LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

INSTITUTE OF AI IN MANAGEMENT

Sharp Bounds for Generalized Causal Sensitivity Analysis

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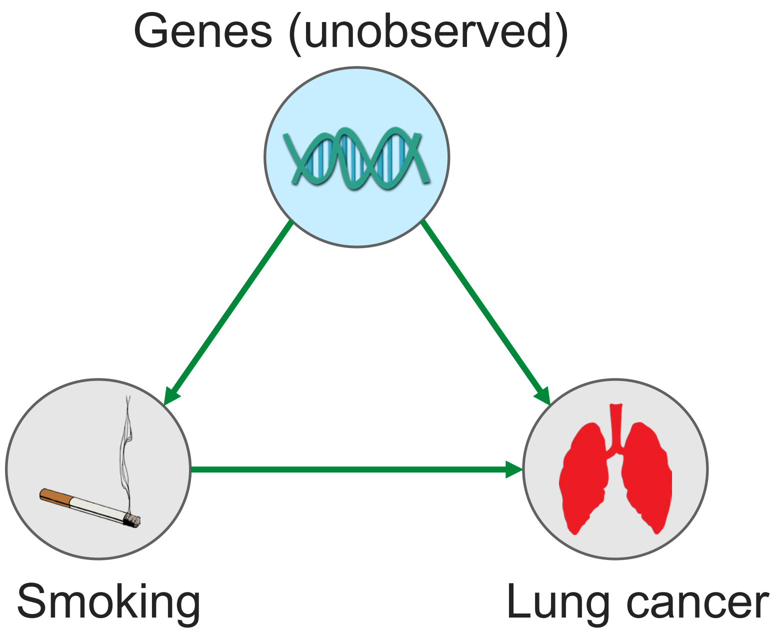
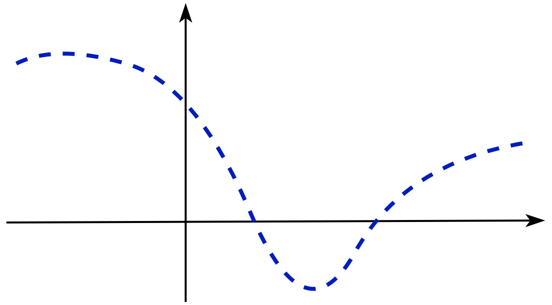
Motivation – Partial identification



Point identification

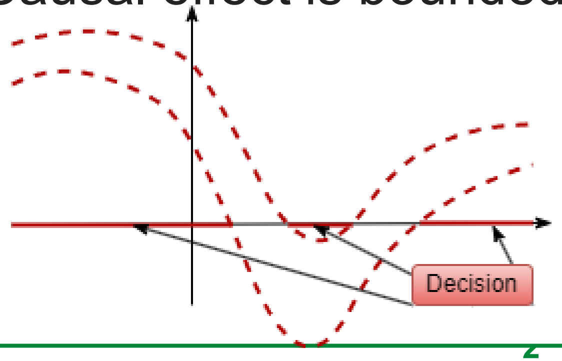
- Causal effect is identifiable



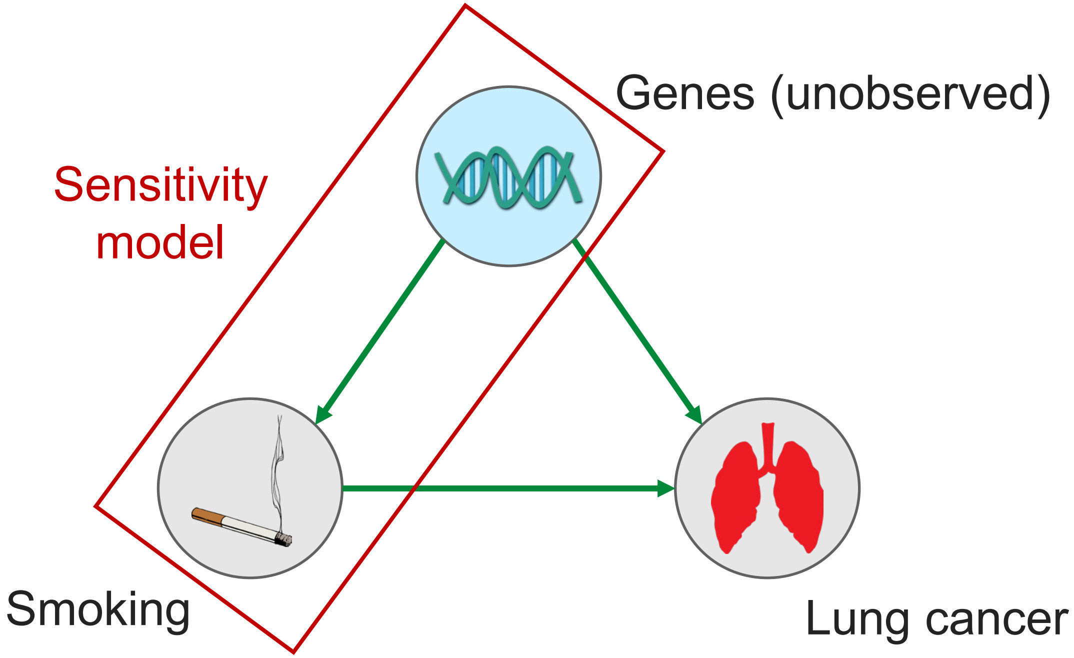
Unobserved confounding

Partial identification

- Causal effect is bounded



Motivation – Sensitivity analysis



- Large effect from smoking on lung cancer in the observational data
- Can the effect be fully explained by unobserved confounders (genes)?
- Cornfield (1959)[1]: No! *To fully explain away the observed effect, the genes would need to have an implausibly large effect on smoking.*

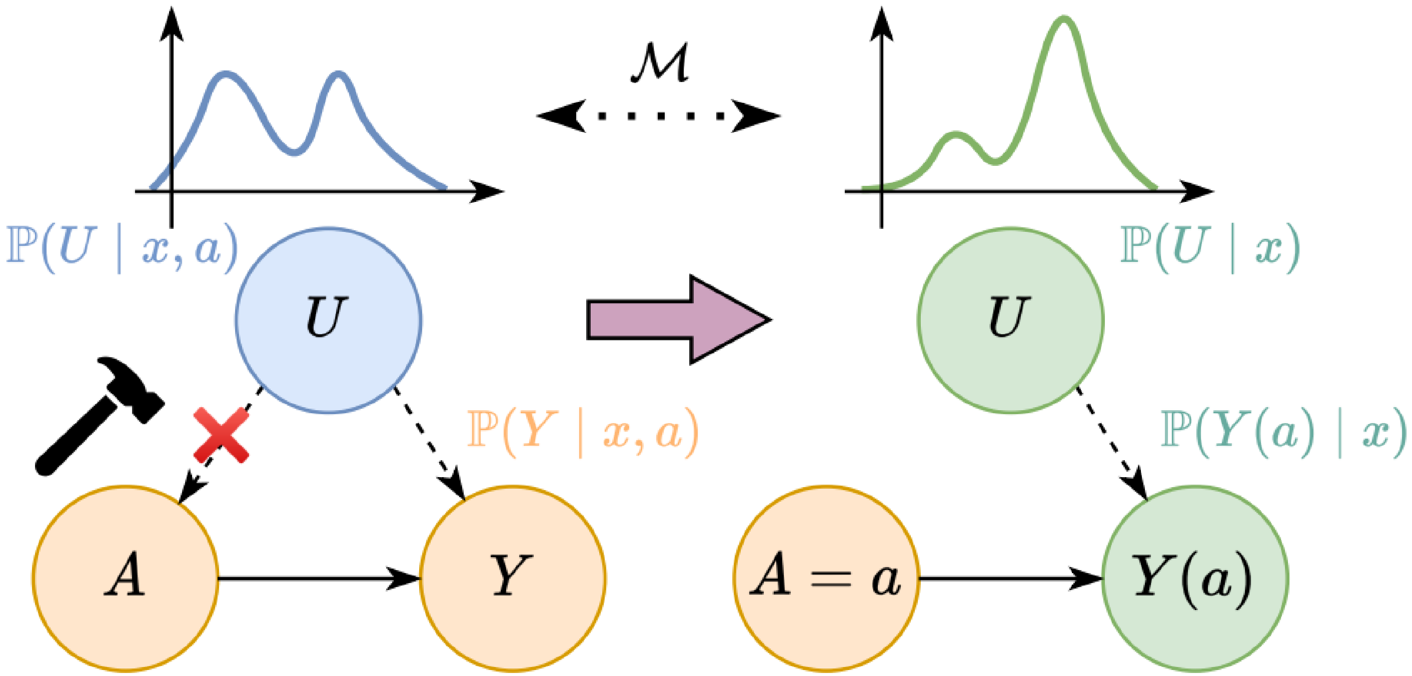
1. J. Cornfield et al., Smoking and lung cancer: Recent evidence and a discussion of some questions. J. Nat. Cancer. Inst. 22. 173-203 (1959)

Existing work & our contributions

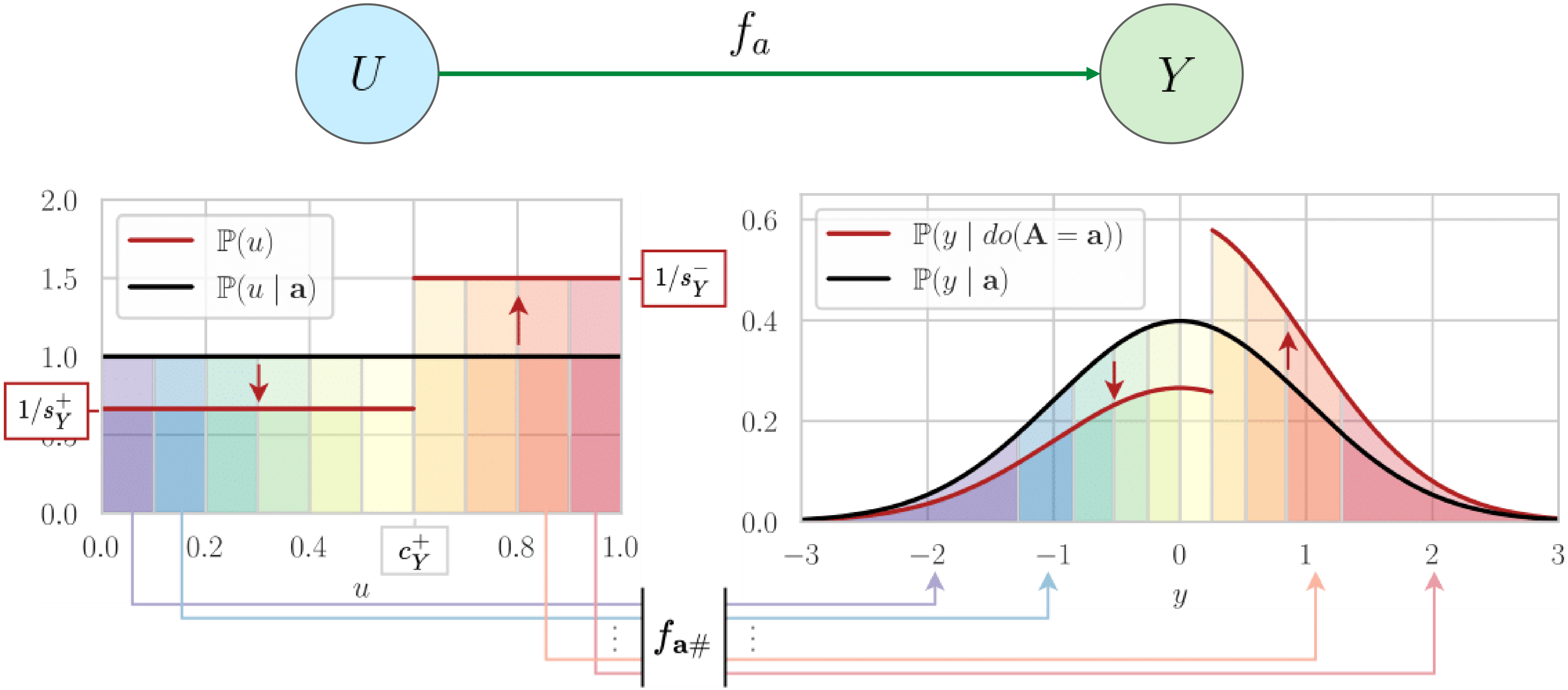
- Existing works derive (often closed-form) bounds for the marginal sensitivity model (MSM)
- Most existing methods only work for **(conditional) average treatment effects** and **binary treatments**
- We propose a novel approach to causal sensitivity analysis
  - Interpretation of the bounding problem via probability mass transport
  - Sharp bounds for a variety of causal queries and treatment types: CATE, dose-response function, distributional effects, mediation/ path analysis

→ **Unified approach to causal sensitivity analysis under MSM**

Basic idea


$\mathcal{M}$
$\mathbb{P}(U \mid x, a)$
U
$\mathbb{P}(Y \mid x, a)$
A
Y
$\mathbb{P}(U \mid x)$
U
$\mathbb{P}(Y(a) \mid x)$
$A = a$
$Y(a)$

Bounding as probability mass transport



MSM bounds in more general settings

- Basic idea extends to more general settings (details in paper):
  - Observed confounders
  - (Multiple) discrete mediators
  - Arbitrary (multidimensional) unobserved confounders
- For binary treatments and (conditional) average treatment effects, we obtain the same bounds as Dorn and Guo (2023)[2]
  - They proved optimality by applying the Neyman-Pearson Lemma from statistical testing theory

Algorithm 1: Causal sensitivity analysis with mediators

Input : Causal query $Q(\mathbf{x}, \bar{\mathbf{a}})$, GMSM $\mathcal{S}$ with bounds s_W^+ and s_W^-.
Output : Upper bound $Q^+(\mathbf{x}, \mathbf{a}, \mathcal{S})$

`// Outcome bound`

$c_W^+ \leftarrow \frac{(1-s_W^-)s_W^+}{s_W^+ - s_W^-}$ for $W \in \mathbf{M} \cup \{Y\}$

$Q_{\ell+1}^+(\bar{\mathbf{m}}_\ell) \leftarrow \mathcal{D}\left(\mathbb{P}_+^Y(\cdot \mid \mathbf{x}, \bar{\mathbf{m}}_\ell, \mathbf{a}_{\ell+1})\right)$ for $\bar{\mathbf{m}}_\ell \in supp(\bar{\mathbf{M}}_\ell)$

`// Adjusting for confounding in mediators`

for $i \in \{\ell, \dots, 1\}$ **do**
  for $\bar{\mathbf{m}}_{i-1} \in \mathrm{Im}(\bar{\mathbf{M}}_{i-1})$ **do**
    $\pi \leftarrow$ Permutation map in ascending order of $\left(Q_{i+1}^+(\bar{\mathbf{m}}_{i-1}, \pi(m_i))\right)_{m_i \in supp(M_i)}$
    $\widetilde{F}(m_i) \leftarrow \sum_{m:\pi(m) \leq m_i} \mathbb{P}(M_i = m \mid \mathbf{x}, \bar{\mathbf{m}}_{i-1}, \mathbf{a}_i)$

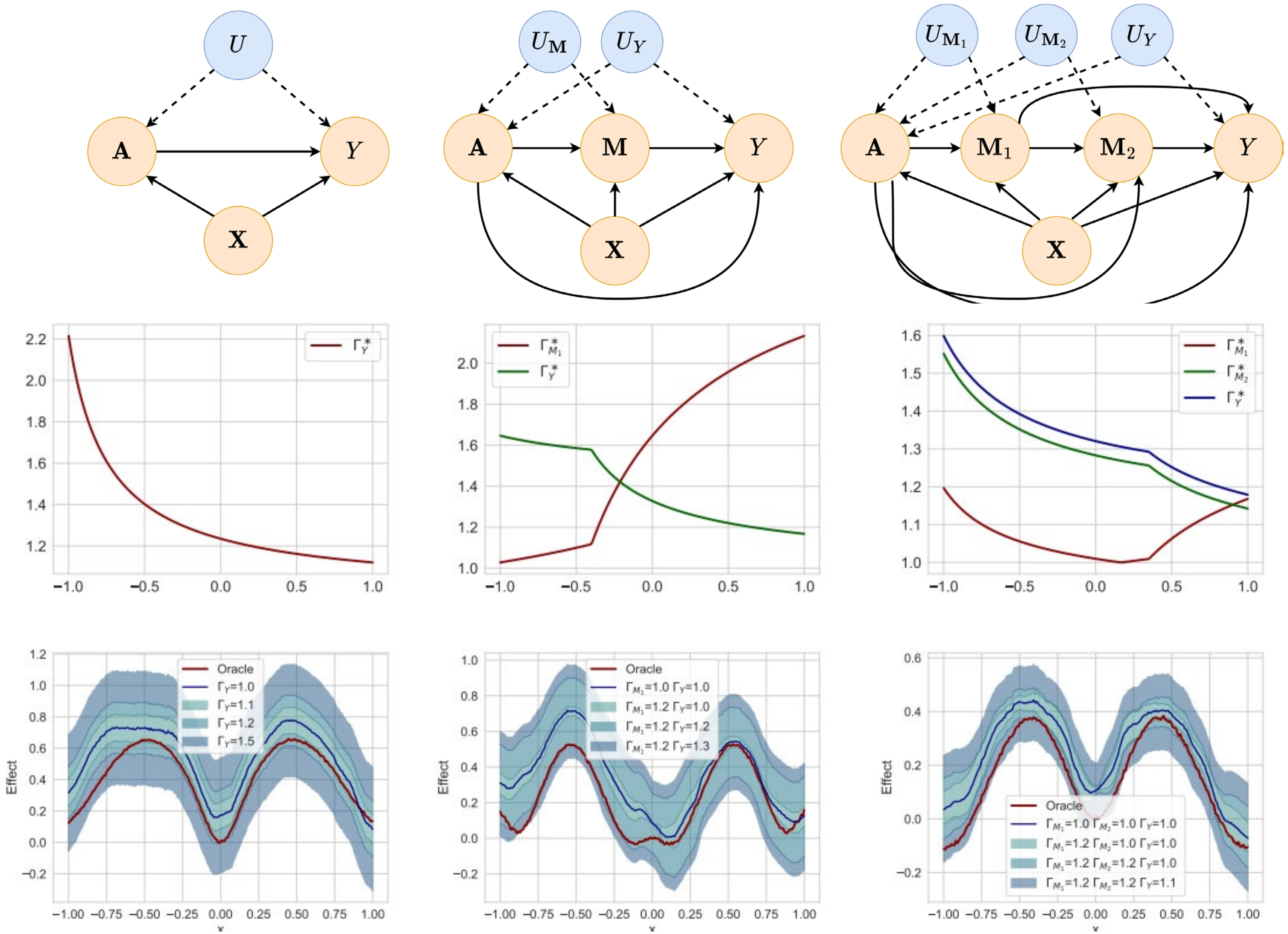
$$\mathbb{P}_+(m_i) \leftarrow \begin{cases} (1/s_{M_i}^+)\mathbb{P}(m_i \mid \mathbf{x}, \bar{\mathbf{m}}_{i-1}, \mathbf{a}_i), & \text{if } \widetilde{F}(\pi(m_i)) < c_{M_i}^+, \\ (1/s_{M_i}^-)\mathbb{P}(m_i \mid \mathbf{x}, \bar{\mathbf{m}}_{i-1}, \mathbf{a}_i), & \text{if } \widetilde{F}(\pi(m_i) - 1) > c_{M_i}^+, \\ (1/s_{M_i}^+)\left(c_{M_i}^+ - \widetilde{F}(\pi(m_i) - 1)\right) + (1/s_{M_i}^-)\left(\widetilde{F}(\pi(m_i)) - c_{M_i}^+\right), & \text{else.} \end{cases}$$

    $Q_i^+(\bar{\mathbf{m}}_{i-1}) \leftarrow \sum_{m_i} Q_{i+1}^+(\bar{\mathbf{m}}_{i-1}, m_i)\, \mathbb{P}_+(m_i)$
  end
end

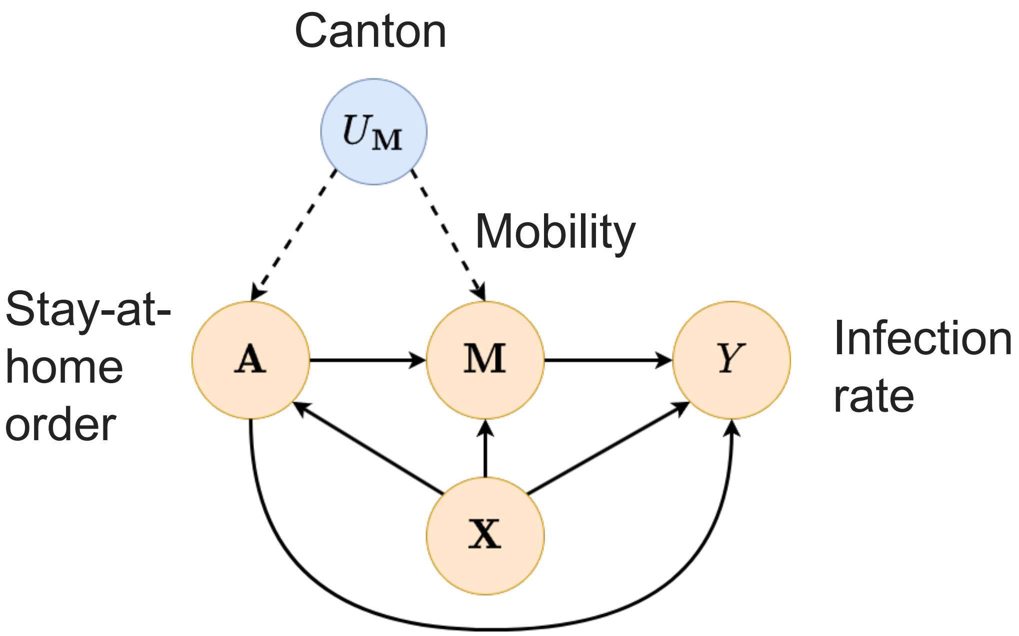
$Q^+(\mathbf{x}, \bar{\mathbf{a}}, \mathcal{S}) \leftarrow Q_1^+$

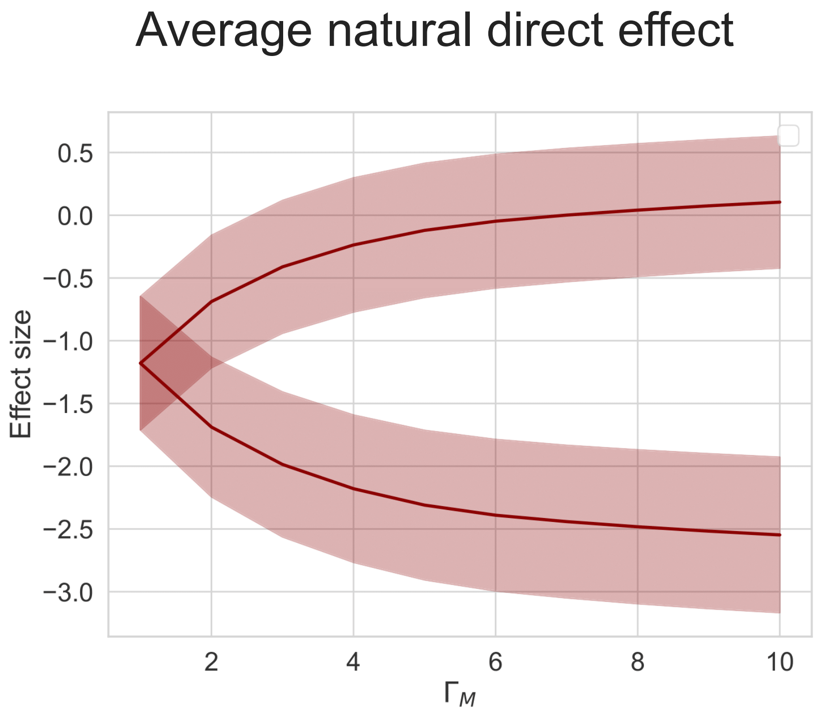
2. J. Dorn and K. Guo, Sharp sensitivity analysis for inverse propensity weighting via quantile balancing. Journal of the American Statistical Association (2023)

Experimental results: synthetic data



Results on real-world data: Covid-19 pandemic in Switzerland





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Link to paper

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