# Parameter-free Dynamic Graph Embedding for Link Prediction

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**Code and Data:** https://github.com/FudanCISL/FreeGEM

# Motivation

- Dynamic interaction graphs
- The evolution of user-item interactions over time
- Two crucial factors
  - collaborative relationship
  - personalized interaction patterns
- Existing methods need time-consuming parameter learning

# FreeGEM

- A parameter-**Free** Dynamic **G**raph **EM**bedding method for link prediction
  - Incremental Graph Embedding Engine
  - Personalized Dynamic Interaction Pattern Modeller

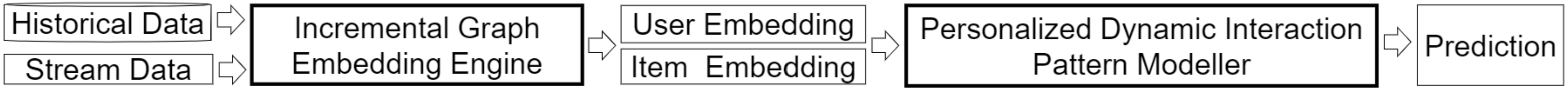



Figure 1: The high-level design of the proposed FreeGEM method.

# Incremental Graph Embedding Engine

- Online-Monitor-Offline Architecture
  - a Online module to approximately embed users/items over time
  - a Monitor module to estimate the approximation error in real time
  - an Offline module to calibrate the user/item embeddings

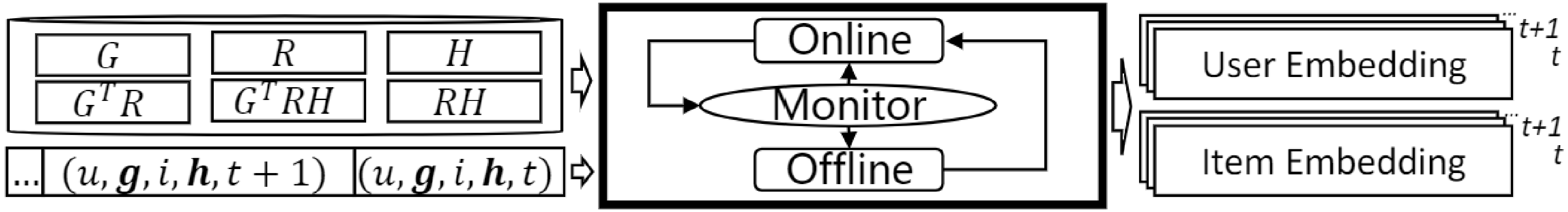



Figure 2: Illustration of the proposed incremental graph embedding engine.

# Incremental Graph Embedding Engine

- Frequency-aware Preference Matrix Reconstruction
  - Normalization
  - Frequency-aware reconstruction
  - Inverse normalization

$$R' = D_U^{-\alpha} R D_I^{-\alpha}$$

$$E_U = US^{\gamma}, E_I = VS^{\gamma} \ (\gamma < 0.5)$$

$$\hat{R} = D_U^{\alpha} \hat{R}' D_I^{\alpha}$$

# Incremental Graph Embedding Engine

- Attribute-integrated SVD

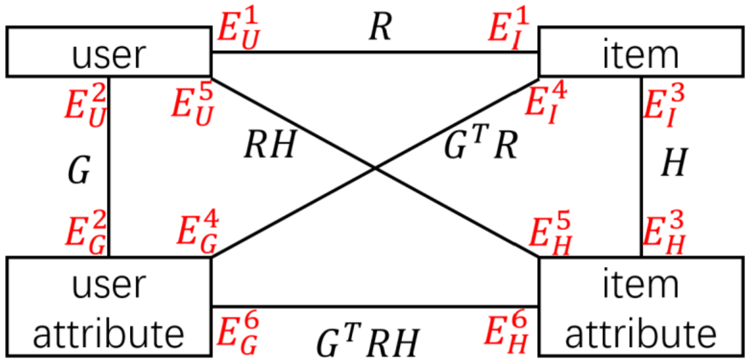



Table 1: Correspondence between the decomposed matrix and the constructed embedded matrix.

| Description of the decomposed matrix | Decomposed matrix | Constructed embedding (left) | Constructed embedding (right) |
|---|---|---|---|
| user-item | $R \in \mathbb{R}^{m\times n}$ | $E_U^1 \in \mathbb{R}^{m\times k_1}$ | $E_I^1 \in \mathbb{R}^{n\times k_1}$ |
| user attribute | $G \in \mathbb{R}^{m\times p}$ | $E_U^2 \in \mathbb{R}^{m\times k_2}$ | $E_G^2 \in \mathbb{R}^{p\times k_2}$ |
| item attribute | $H \in \mathbb{R}^{n\times q}$ | $E_I^3 \in \mathbb{R}^{n\times k_3}$ | $E_H^3 \in \mathbb{R}^{q\times k_3}$ |
| user_attribute-item | $G^\top R \in \mathbb{R}^{p\times n}$ | $E_G^4 \in \mathbb{R}^{p\times k_4}$ | $E_I^4 \in \mathbb{R}^{n\times k_4}$ |
| user-item_attribute | $RH \in \mathbb{R}^{m\times q}$ | $E_U^5 \in \mathbb{R}^{m\times k_5}$ | $E_H^5 \in \mathbb{R}^{q\times k_5}$ |
| user_attribute-item_attribute | $G^\top RH \in \mathbb{R}^{p\times q}$ | $E_G^6 \in \mathbb{R}^{p\times k_6}$ | $E_H^6 \in \mathbb{R}^{q\times k_6}$ |

Table 2: Correspondence between path and embedding matrix.

| No. | Path | User embedding | Item embedding |
|---|---|---|---|
| 1 | user-item | $E_{U_1} = E_U^1 \in \mathbb{R}^{m\times k_1}$ | $E_{I_1} = E_I^1 \in \mathbb{R}^{n\times k_1}$ |
| 2 | user-user_attribute-item | $E_{U_2} = E_U^2(E_G^2)^\top \in \mathbb{R}^{m\times p}$ | $E_{I_2} = E_I^4(E_G^4)^\top \in \mathbb{R}^{n\times p}$ |
| 3 | user-item_attribute-item | $E_{U_3} = E_U^5(E_H^5)^\top \in \mathbb{R}^{m\times q}$ | $E_{I_3} = E_I^3(E_H^3)^\top \in \mathbb{R}^{n\times q}$ |
| 4 | user-user_attribute-item_attribute-item | $E_{U_4} = E_U^2(E_G^2)^\top E_G^6 \in \mathbb{R}^{m\times k_6}$ | $E_{I_4} = E_I^3(E_H^3)^\top E_H^6 \in \mathbb{R}^{n\times k_6}$ |
| 5 | user-item_attribute-user_attribute-item | $E_{U_5} = E_U^5(E_H^5)^\top E_H^6 \in \mathbb{R}^{m\times k_6}$ | $E_{I_5} = E_I^4(E_G^4)^\top E_G^6 \in \mathbb{R}^{n\times k_6}$ |

$$E_U = (\alpha_1 E_{U_1})||(\alpha_2 E_{U_2})||(\alpha_3 E_{U_3}) \in \mathbb{R}^{m\times(k_1+p+q)}.$$

$$E_I = (\alpha_1 E_{I_1})||(\alpha_2 E_{I_2})||(\alpha_3 E_{I_3}) \in \mathbb{R}^{n\times(k_1+p+q)}.$$

# Incremental Graph Embedding Engine

- Online Module

$$\mathbf{m} = U^\top \mathbf{u},\ \mathbf{n} = V^\top \mathbf{i},\ \mathbf{p} = \mathbf{u} - U\mathbf{m},\ \mathbf{q} = \mathbf{i} - V\mathbf{n},\ P = ||\mathbf{p}||^{-1}\mathbf{p},\ Q = ||\mathbf{q}||^{-1}\mathbf{q}.$$

$$K = \begin{bmatrix} S & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} + \begin{bmatrix} \mathbf{m} \\ ||\mathbf{p}|| \end{bmatrix} \begin{bmatrix} \mathbf{n} \\ ||\mathbf{q}|| \end{bmatrix}^\top$$

$$R_{t+1} = R_t + \mathbf{u}\mathbf{i}^\top \approx ([U\ P]U_K)S_K([V\ Q]V_K))^\top$$

# Incremental Graph Embedding Engine

- Monitor Module

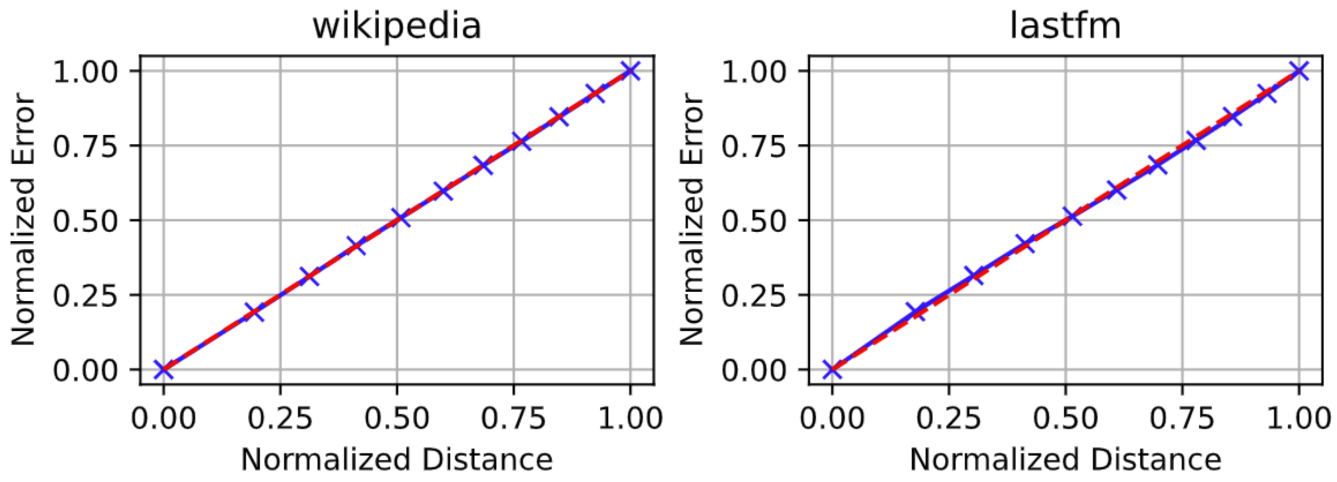
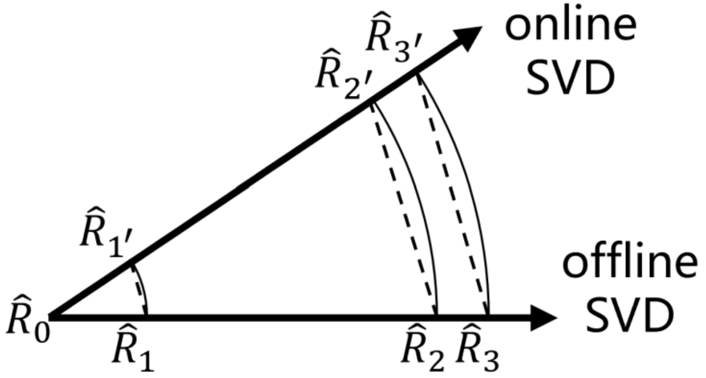



Figure 9: There is a strong positive correlation between *distance* (the F-norm distance between the reconstructed matrix by online SVD at the current time step and the initial reconstructed matrix) and online approximation error.




$$e = ||\hat{R}_i - \hat{R}_{i'}||_F$$

$$d = ||\hat{R}_{i'} - \hat{R}_0||_F$$

# Personalized Dynamic Interaction Pattern Modeller

- Dynamic Time Decay $f_i(t) = exp\{\beta_i(t/T_i - 1)\}$ $\beta_i/\beta_j = T_i/T_j$
- Attention Module

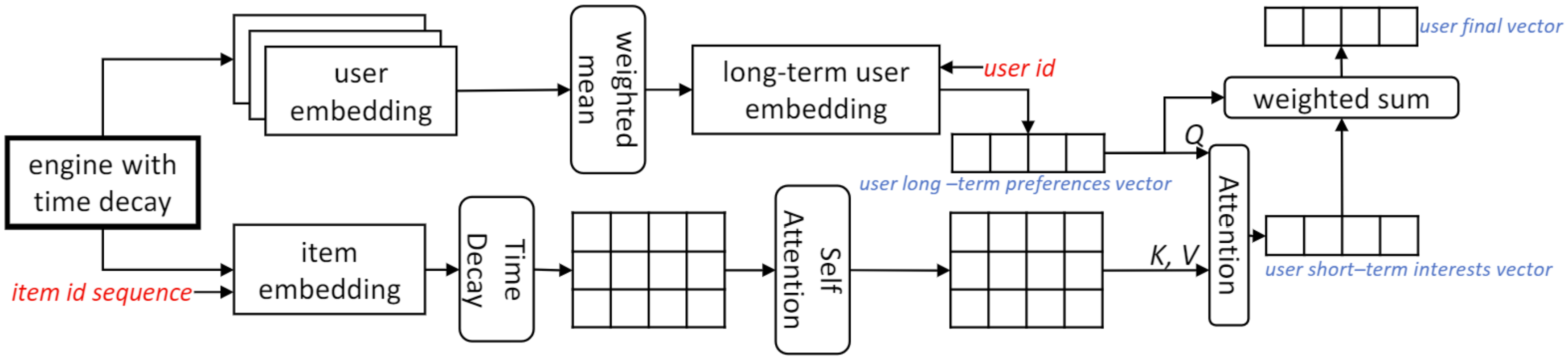



Figure 5: Personalized dynamic interaction pattern modeling module.

$$\mathbf{e}_{long} = \sum_{r=1}^{a} \frac{1}{r}\mathbf{e}_u^{(r)} \qquad S_u = \mathbf{e}_i^{(1)}||\cdots||\mathbf{e}_i^{(b)}, \;\; S'_u = \frac{S_u^\top S_u}{\sqrt{k}} S_u^\top, \;\; \mathbf{e}_{short} = (S'_u)^\top \frac{S'_u \mathbf{e}_{long}}{\sqrt{k}} \qquad \mathbf{e} = \lambda \mathbf{e}_{short} + (1-\lambda)\mathbf{e}_{long}$$

# Main results

- Future item recommendation
- Next interaction prediction

Table 3: Accuracy comparison with state-of-the-art methods on two link prediction tasks.

(a) Future item recommendation

| | Video Recall | Game Recall | ML-100K Recall | ML-1M Recall |
|---|---|---|---|---|
| LightGCN | 0.036 | 0.026 | 0.025 | 0.029 |
| Time-LSTM | 0.044 | 0.020 | 0.058 | 0.033 |
| RRN | 0.068 | 0.029 | 0.065 | 0.043 |
| DeepCoevolve | 0.050 | 0.027 | 0.069 | 0.030 |
| JODIE* | 0.078 | 0.035 | 0.074 | 0.035 |
| CoPE* | 0.088 | 0.047 | 0.081 | 0.049 |
| **FreeGEM *(no attr)** | **0.113** | **0.059** | **0.114** | **0.053** |
| **FreeGEM *(with attr)** | - | - | **0.149** | **0.065** |

(b) Next interaction prediction

| | Wikipedia | | LastFM | |
|---|---|---|---|---|
| | MRR | Hit | MRR | Hit |
| Time-LSTM | 0.247 | 0.342 | 0.068 | 0.137 |
| RRN | 0.522 | 0.617 | 0.089 | 0.182 |
| LatentCross | 0.424 | 0.481 | 0.148 | 0.227 |
| CTDNE | 0.035 | 0.056 | 0.010 | 0.010 |
| DeepCoevolve | 0.515 | 0.563 | 0.019 | 0.039 |
| JODIE | 0.746 | 0.822 | 0.195 | 0.307 |
| CoPE | 0.750 | **0.890** | **0.200** | 0.446 |
| **FreeGEM** | **0.786** | 0.852 | 0.195 | **0.453** |

# Ablation Studies

- Future item recommendation
- Next interaction prediction

Table 4: The ablation studies on the future item recommendation task.

| | Pattern Modeller | Matrix Reconstruction | Video | Game | ML-100K | ML-1M |
|---|---|---|---|---|---|---|
| A | ✗ | ✗ | 0.073 | 0.023 | 0.051 | 0.045 |
| B | ✗ | ✓ | 0.107 | 0.041 | 0.074 | 0.051 |
| C | ✓ | ✗ | 0.080 | 0.025 | 0.113 | 0.052 |
| **FreeGEM *(no attr)** | ✓ | ✓ | 0.113 | 0.059 | 0.114 | 0.053 |

Table 5: The ablation studies on the next interaction prediction task.

| | | | | | Wikipedia | | LastFM | |
|---|---|---|---|---|---|---|---|---|
| | Offline | Online | Decay | Attention | MRR | Hit@10 | MRR | Hit@10 |
| Last-1 | - | - | - | - | 0.775* | 0.775* | 0.098* | 0.098* |
| Last-10 | - | - | - | - | 0.792 | 0.842 | 0.139 | 0.263 |
| D | ✗ | ✗ | ✗ | ✗ | 0.441 | 0.620 | 0.074 | 0.186 |
| E | ✓ | ✗ | ✗ | ✗ | 0.510 | 0.703 | 0.089 | 0.222 |
| F | ✗ | ✓ | ✗ | ✗ | 0.497 | 0.715 | 0.104 | 0.251 |
| G | ✓ | ✓ | ✗ | ✗ | 0.530 | 0.739 | 0.11 | 0.256 |
| H | ✓ | ✓ | ✓ | ✗ | 0.779 | 0.851 | 0.163 | 0.348 |
| I | ✓ | ✓ | ✗ | ✓ | 0.541 | 0.747 | 0.190 | 0.446 |
| **FreeGEM** | ✓ | ✓ | ✓ | ✓ | 0.786 | 0.852 | 0.195 | 0.453 |

# Ablation Studies

- Monitor

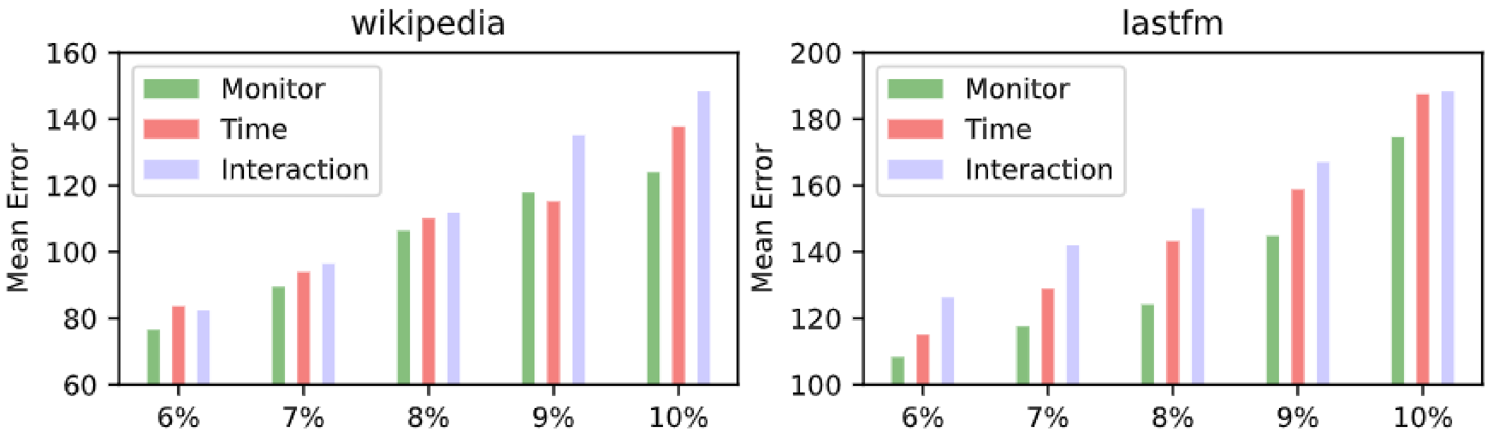



Figure 6: Performance comparison of three restart methods.

# Other Studies

- Running time

Table 6: Total running time comparison. The times of speedup is shown in the parentheses.

| Wikipedia | | | Lastfm | | |
|---|---|---|---|---|---|
| FreeGEM | JODIE | CoPE | FreeGEM | JODIE | CoPE |
| 9.7min | 350.0min (36.1X) | 3,589.1min (370.0X) | 54.8min | 15,790.0min (288.1X) | 51,212.5min (934.5X) |

# Other Studies

- Cold-start
  - For those who Recall is not 0, achieving a relative increase of 166%
  - For those who Recall is 0, increases the average Recall@10 to 0.100
- Under represented groups
  - Before using attribute information, male is 45.9% higher than female
  - After using attribute information, male is 22.8 % higher than female

# Other Studies

- Robustness

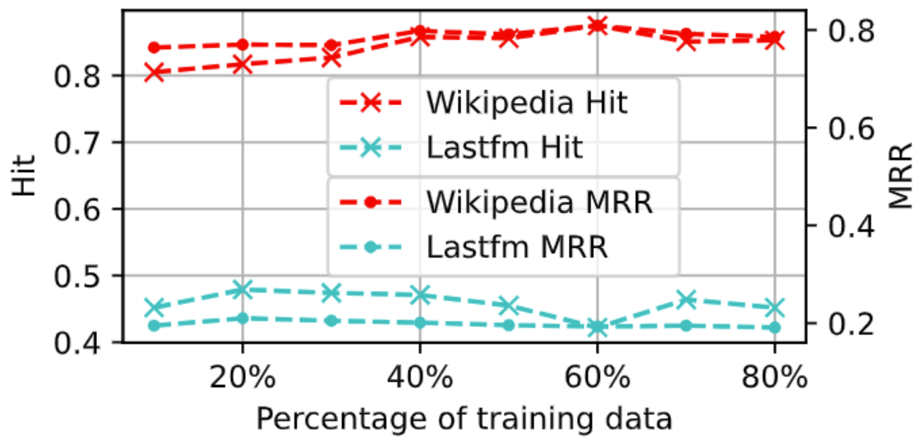



Figure 7: Robustness.

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**Code and Data:** https://github.com/FudanCISL/FreeGEM