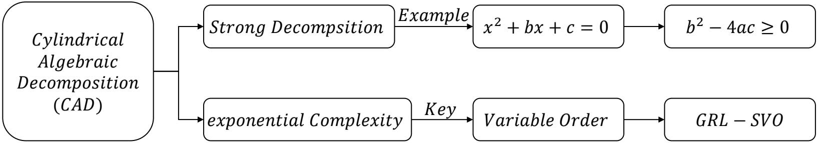
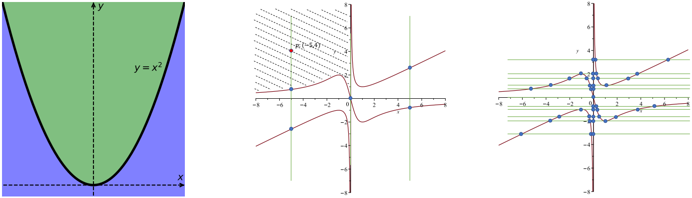
Suggesting Variable Order for Cylindrical Algebraic Decomposition via Reinforcement Learning

Fuqi Jia*, Yuhang Dong*, Minghao Liu, Pei Huang, Feifei Ma, Jian Zhang

November 13, 2023


Cylindrical Algebraic Decomposition (CAD)
Strong Decompsition
Example
$x^2 + bx + c = 0$
$b^2 - 4ac \geq 0$
exponential Complexity
Key
Variable Order
GRL − SVO

Cylindrical Algebraic Decomposition (CAD)



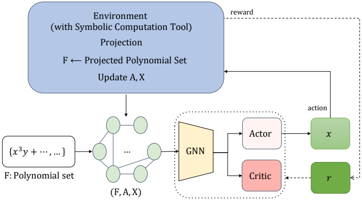
(a) 3 cells (b) $x \prec y$: 13 cells (c) $y \prec x$: 89 cells

Figure 1: Examples of CAD. Figure 1a shows cells of $\{y - x^2\}$. Figure 1b and 1c are different variable orders on $\{x^3y + 4x^2 + xy, -x^2 + 2xy - 1\}$.

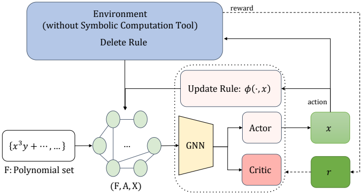
Existing Heuristics

Table 1: Classification on Existing Heuristics

	EB (Expert-Based)	LB (Learning-Based)
UP (Utilizing *project*)	*sotd* [7], *ndrr* [8], *gmods* [9]	***GRL-SVO(UP)***
NUP (Not Utilizing *project*)	*brown* [10], *triangular* [11], *chord* [12]	*EMLP* [13], *PVO* [14], ***GRL-SVO(NUP)***

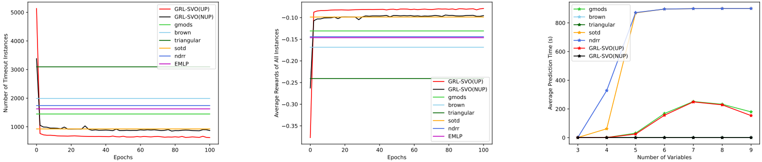


(a) GRL-SVO(UP)



(b) GRL-SVO(NUP)

Figure 2: The architecture of GRL-SVO(UP) and GRL-SVO(NUP) where $\phi(\cdot, x) = MLP(CONCAT(\cdot, x))$ for updating the embedding for the neighbours of x. The dashed lines represent that it will be only utilized in training mode.



(a) (b) (c)

Figure 3: The performance of all heuristics. Figure 3a and 3b correspond to the training phase, and the horizontal lines represent the timeout instances of corresponding heuristics on the training set. Figure 3c is the PT graph over number of variables.

Table 2: The performance of all heuristics. The dash "-" indicates that the method does not support the category.

Categories		NUP						UP			
		EB		LB				EB			LB
		brown	*triangular*	*EMLP*	*PVO(brown)*	*PVO(triangular)*	*GRL-SVO(NUP)*	*sotd*	*ndrr*	*gmods*	*GRL-SVO(UP)*
3-var(test)	*#SI*	1620	1504	1686	-	-	1772	1784	1663	1693	**1798**
	AVG.T	171.41	228.32	140.87	-	-	94.87	91.47	148.92	124.06	**78.06**
	AVG.N	2427.74	2669.67	2390.68	-	-	2166.67	2149.07	2007.98	2195.80	2089.68
4-var	*#SI*	415	376	-	408	392	443	**625**	488	513	533
	AVG.T	352.87	394.71	-	360.33	376.71	314.57	**85.12**	292.06	215.48	191.45
	AVG.N	5241.95	5585.90	-	5323.83	5582.46	5131.40	**3925.28**	4248.45	4849.18	4764.50
5-var	*#SI*	236	202	-	242	218	238	43	27	329	**346**
	AVG.T	434.52	494.37	-	418.34	465.43	420.51	827.47	853.75	238.01	**207.79**
	AVG.N	12310.70	13224.18	-	11795.90	12555.82	12090.49	14538.69	14845.67	10466.79	**9744.58**
6-var	*#SI*	175	149	-	180	160	202	5	5	273	**306**
	AVG.T	501.75	552.14	-	490.73	527.86	439.62	889.16	889.97	284.40	**214.55**
	AVG.N	20639.07	20440.23	-	20181.98	19290.37	19302.97	23298.50	23329.10	17561.67	**16715.20**
7-var	*#SI*	163	118	-	-	-	153	1	1	270	**297**
	AVG.T	548.15	631.85	-	-	-	552.47	897.75	897.79	313.73	**245.57**
	AVG.N	27790.31	27795.79	-	-	-	27302.28	30452.64	30456.31	24465.89	**22432.30**
8-var	*#SI*	173	138	-	-	-	172	0	0	310	**345**
	AVG.T	601.90	654.20	-	-	-	597.09	900.00	900.00	372.34	**322.80**
	AVG.N	39382.26	40679.57	-	-	-	38815.98	43112.02	43112.02	34016.98	**33505.21**
9-var	*#SI*	151	125	-	-	-	158	0	0	286	**325**
	AVG.T	649.41	690.29	-	-	-	625.69	900.00	900.00	431.11	**374.78**
	AVG.N	48273.67	49832.78	-	-	-	46946.09	52173.03	52173.03	42594.25	**42270.91**
SMT-LIB (3-var)	*#SI*	1770	1763	1675	-	-	1766	1750	1694	**1772**	**1772**
	AVG.T	20.33	23.68	83.09	-	-	22.38	34.38	65.10	**18.32**	18.53
	AVG.N	4449.79	5070.46	7661.07	-	-	4104.43	**3672.21**	4140.72	3873.22	3946.84
SMT-LIB (4-var to 6-var)	*#SI*	374	372	-	372	372	364	356	339	**379**	**379**
	AVG.T	86.03	89.95	-	88.32	88.98	91.17	105.18	142.32	**59.96**	67.51
	AVG.N	24596.20	24260.88	-	23039.49	22730.34	21040.09	**16896.16**	21013.25	17388.52	18894.51
SMT-LIB (7-var to 9-var)	*#SI*	13	12	-	-	-	12	11	11	**16**	14
	AVG.T	308.14	377.32	-	-	-	339.90	541.53	588.33	**260.53**	329.91
	AVG.N	53971.24	58675.94	-	-	-	51570.88	51470.41	62185.82	**50381.12**	56312.41

Thanks!

References I

[1] George E Collins. "Quantifier elimination for real closed fields by cylindrical algebraic decompostion". In: *Automata theory and formal languages*. Springer, 1975, pp. 134–183.

[2] Mark de Berg et al. *Computational Geometry: Algorithms and Applications, 3rd Edition*. Springer, 2008. ISBN: 9783540779735. URL: https://www.worldcat.org/oclc/227584184.

[3] Steven M. LaValle. *Planning Algorithms*. Cambridge University Press, 2006. ISBN: 9780511546877. DOI: 10.1017/CBO9780511546877. URL: http://planning.cs.uiuc.edu/.

References II

[4] Dejan Jovanovic and Leonardo Mendonça de Moura. "Solving Non-linear Arithmetic". In: *IJCAR 2012*. Vol. 7364. Lecture Notes in Computer Science. Springer, 2012, pp. 339–354. DOI: 10.1007/978-3-642-31365-3_27. URL: https://doi.org/10.1007/978-3-642-31365-3_27.

[5] Dejan Jovanovic. "Solving Nonlinear Integer Arithmetic with MCSAT". In: *VMCAI 2017*. Vol. 10145. Lecture Notes in Computer Science. Springer, 2017, pp. 330–346. DOI: 10.1007/978-3-319-52234-0_18. URL: https://doi.org/10.1007/978-3-319-52234-0_18.

[6] Fuqi Jia et al. "Improving Bit-Blasting for Nonlinear Integer Constraints". In: *ISSTA 2023*. ACM, 2023, pp. 14–25. DOI: 10.1145/3597926.3598034. URL: https://doi.org/10.1145/3597926.3598034.

References III

[7] Andreas Dolzmann, Andreas Seidl, and Thomas Sturm. "Efficient Projection Orders for CAD". In: *ISSAC 2004*. ACM, 2004, pp. 111–118. DOI: 10.1145/1005285.1005303. URL: https://doi.org/10.1145/1005285.1005303.

[8] Russell J. Bradford et al. "Optimising Problem Formulation for Cylindrical Algebraic Decomposition". In: *CICM 2013*. Vol. 7961. Lecture Notes in Computer Science. Springer, 2013, pp. 19–34. DOI: 10.1007/978-3-642-39320-4_2. URL: https://doi.org/10.1007/978-3-642-39320-4_2.

References IV

[9] Tereso del Río and Matthew England. “New Heuristic to Choose a Cylindrical Algebraic Decomposition Variable Ordering Motivated by Complexity Analysis”. In: *CASC 2022*. Vol. 13366. Lecture Notes in Computer Science. Springer, 2022, pp. 300–317. DOI: 10.1007/978-3-031-14788-3_17. URL: https://doi.org/10.1007/978-3-031-14788-3_17.

[10] Christopher W Brown. “Companion to the Tutorial: Cylindrical Algebraic Decomposition”. In: *United States Naval Academy* (2004).

References V

[11] Changbo Chen et al. "Computing the Real Solutions of Polynomial Systems with the RegularChains Library in Maple". In: *ACM Commun. Comput. Algebra* 45.3/4 (2011), pp. 166–168. DOI: 10.1145/2110170.2110174. URL: https://doi.org/10.1145/2110170.2110174.

[12] Haokun Li et al. "Choosing Better Variable Orderings for Cylindrical Algebraic Decomposition via Exploiting Chordal Structure". In: *J. Symb. Comput.* 116 (2023), pp. 324–344. DOI: 10.1016/j.jsc.2022.10.009. URL: https://doi.org/10.1016/j.jsc.2022.10.009.

References VI

[13] Matthew England and Dorian Florescu. "Comparing Machine Learning Models to Choose the Variable Ordering for Cylindrical Algebraic Decomposition". In: *CICM 2019*. Vol. 11617. Lecture Notes in Computer Science. Springer, 2019, pp. 93–108. DOI: 10.1007/978-3-030-23250-4_7. URL: https://doi.org/10.1007/978-3-030-23250-4_7.

[14] Changbo Chen, Zhangpeng Zhu, and Haoyu Chi. "Variable Ordering Selection for Cylindrical Algebraic Decomposition with Artificial Neural Networks". In: *ICMS 2020*. Vol. 12097. Lecture Notes in Computer Science. Springer, 2020, pp. 281–291. DOI: 10.1007/978-3-030-52200-1_28. URL: https://doi.org/10.1007/978-3-030-52200-1_28.