Neural Ideal Large Eddy Simulation: Modeling Turbulence with Neural Stochastic Differential Equations

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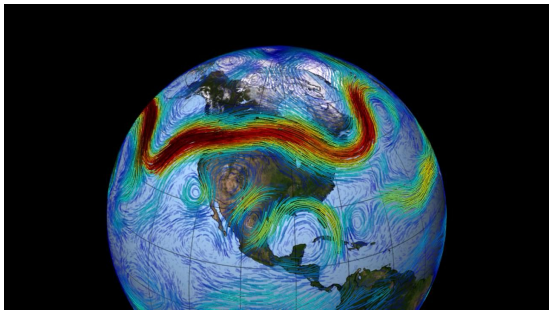
Closure modeling: motivation

- Chaotic physical flows are **expensive** to simulate accurately
  - Since they require resolving **multiple scales** spanning several orders of magnitude
- Practitioners resort to **coarse-grained** simulations
  - However they **veer off-course** since don't account for the effect of small scales on the large scales



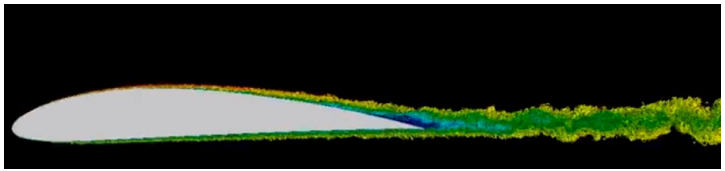


Image courtesy: NASA, Nek5000

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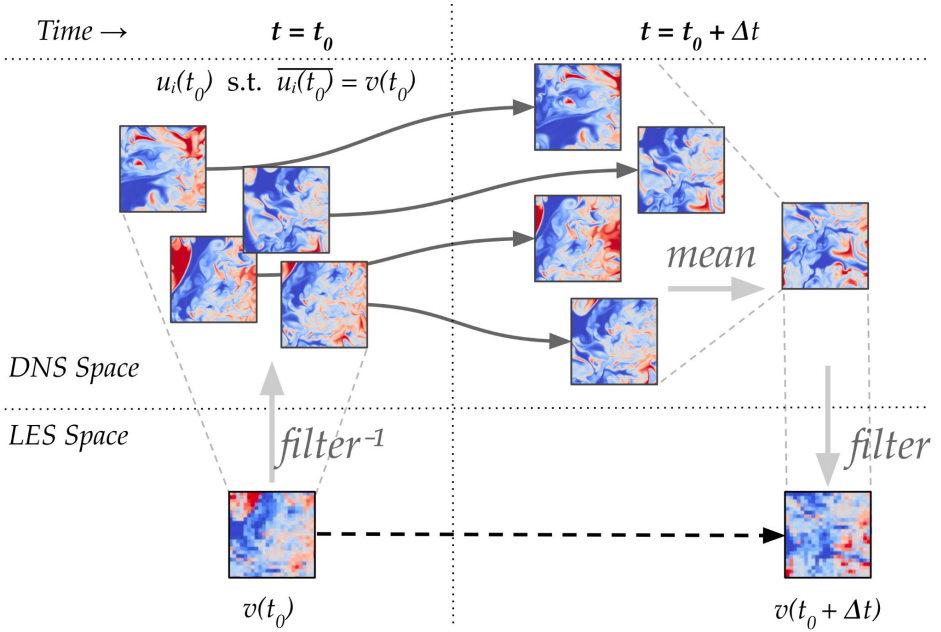
Ideal Large Eddy Simulation (LES)

- An LES, or reduced order, field v could represent any full-order field u such that $\overline{u} = v$
- Ideal LES (Langford and Moser 1999) takes the conditional average of *all possible* small scale effects consistent with the large scale state v
- Mathematically,

$$\frac{\partial v}{\partial t} = \mathbb{E}_{\pi_t}\left[\overline{\frac{\partial u}{\partial t}} \,\middle|\, \overline{u} = v\right]$$



Can we make architectural choices which bake in the inductive bias of Ideal LES?

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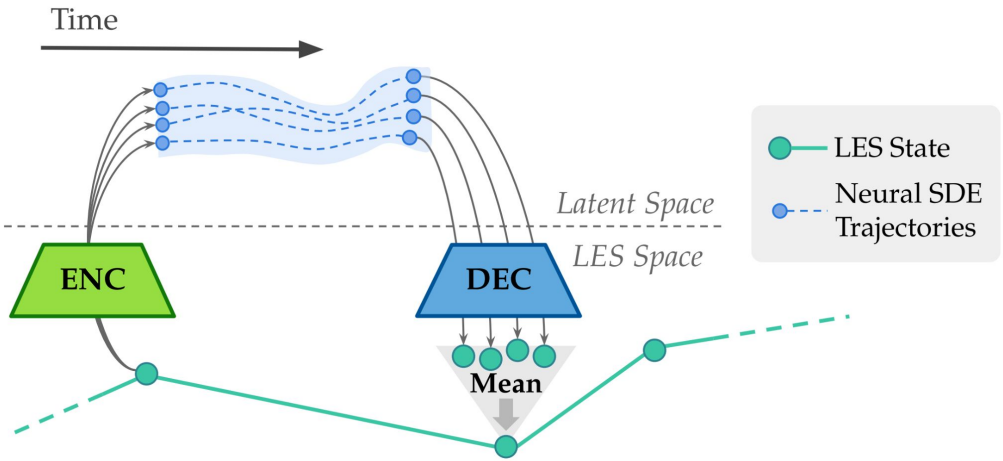
Neural ideal LES (niLES)

- Given LES states $\overline{u(t_0)}$ and $\overline{u(t_1)}$, model distribution of full-order trajectories on $[t_0, t_1]$
- Learn a data-driven neural SDE parameterized by a neural network evolving on a *fast* timescale τ
- SDE evolves a latent state much smaller dimensionality
- With enough data, SDE should discover the underlying solution manifold



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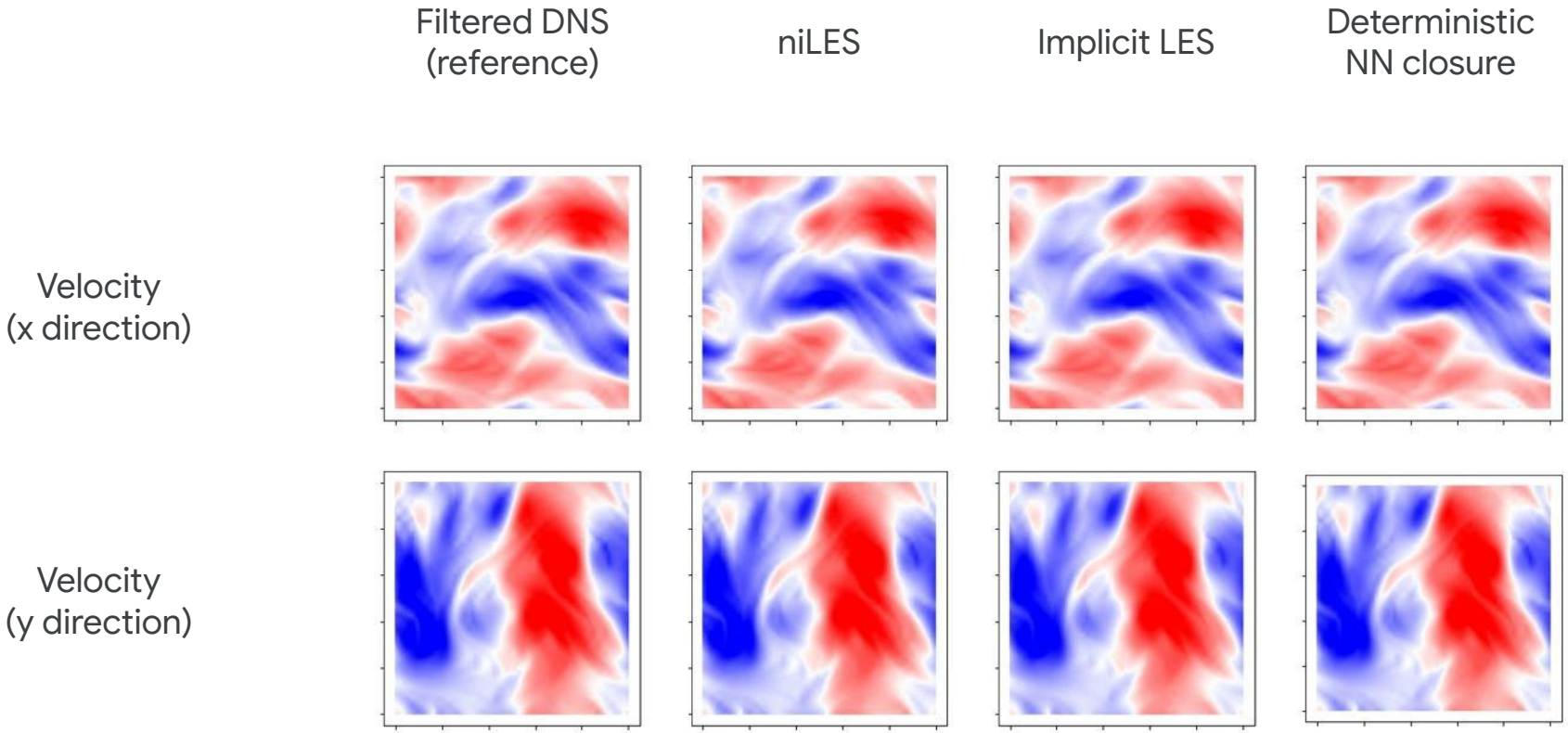
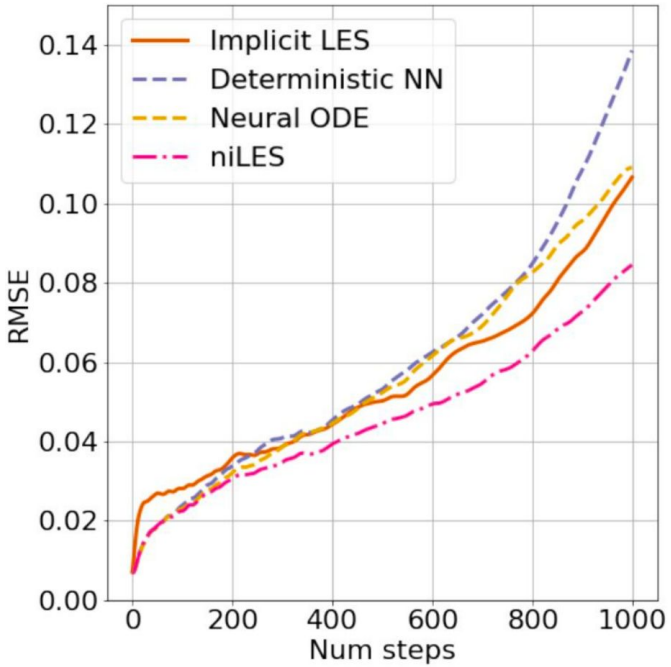
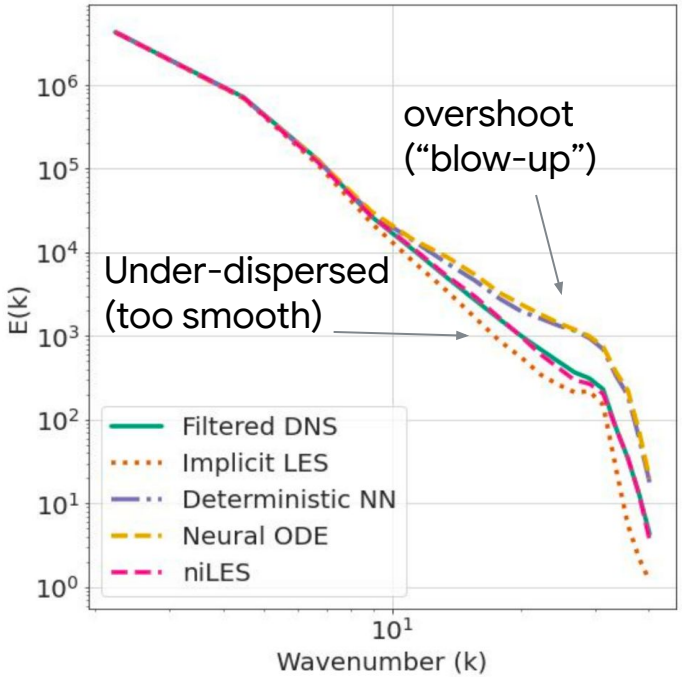
Results


Filtered DNS (reference)
niLES
Implicit LES
Deterministic NN closure
Velocity (x direction)
Velocity (y direction)

Accuracy and long-term statistics



RMSE over 1000 steps



TKE spectrum over 2500 steps

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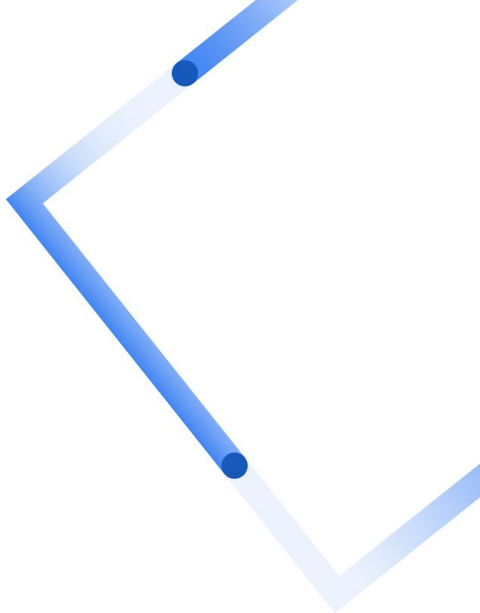
Thank You



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