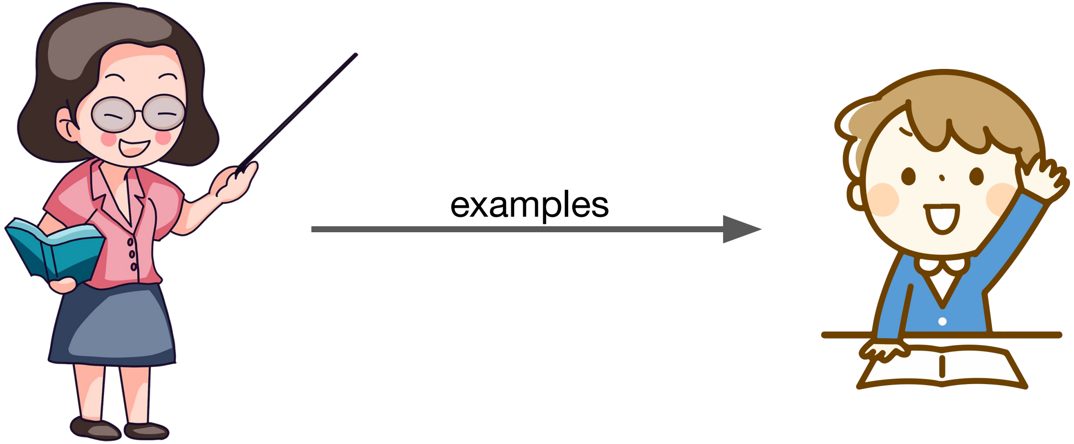
On Batch Teaching with Sample Complexity Bounded by VCD

Farnam Mansouri, Hans U. Simon, Adish Singla, Sandra Zilles

Thirty-sixth Conference on Neural Information Processing Systems, 2022

Motivation

Q: Teaching Complexity $\overset{?}{<}$ VC-Dimension



GM-Collusion-freeness

- GM-Collusion-free :
  - (informally) Adding more consistent examples shouldn't change learner's hypothesis [Goldman, Mathias 1996]
  - Smallest: NCTD [Kirkpatrick et al. 2019] $\overset{?}{\lesssim}$ VC-Dimension (**open question**)

- Beyond GM-Collusion-freeness
  - What about the importance of examples?

Importance: example [Zilles, Lange, Holte, Zinkevich 2011]

concept in $\mathcal{C}_3^{pair}$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
C_1	0	0	0	0	0	0	0	0	**0**	**0**	0
C_2	**1**	0	0	0	0	0	0	0	0	0	0
C_3	0	0	0	0	0	0	0	0	**0**	0	**1**
C_4	0	**1**	0	0	0	0	0	0	0	0	1
C_5	0	0	0	0	0	0	0	0	**0**	1	**0**
C_6	0	0	**1**	0	0	0	0	0	0	1	0
C_7	0	0	0	0	0	0	0	0	**0**	**1**	1
C_8	0	0	0	**1**	0	0	0	0	0	1	1
C_9	0	0	0	0	0	0	0	0	**1**	**0**	0
C_{10}	0	0	0	0	**1**	0	0	0	1	0	0
C_{11}	0	0	0	0	0	0	0	0	**1**	0	**1**
C_{12}	0	0	0	0	0	**1**	0	0	1	0	1
C_{13}	0	0	0	0	0	0	0	0	**1**	1	**0**
C_{14}	0	0	0	0	0	0	**1**	0	1	1	0
C_{15}	0	0	0	0	0	0	0	0	**1**	**1**	1
C_{16}	0	0	0	0	0	0	0	**1**	1	1	1

Table 1

Desirable Properties of Teaching Models

1. Class monotonicity
2. Domain monotonicity
3. Antichain property

Subset Teaching Dimension [Zilles, Lange, Holte, Zinkevich 2011]

STD Definition (informal): Define $\mathrm{STS}^0(C, \mathcal{C}) := \{\{(x, C(x)) \mid x \in X\}\}$. Iteratively define a collection $\mathrm{STS}^{k+1}(C, \mathcal{C})$ of *subset teaching sets* as the collection that contains all smallest-size sets S that satisfy the following:

1. $S \subseteq S'$ for some $S' \in \mathrm{STS}^k(C, \mathcal{C})$;
2. $S \not\subseteq S'$ for all $S' \in \mathrm{STS}^k(C', \mathcal{C})$ where $C' \in \mathcal{C}$, $C' \neq C$.

The Subset Teaching Dimension $\mathrm{STD}(\mathcal{C})$, is defined as $\mathrm{STD}(\mathcal{C}) := \max_C \min_{S \in \mathrm{STS}^\infty} |S|$.

Class Monotonicity	Domain Monotonicity	Anti Chain Property	≦ VCD	≦ RTD
no	no	yes	no	no

Table 2: STD properties

A Variant of Subset Teaching Dimension

Definition of $\mathrm{STD}_{\min}$ **(informal):** A sequence $\mathcal{T} = (T_k)_{k \in \mathbb{N}}$ of teachers for $\mathcal{C}$ is called a subset teaching sequence for $\mathcal{C}$ if, for all $C \in \mathcal{C}$ and all $k \in \mathbb{N}$:

$$
\begin{aligned}
T_0(C) &= \{(x, C(x)) \mid x \in X\}, \\
T_{k+1}(C) &\subseteq T_k(C), \\
T_{k+1}(C) &\not\subseteq T_k(C') \text{ for all } C' \in \mathcal{C},\ C' \neq C .
\end{aligned}
$$

Define $\mathrm{STD}_{\min}(\mathcal{C}) = \min_{\mathcal{T}^{\infty}} \max_C |\mathcal{T}^{\infty}(C)|$.

Class Monotonicity	Domain Monotonicity	Anti Chain Property	≦ VCD	≦ RTD
yes	yes	yes	yes	yes

Table 3: $\mathrm{STD}_{\min}$ properties