Tensor Wheel Decomposition and Its Tensor Completion Application

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Outline

➤ Background and Related Works

➤ Motivation and TW Decomposition

➤ Numerical Application to Tensor Completion

➤ Experimental Results

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Background: Tensor Decompositions
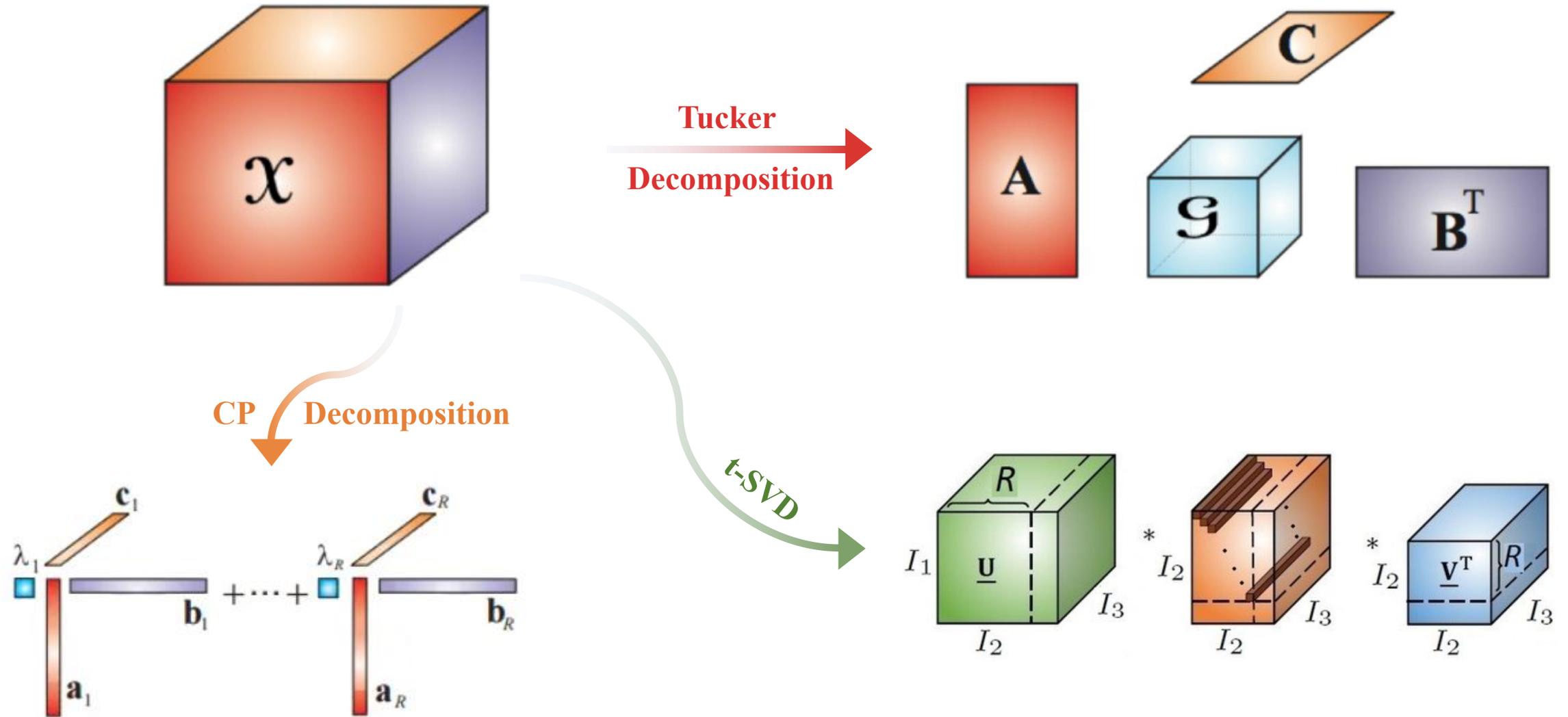
Related Works: Tensor Networks
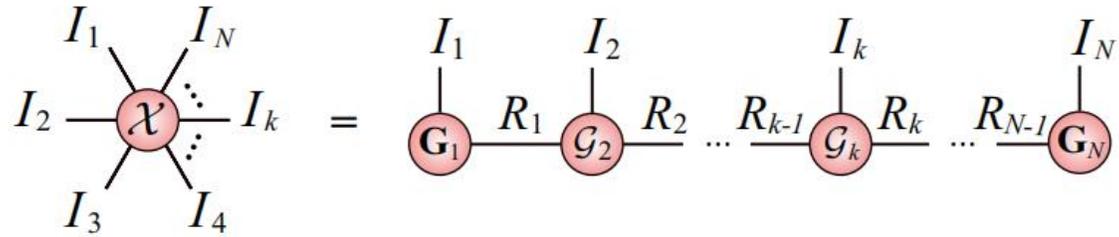
(a) TT decomposition
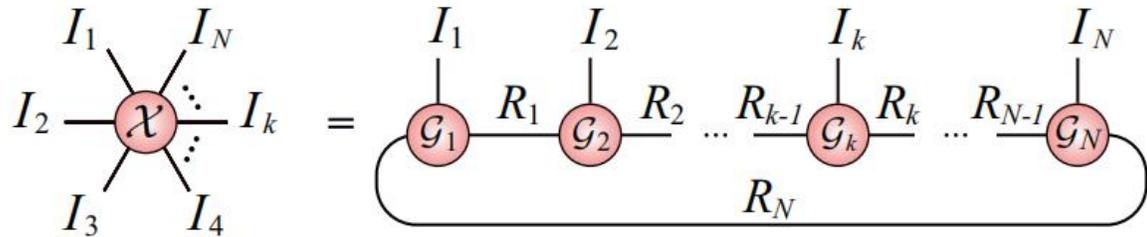
(b) TR decomposition
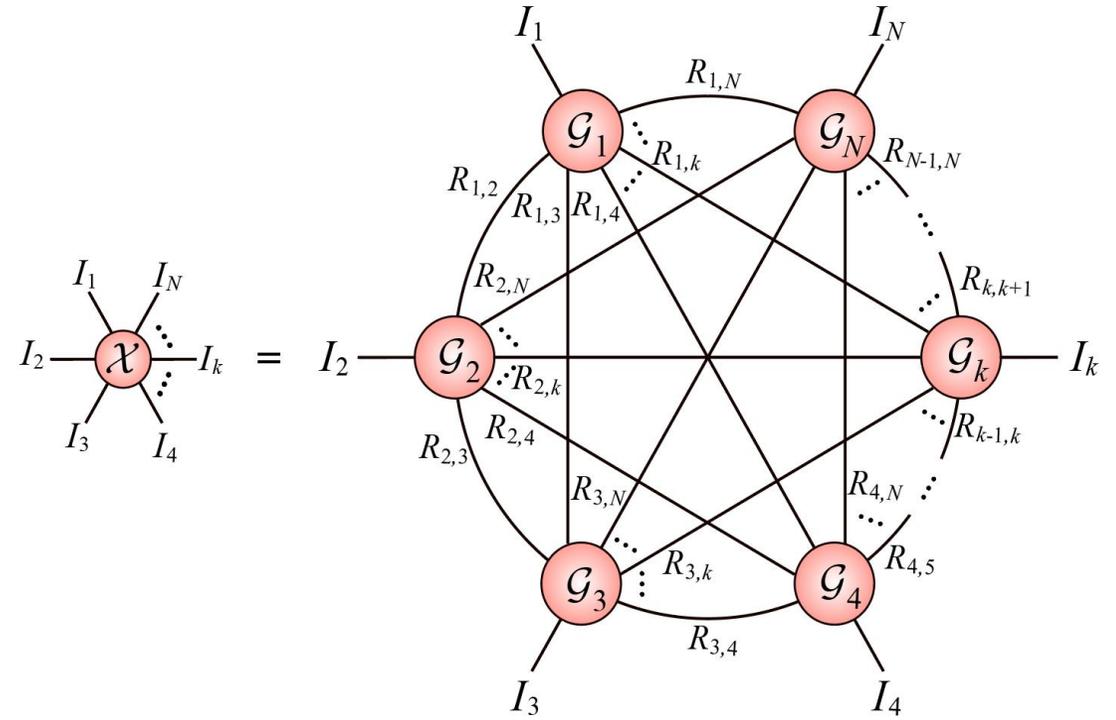
(c) FCTN decomposition

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Motivation

TT format
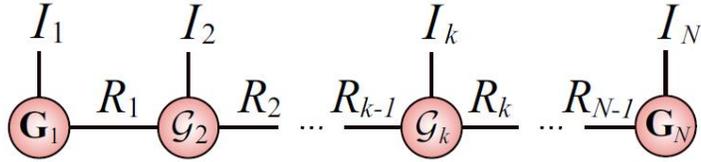
Generalization

TR format
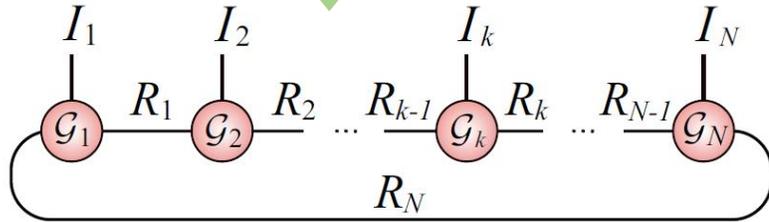
FCTN format
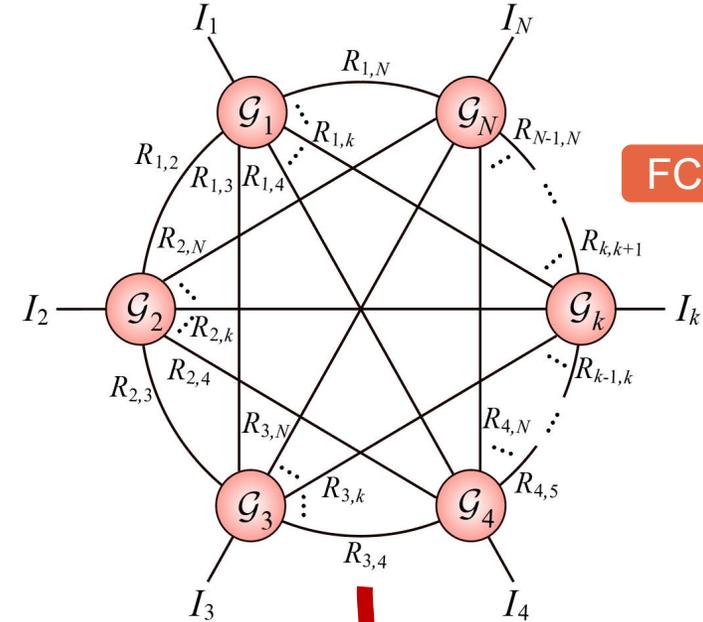
What topology?

If higher characterization capability

If smaller edge scaling

Tensor Wheel (TW) Decomposition
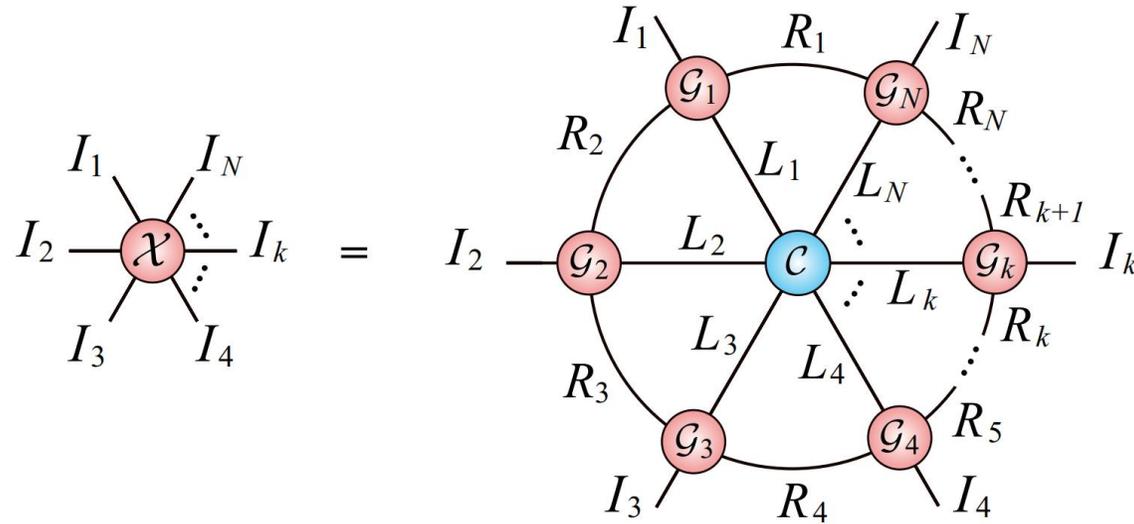
Wheel topology

● **element-wise relation:**

$$\mathcal{X}(i_1, i_2, \cdots, i_N) = \sum_{r_1=1}^{R_1} \sum_{r_2=1}^{R_2} \cdots \sum_{r_N=1}^{R_N} \sum_{l_1=1}^{L_1} \cdots \sum_{l_N=1}^{L_N} \{ \mathcal{G}_1(r_1, i_1, l_1, r_2) \mathcal{G}_2(r_2, i_2, l_2, r_3) \cdots$$
$$\mathcal{G}_k(r_k, i_k, l_k, r_{k+1}) \cdots \mathcal{G}_N(r_N, i_N, l_N, r_1) \mathcal{C}(l_1, l_2, \cdots, l_N) \}.$$

Tensor Wheel (TW) Decomposition
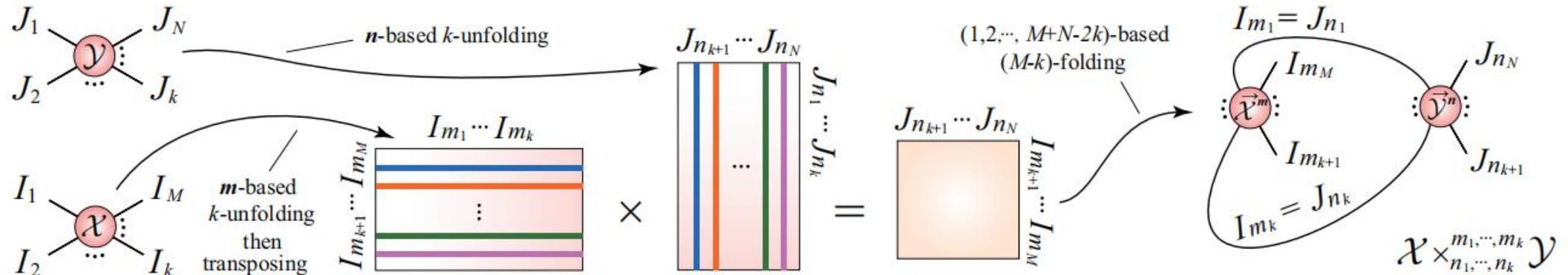
● **tensor-form relation:**

$$\mathcal{X} = \mathcal{G}_1 \times_1^4 \mathcal{G}_2 \times_1^6 \cdots \times_1^{2k} \mathcal{G}_k \times_1^{2k+2} \cdots \times_{1,4}^{2N,1} \mathcal{G}_N \times_{1,2,\cdots,N}^{2,4,\cdots,2N} \mathcal{C}.$$

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TW-TC Model and PAM-based Algorithm

● **TW-TC model:**

$$\min_{\mathcal{X},\mathcal{G}_{1:N},\mathcal{C}} \frac{1}{2}\|\mathcal{X} - \mathrm{TW}[\![\{\mathcal{G}_k\}_{k=1}^N;\mathcal{C}]\!]\|_F^2 + \iota(\mathcal{X}) \text{ with } \iota(\mathcal{X}) := \begin{cases} 0, & \mathcal{X} \in \{\mathcal{L} : \mathcal{P}_\Omega(\mathcal{L}) = \mathcal{P}_\Omega(\mathcal{F})\}; \\ \infty, & \text{otherwise} \end{cases}.$$

● **Iterative algorithm:**

$$\begin{cases} \mathcal{G}_k^{(t+1)} \in \underset{\mathcal{G}_k}{\arg\min} \left\{ \frac{1}{2}\|\mathcal{X}^{(t)} - \mathrm{TW}[\![\mathcal{G}_{1:k-1}^{(t+1)}, \mathcal{G}_k, \mathcal{G}_{k+1:N}^{(t)};\mathcal{C}^{(t)}]\!]\|_F^2 + \frac{\rho}{2}\|\mathcal{G}_k - \mathcal{G}_k^{(t)}\|_F^2 \right\}, \\ \mathcal{C}^{(t+1)} \in \underset{\mathcal{C}}{\arg\min} \left\{ \frac{1}{2}\|\mathcal{X}^{(t)} - \mathrm{TW}[\![\mathcal{G}_{1:N}^{(t+1)};\mathcal{C}]\!]\|_F^2 + \frac{\rho}{2}\|\mathcal{C} - \mathcal{C}^{(t)}\|_F^2 \right\}, \\ \mathcal{X}^{(t+1)} \in \underset{\mathcal{X}}{\arg\min} \left\{ \frac{1}{2}\|\mathcal{X} - \mathrm{TW}[\![\mathcal{G}_{1:N}^{(t+1)};\mathcal{C}^{(t+1)}]\!]\|_F^2 + \frac{\rho}{2}\|\mathcal{X} - \mathcal{X}^{(t)}\|_F^2 + \iota(\mathcal{X}) \right\}. \end{cases}$$

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Synthetic Data Completion
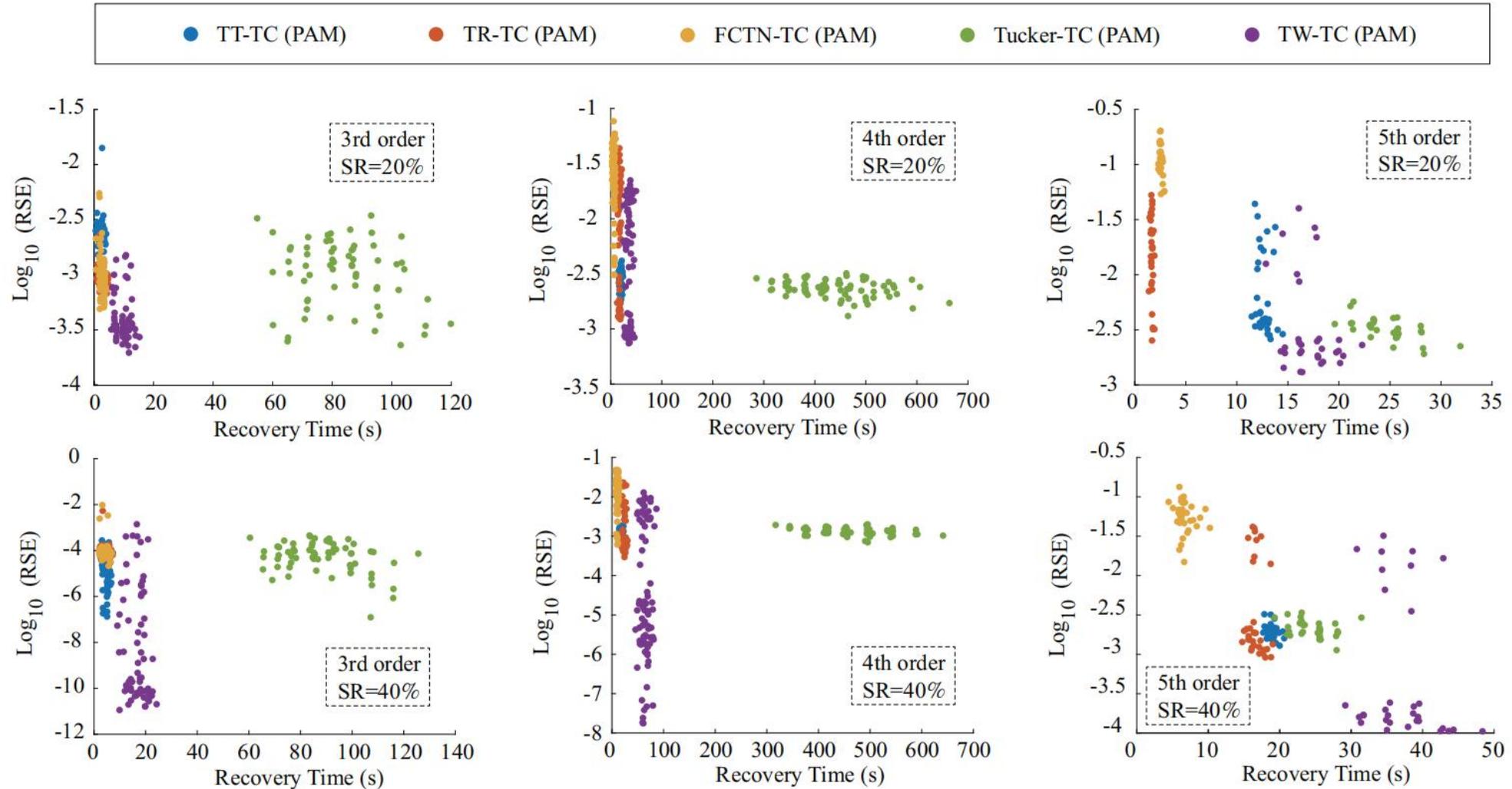
Real-world Data Completion: Visualization
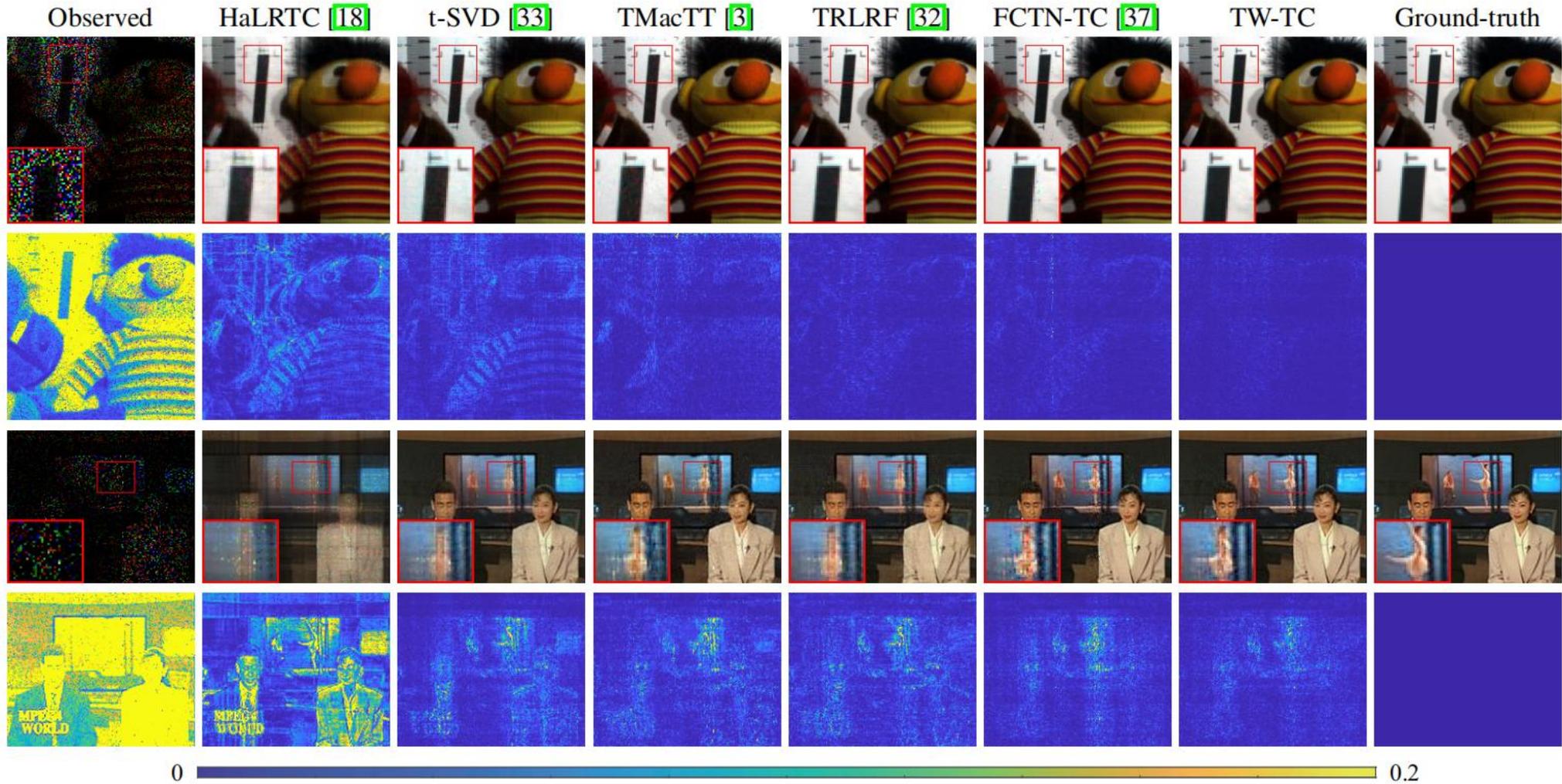
Real-world Data Completion: Numeralization

Data Index	–	Observed	HaLRTC [18]	t-SVD [33]	TMacTT [3]	TRLRF [32]	FCTN-TC [37]	TW-TC
Toy	5%	11.156	19.446	25.379	27.596	30.126	29.446	**30.689**
	10%	11.391	24.087	29.527	32.134	35.428	34.169	**37.121**
	20%	11.904	29.944	35.097	36.791	41.010	40.453	**44.009**
	Time (s)	–	4.74	15.72	63.50	181.25	26.27	154.67
News	5%	8.806	15.185	26.791	25.972	26.942	25.064	**28.887**
	10%	9.041	19.579	28.748	29.213	29.305	30.272	**32.551**
	20%	9.553	23.935	34.533	32.367	32.923	34.897	**36.206**
	Time (s)	–	7.64	31.76	36.40	521.30	128.62	342.36
Container	5%	4.600	18.273	27.979	23.681	27.333	28.364	**29.473**
	10%	4.834	21.198	31.690	26.646	28.834	33.721	**34.459**
	20%	5.344	24.901	35.460	35.204	35.726	37.536	**38.259**
	Time (s)	–	8.88	23.70	71.35	204.78	121.31	339.04
HSV	5%	7.494	11.579	39.736	42.459	42.501	42.675	**48.999**
	10%	7.729	22.795	44.756	47.563	47.132	49.067	**52.743**
	20%	8.240	32.631	50.123	51.065	51.961	53.566	**54.678**
	Time (s)	–	9.32	19.57	144.98	459.79	101.77	493.38

Discussions
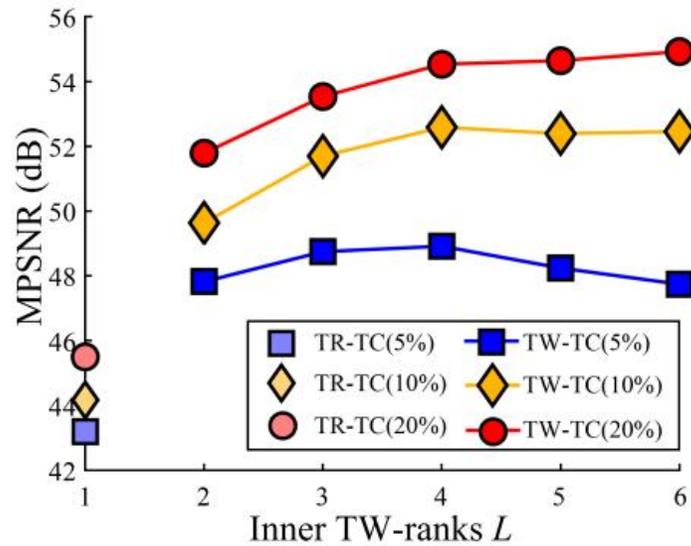
(a) MPSNR versus inner TW-ranks when outer TW-ranks and all TR-ranks are 6.
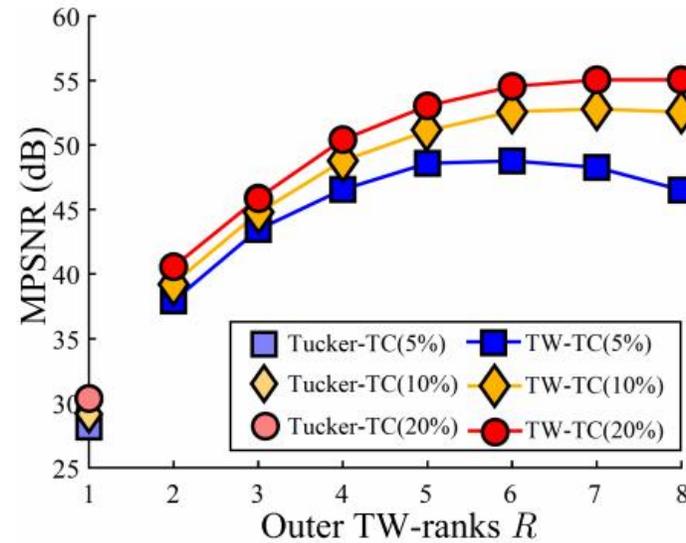
(b) MPSNR versus outer TW-ranks when inner TW-ranks and all TR-ranks are 4.
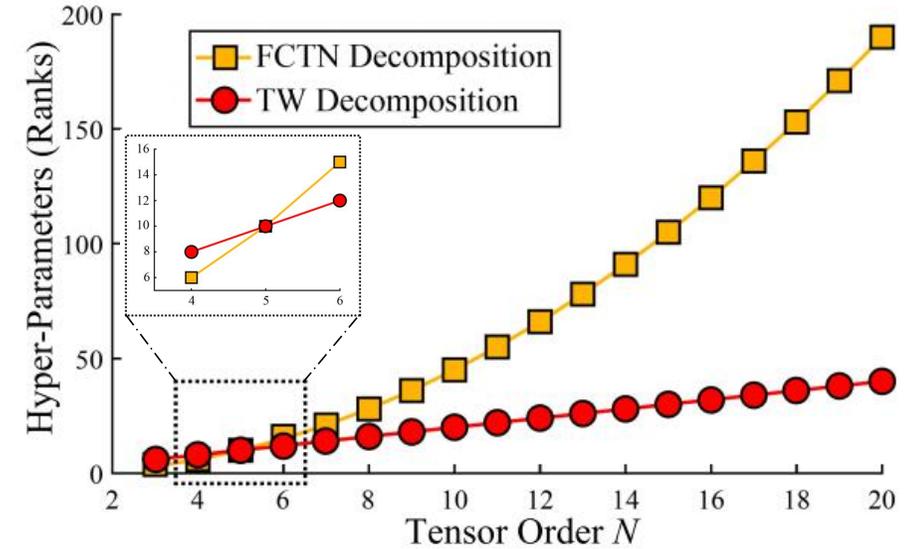
(c) The number of hyper-parameters of FCTN and TW decompositions against tensor dimension

Conclusions

□ **Propose a novel tensor wheel (TW) decomposition.**

□ **Provide one numerical application (i.e., tensor completion) of TW.**

Thank you!

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Homepage: https://zhongchengwu.github.io

Code: https://github.com/zhongchengwu/code_TWDec