

# Near-Optimal Multi-Agent Learning for Safe Coverage Control



**Manish  
Prajapat**



**Matteo  
Turchetta**



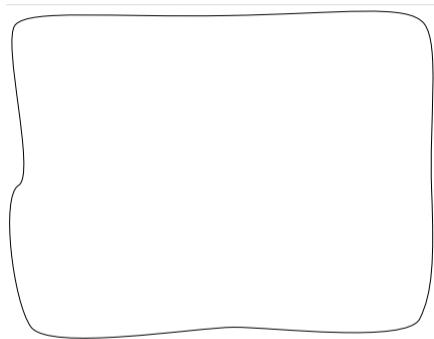
**Melanie N.  
Zeilinger<sup>†</sup>**



**Andreas  
Krause<sup>†</sup>**

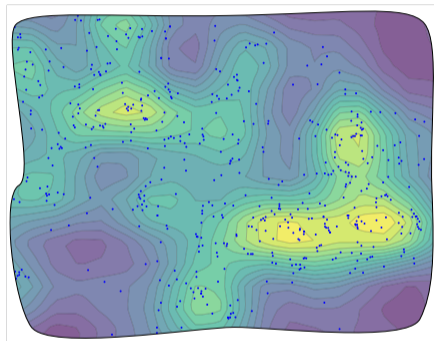


# What is Safe Multi-agent Coverage Control?



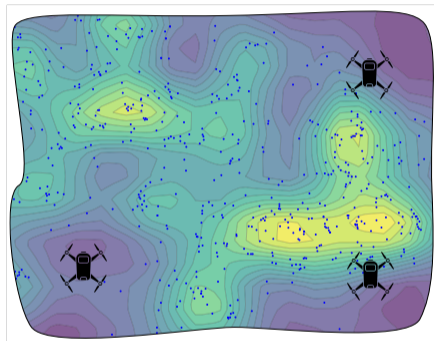
# What is Safe Multi-agent Coverage Control?

- Density
  - Spatially distributed events



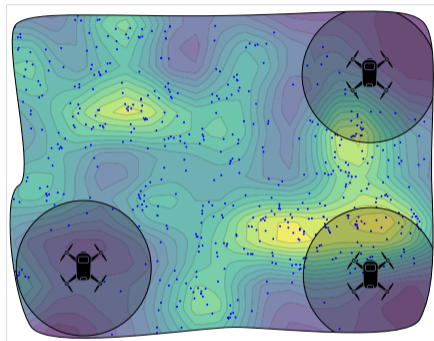
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  - A set of agents coordinate in the process



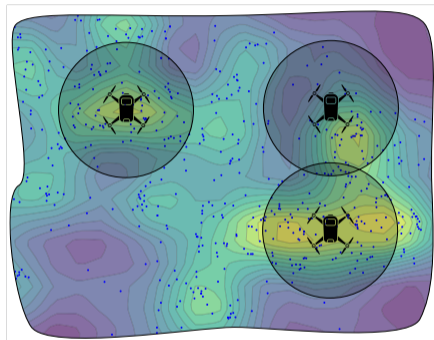
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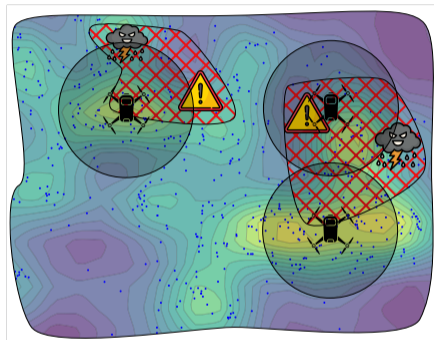
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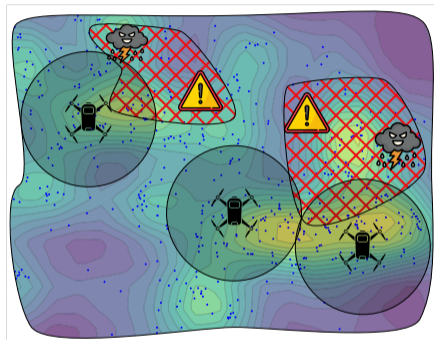
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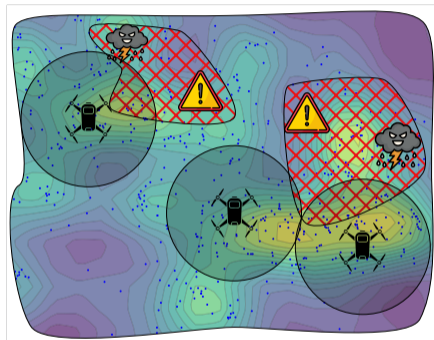
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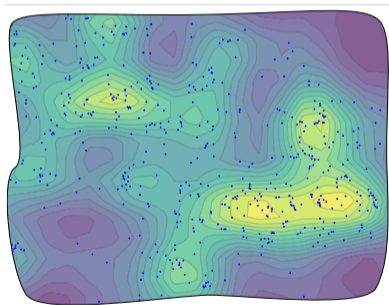


Real-world applications: Bio-diversity monitoring, Swarm robots, 3D scene reconstruction, etc.

# Problem definition

Coverage function:

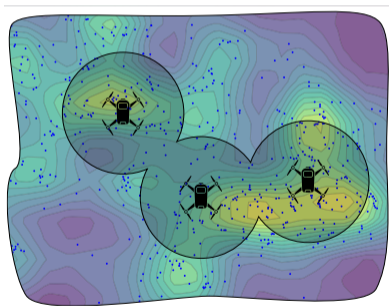
$$\underbrace{F(X; \rho)}_{\text{coverage}} = \sum_{\underbrace{x^i \in X}_{\text{agents' location}}} \sum_{\underbrace{v \in D^i}_{\text{disc of agent } i}} \underbrace{\rho(v)}_{\text{density}}$$



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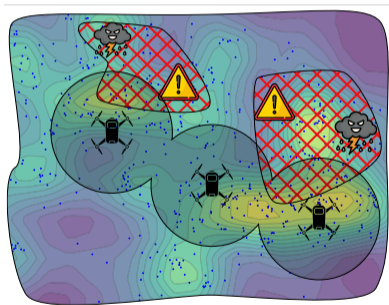


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*agents' location*      *density*



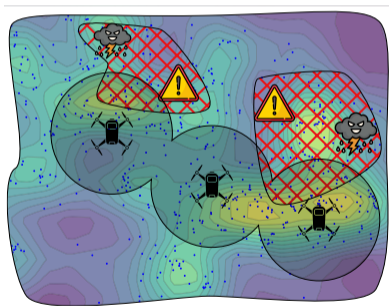
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Challenges

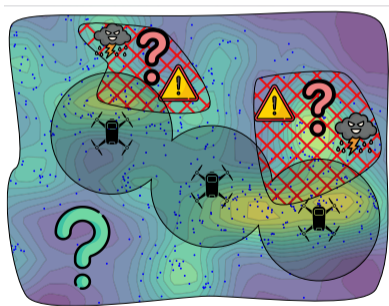


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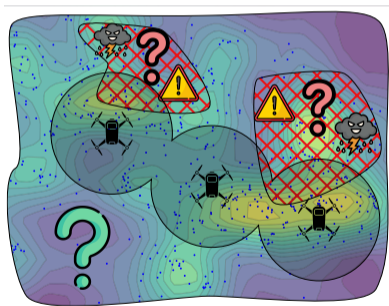
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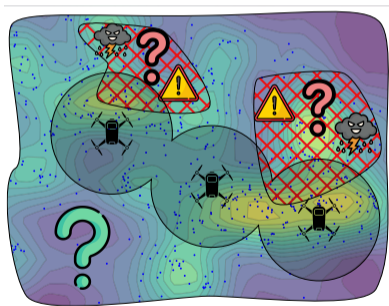
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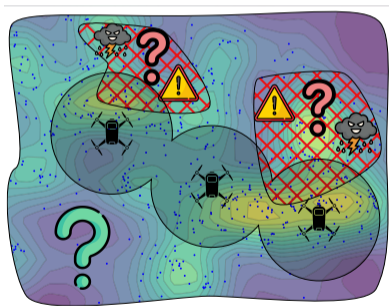
## Submodular function



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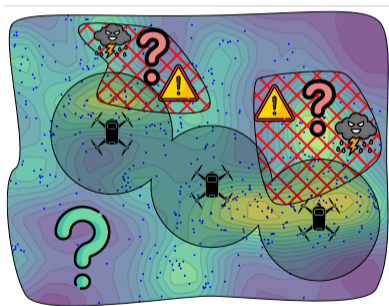
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*agents' location*      *disc of agent i*



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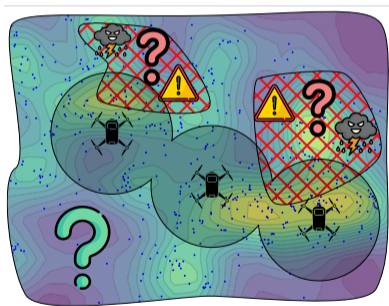
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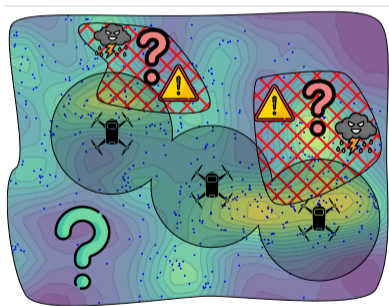
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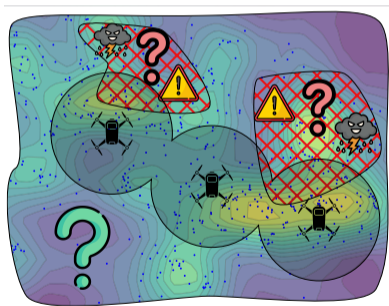
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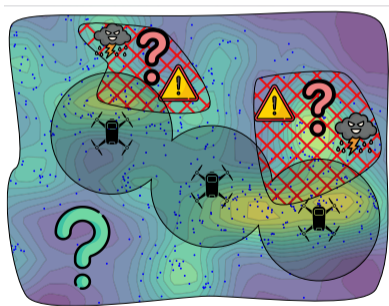
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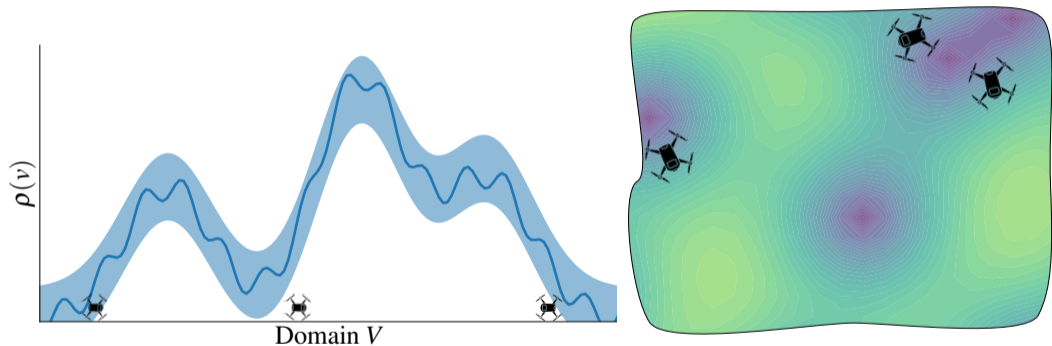
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- How far are we from the optimal solution?

# MACOPT: Multi-agent Coverage Control

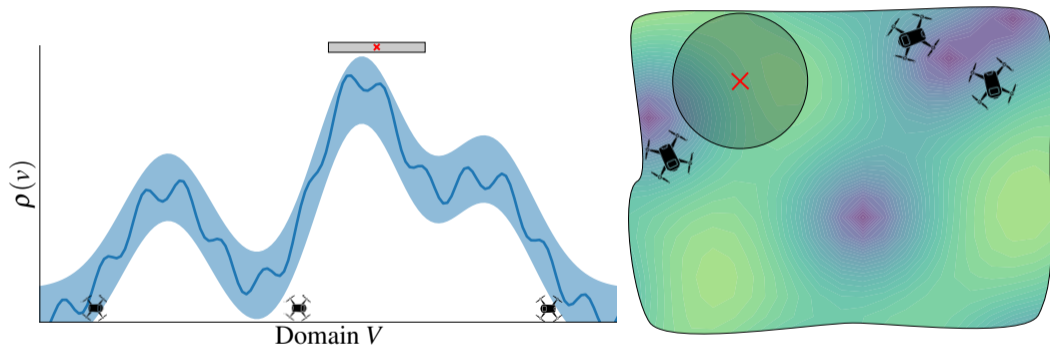
Unconstrained case



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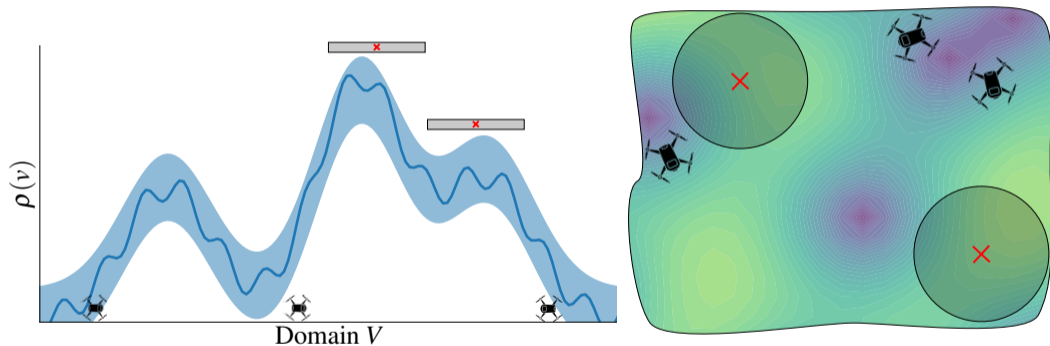




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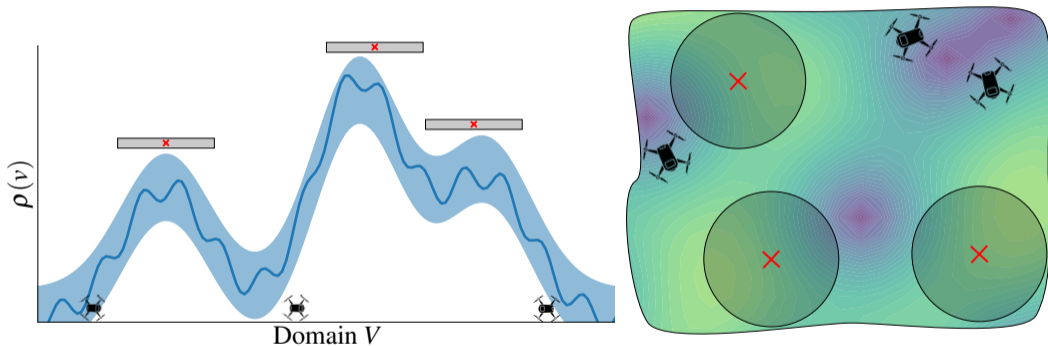
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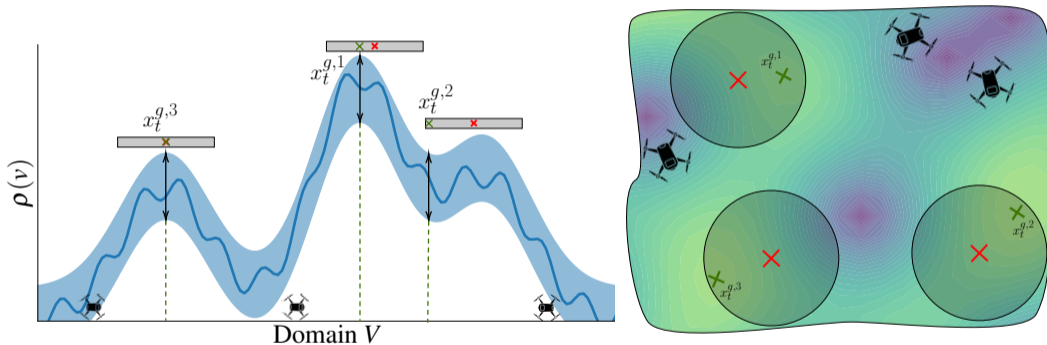
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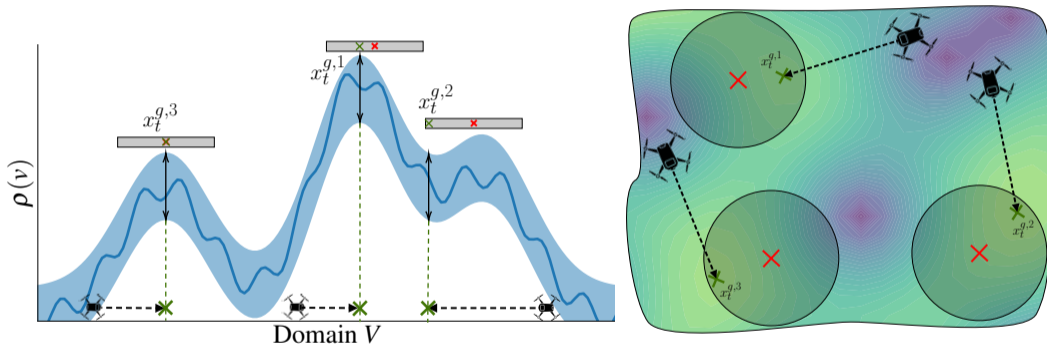
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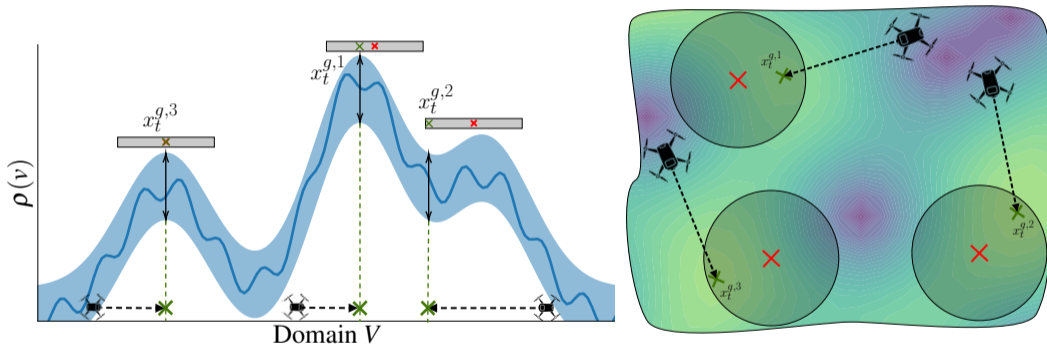
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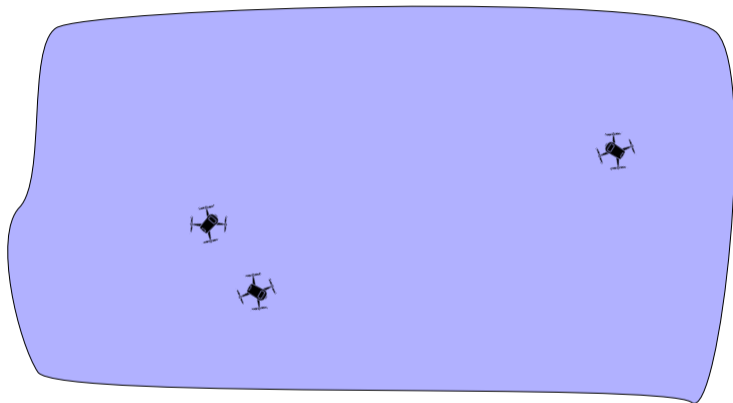
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Theoretical result: **Cumulative regret grows sublinear with time**

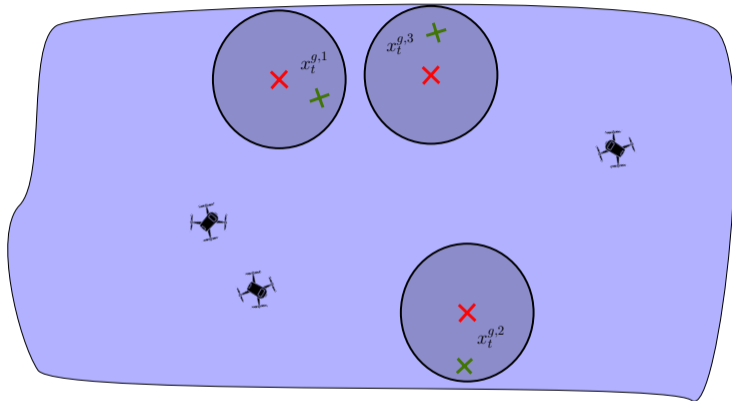
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Multi-Agent extension of Goal-oriented Safe Exploration (GoOSE) (Turchetta et al., 2019)



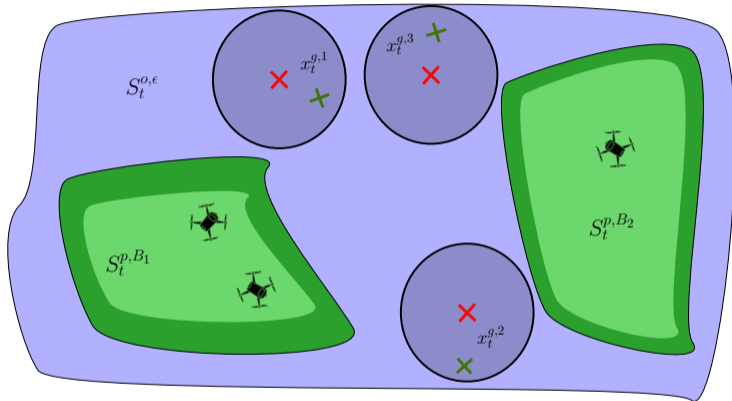
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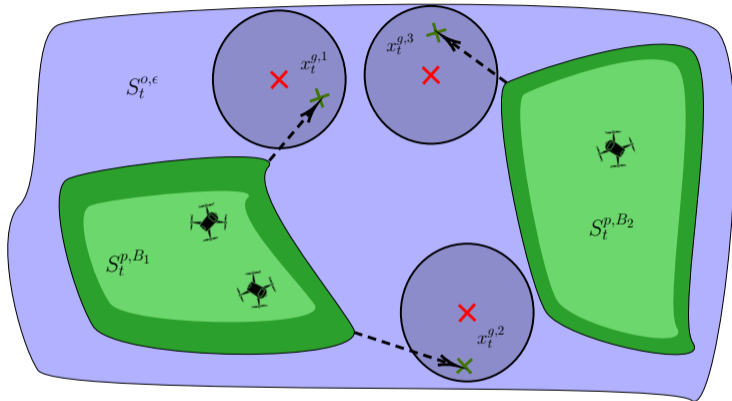
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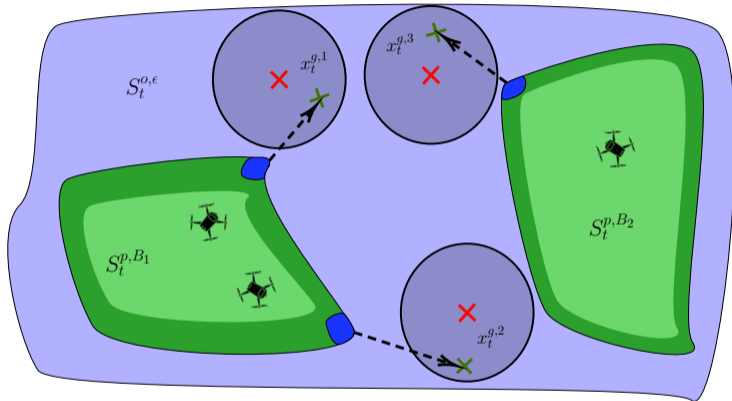
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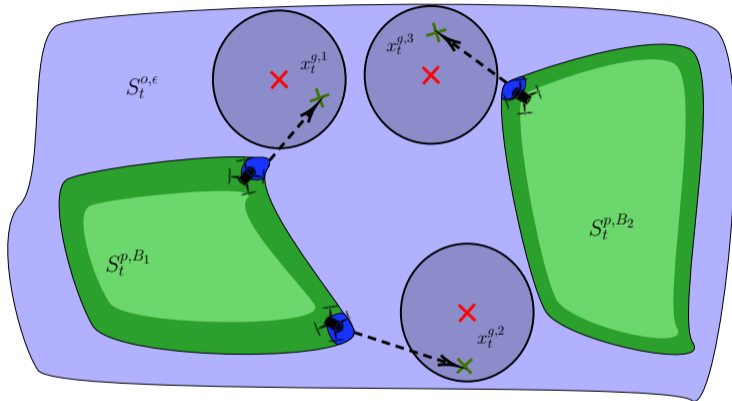
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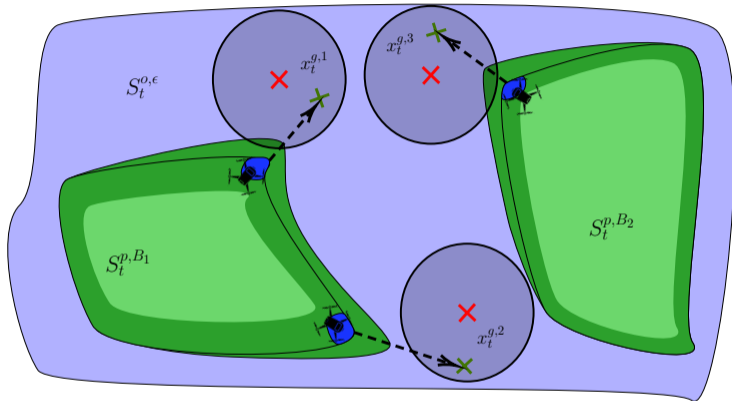
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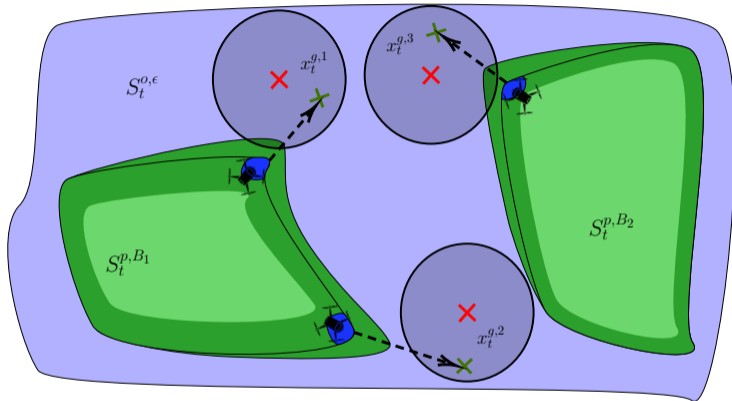
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Theoretical Results:

- Guarantees that **SAFEMAC** is safe with high probability
- Achieves near-optimal coverage in finite time

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Experiments on biodiversity monitoring and obstacle avoidance environments

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- **SAFEMAC up to 50%** more sample efficient as compared to two-stage algorithm

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Scan for paper !!!



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