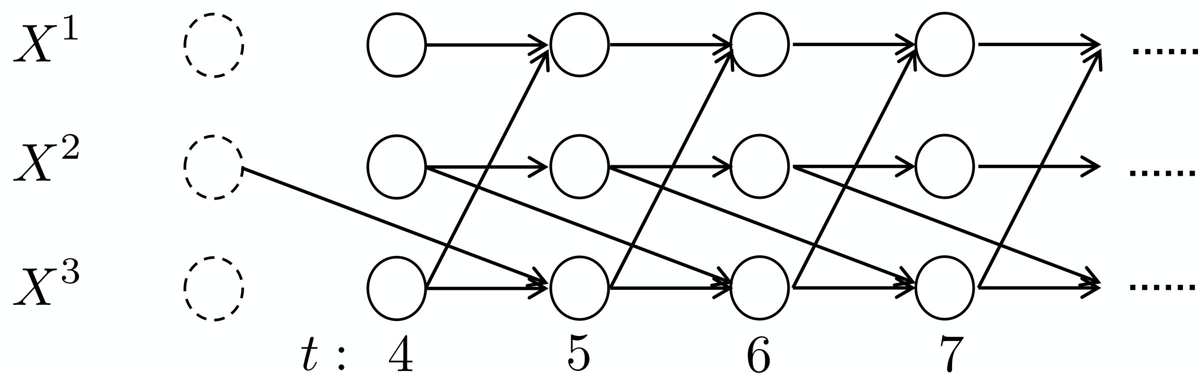

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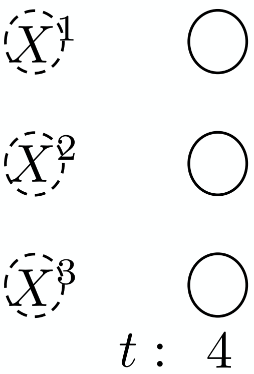

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For each $j \in [n]$, there exists an $\omega \in \mathbb{N}^+$ such that :

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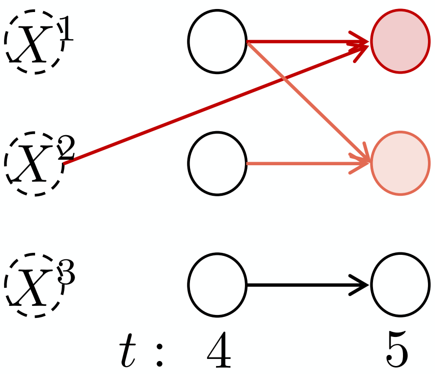

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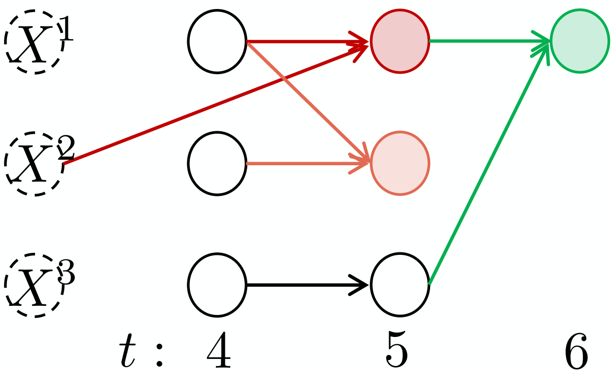

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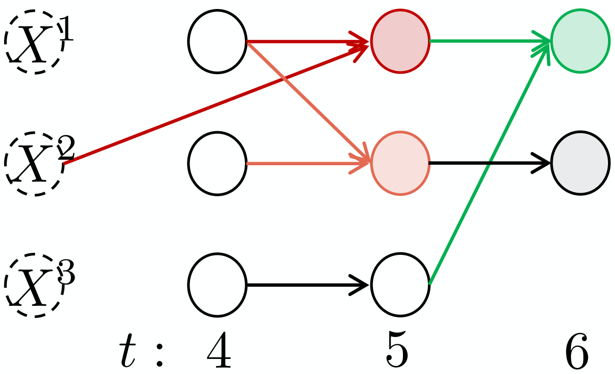

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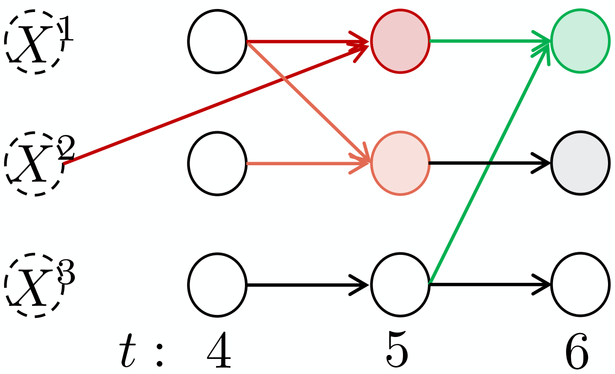

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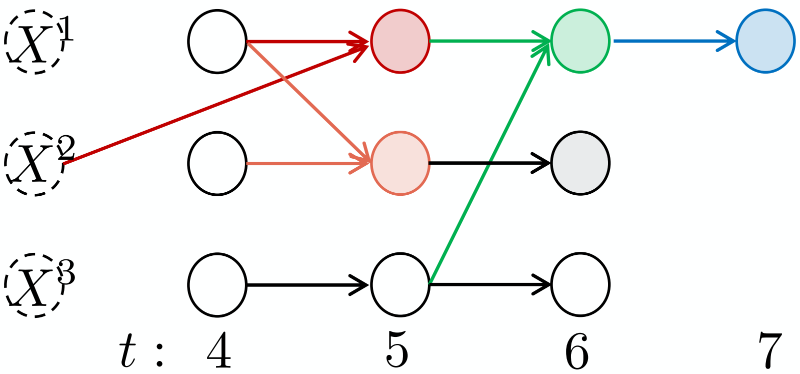

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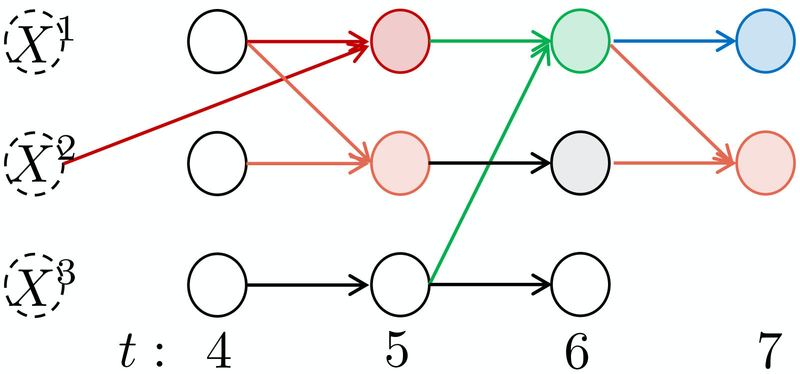

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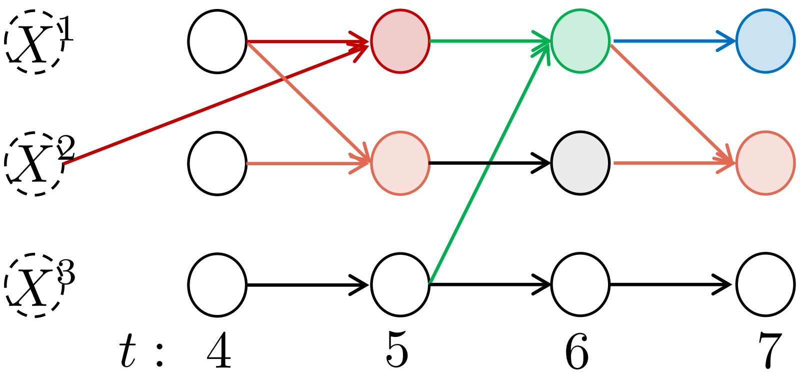

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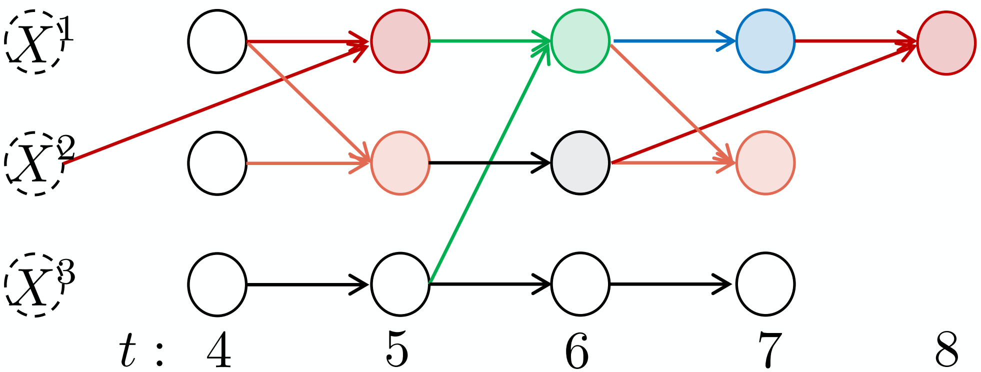

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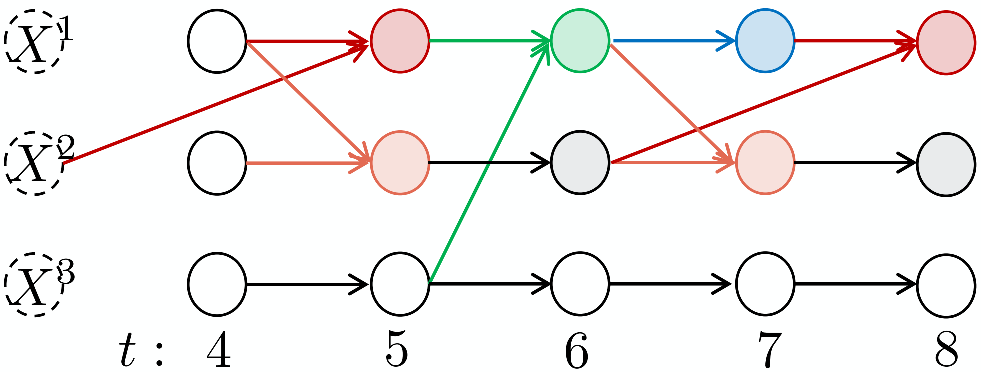

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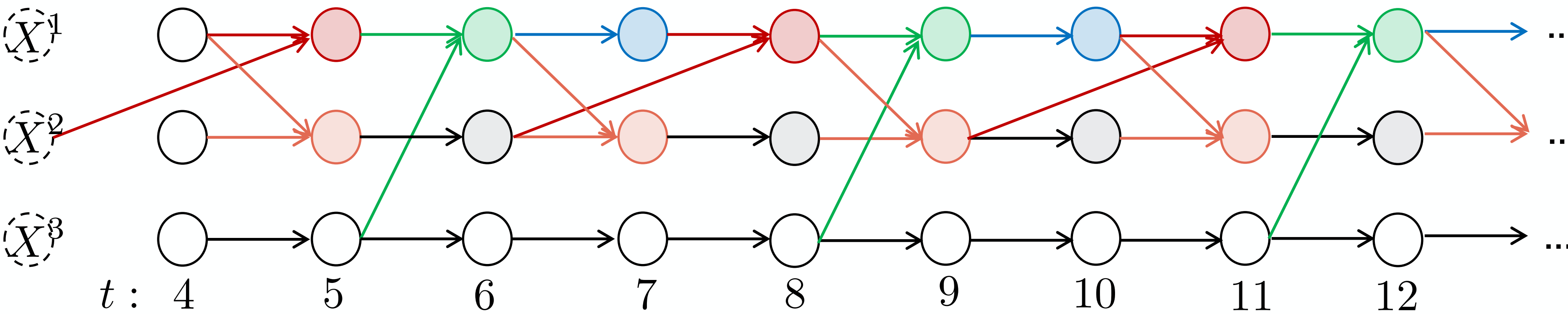

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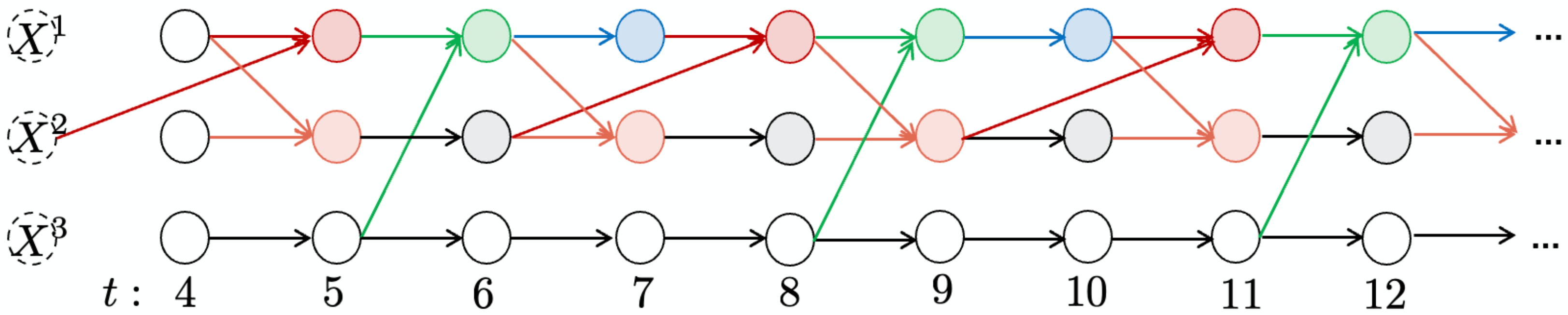
PCMCI$_\Omega$

❖ A constraint-based method designed for Semi-Stationary SCM:



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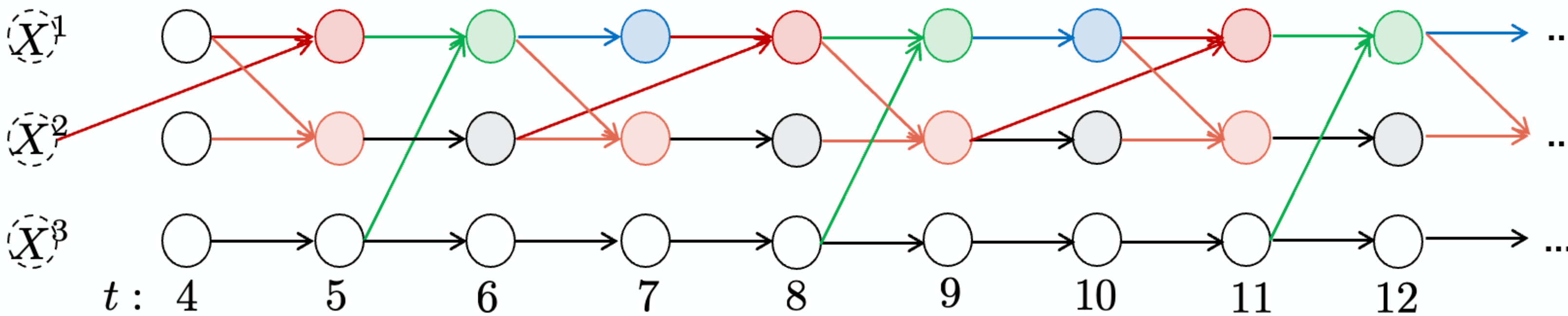

Definition. *Time Partition*

A *time partition* $\Pi^j(T)$ of $X^j \in V$ in Semi-Stationary SCM with periodicity ω_j is a way of dividing all time points $t \in [T]$ into a collection of non-overlapping non-empty subsets $\{\Pi_k^j(T)\}_{k \in [\omega_j]}$ such that:

$$\Pi_k^j(T) := \{t : \tau_{\max} + 1 \leq t \leq T, (t \bmod \omega_j) + 1 = k\}$$

Variables in $\{X_t^j\}_{t \in \Pi_k^j(T)}$ share the same causal mechanism.

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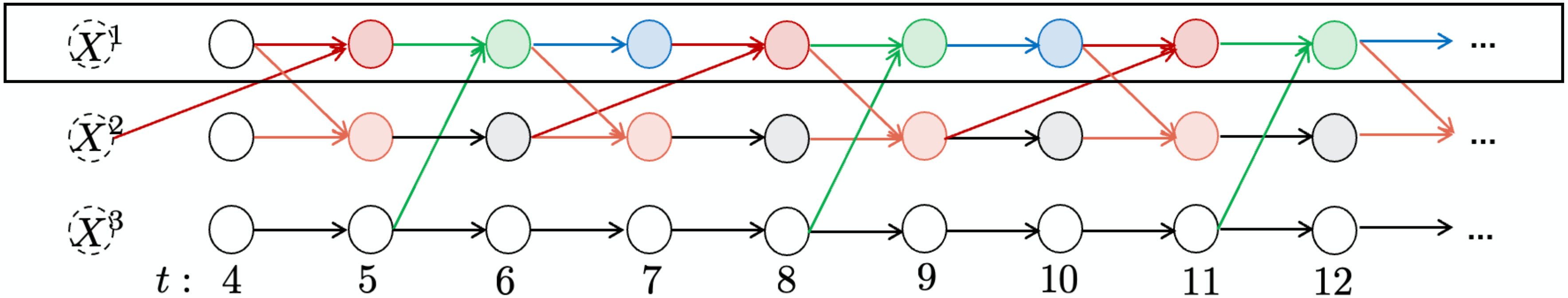

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$\Pi_1^1(T)$

$\omega_1 = 3$ $\Pi_2^1(T)$

$\Pi_3^1(T)$

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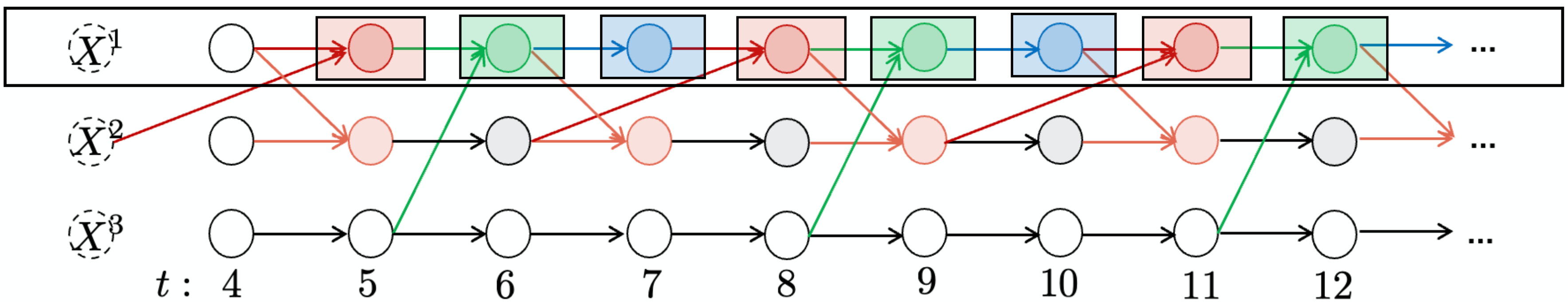

E.g., find $Pa(X_t^1)$ on the correct time partition with $\omega_1 = 3$.

$\{X_t^1\}_{t\in\Pi_1^1(T)} \xrightarrow{Pa(X_t^1)} \{X_{t-1}^1, X_{t-2}^2\}_{t\in\Pi_1^1(T)}$

$\{X_t^1\}_{t\in\Pi_2^1(T)} \xrightarrow{Pa(X_t^1)} \{X_{t-1}^1, X_{t-1}^3\}_{t\in\Pi_2^1(T)}$

$\{X_t^1\}_{t\in\Pi_3^1(T)} \xrightarrow{Pa(X_t^1)} \{X_{t-1}^1\}_{t\in\Pi_3^1(T)}$

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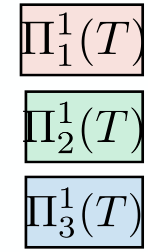
$\omega_1 = 3$



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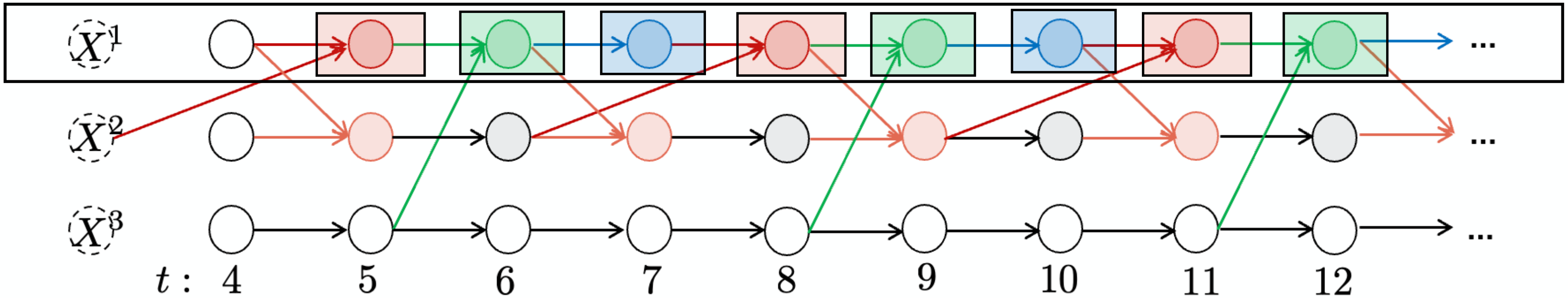

Question: How to estimate ω_j?

Claim: For a univariate time series $X^j \in V$ in a Semi-Stationary SCM with periodicity ω_j, $\hat{\omega}_j \neq \omega_j$ will lead to a denser graph.

PCMCI$_\Omega$

$\omega_1 = 3$

$\hat{\omega}_1 = 2$ $\hat{\Pi}_1^1(T)$

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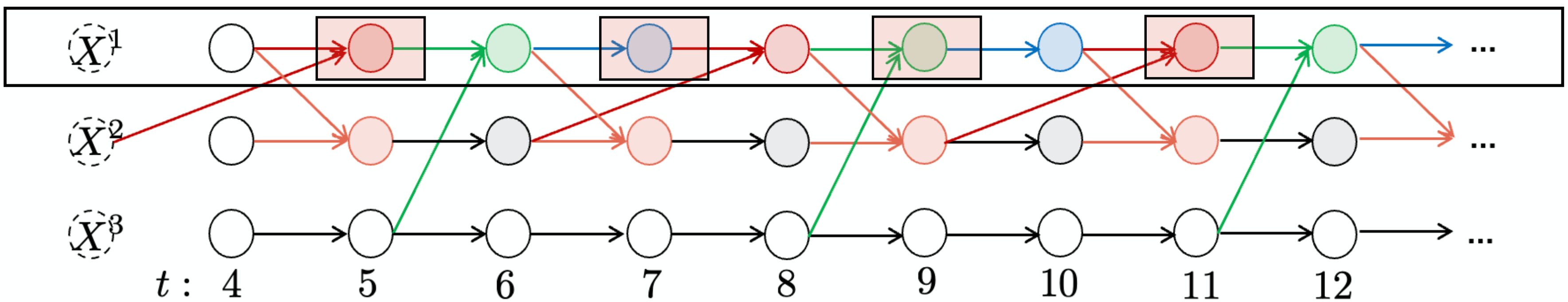

For a univariate time series $X^j \in V$ in a Semi-Stationary SCM with periodicity ω_j, $\hat{\omega}_j \neq \omega_j$ will lead to a denser graph.

E.g., find $Pa(X_t^1)$ on the wrong time partition with $\hat{\omega}_1 = 2$.

Use samples $\{X_t^1\}_{t \in \hat{\Pi}_1^1(T)}$ $\xrightarrow{Pa(X_t^1)}$

- $\{X_{t-1}^1, X_{t-2}^2\}_{t \in \hat{\Pi}_1^1(T)}$
- $\{X_{t-1}^1, X_{t-1}^3\}_{t \in \hat{\Pi}_1^1(T)}$
- $\{X_{t-1}^1\}_{t \in \hat{\Pi}_1^1(T)}$
- ...

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Thank You

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