NEURAL INFORMATION PROCESSING SYSTEMS

PlasticityNet

Learning to Simulate Metal, Sand, and Snow for Optimization Time Integration

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- Overview
- Technical Details
- Experiment Results

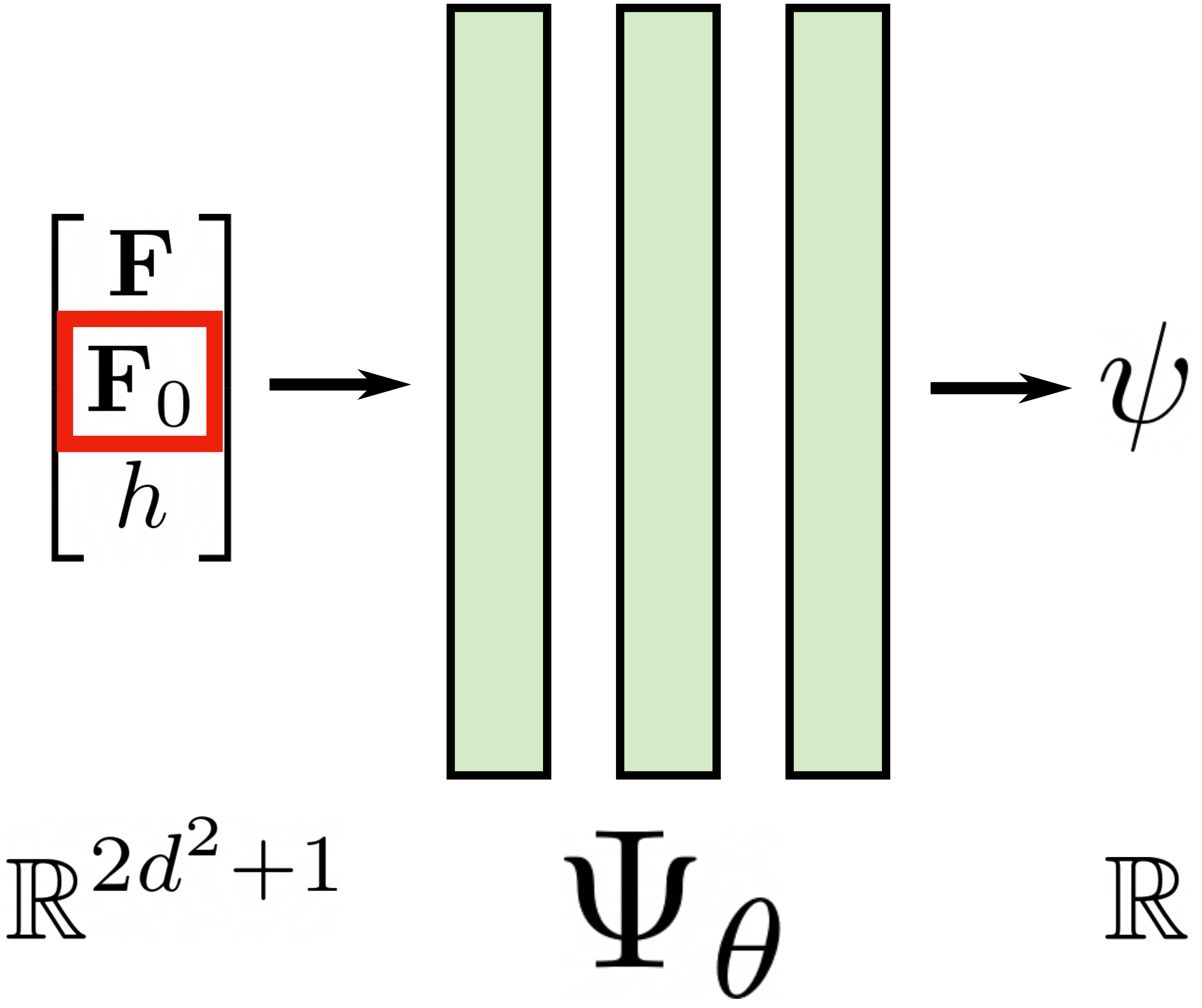
Motivation

- Optimization time integrator is stable but requires $f^{\text{int}} = -\frac{\partial \Psi}{\partial x}$.

- Implicit plasticity leads to $\frac{\partial f_i}{\partial x_j} \neq \frac{\partial f_j}{\partial x_i}$ in general.

- Goal: use optimization time integrators to simulate general plasticities with **large time steps**.

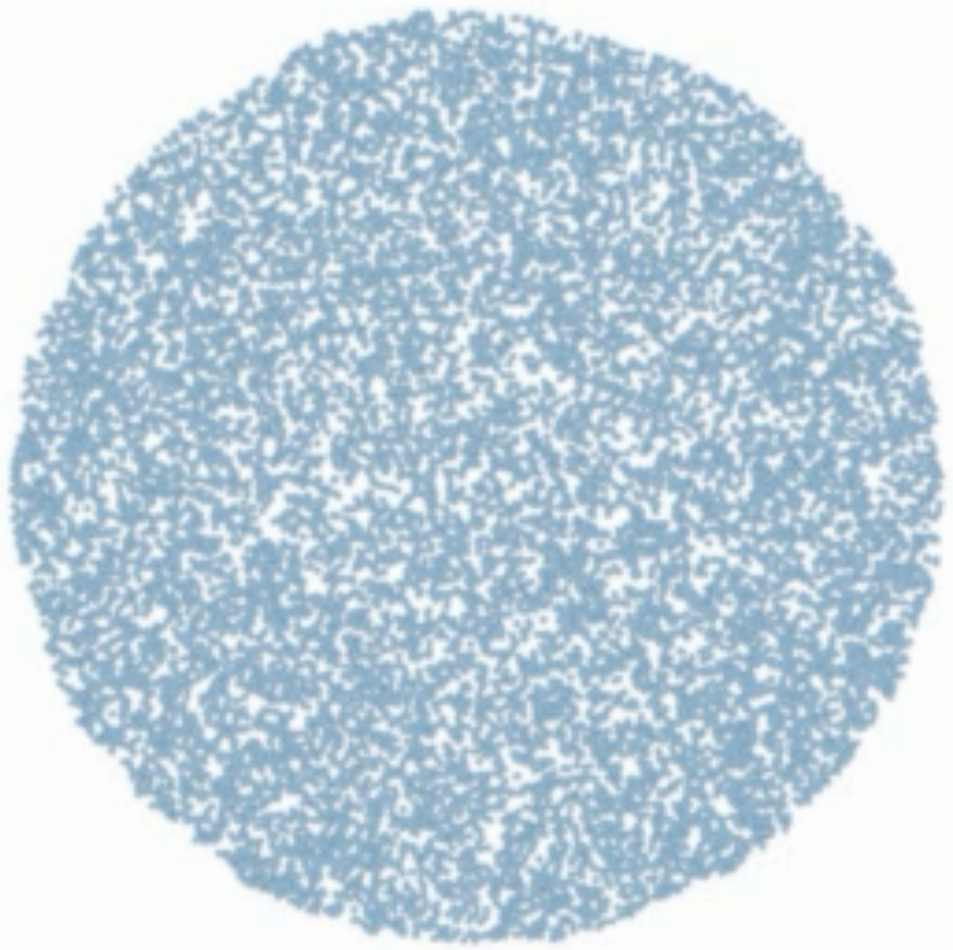
PlasticityNet



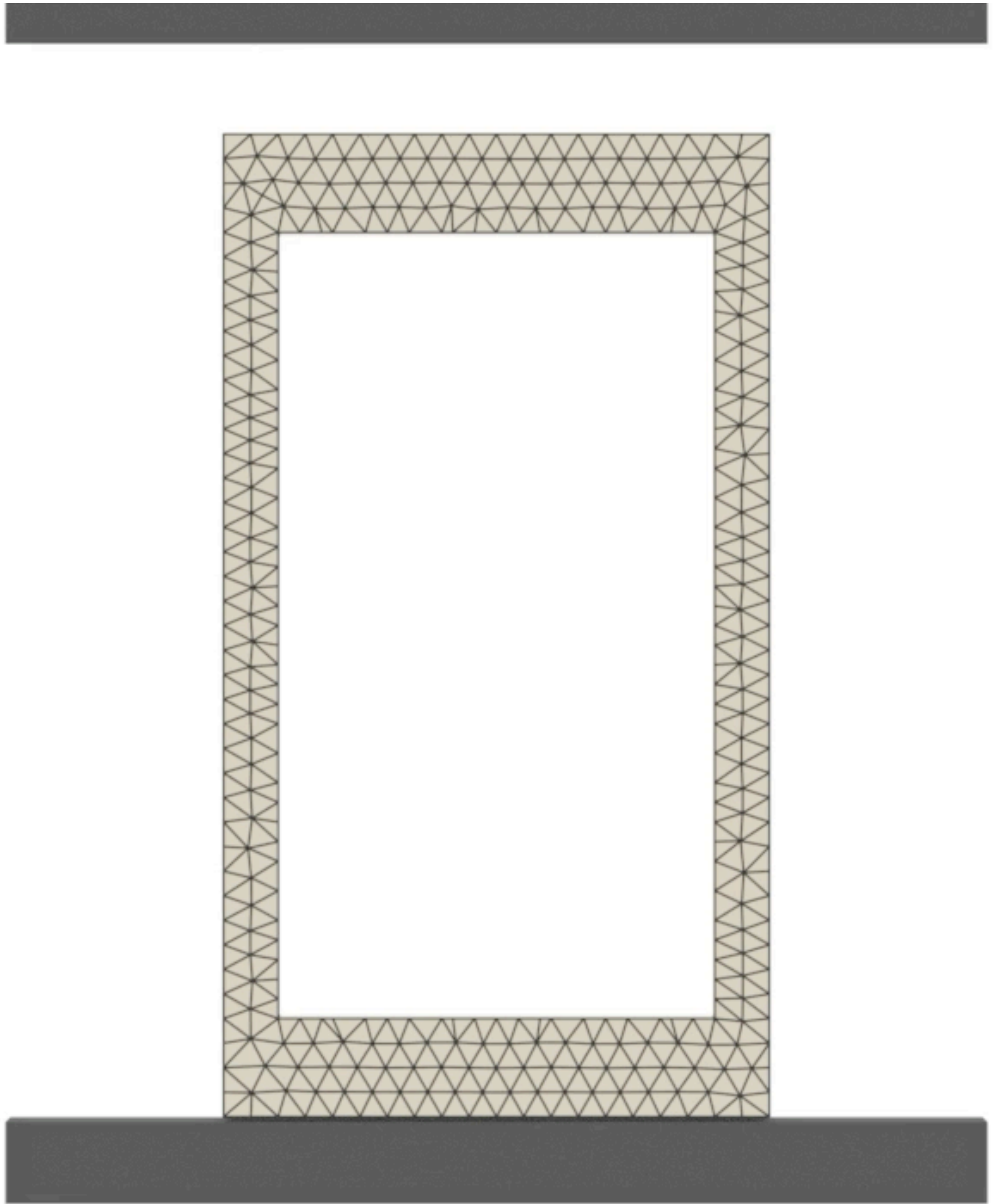
- Learning an **energy** for plastic forces.
- Compatible with **optimization time integrators**.

Sand

Snow



Metal



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Implicit Time Integration

Momentum conservation

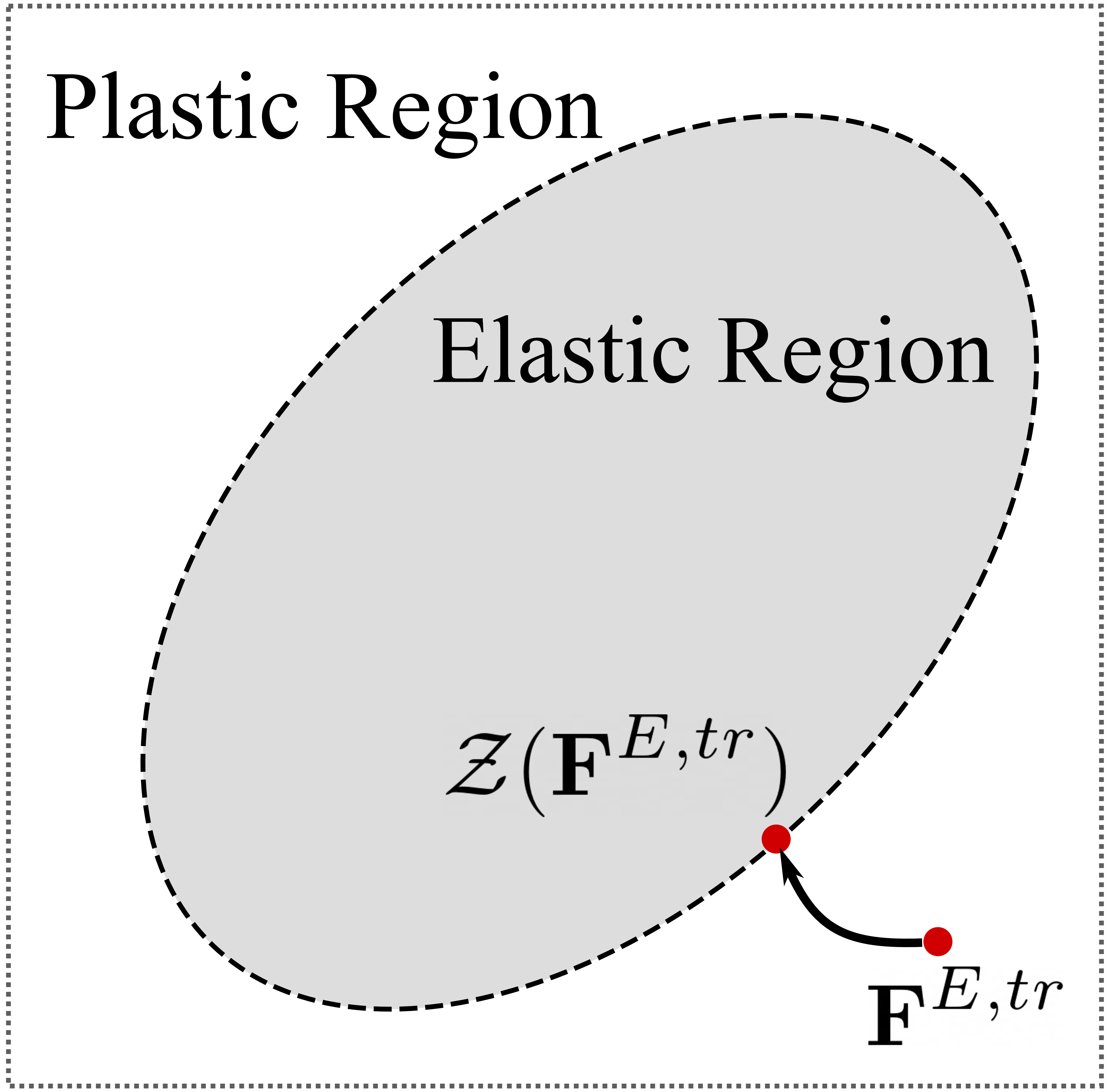
$$\mathbf{M}(\mathbf{v}^{n+1} - (\mathbf{v}^n + \mathbf{g}\Delta t)) = \Delta t \mathbf{f}^{n+1}$$

$$\mathbf{f}_i^{n+1} = -\sum_q V_q^0 \boldsymbol{\tau}(\mathcal{Z}(\mathbf{F}_q^{E,tr}))\mathbf{F}_q^{E,tr^{-\top}} \mathbf{F}^{P,n^{-\top}} \nabla w_{iq}$$

Plasticity

Elasticity

Return mapping for plasticity



Optimization Time Integrator

Integrability Condition (Li et al. 2022)

$$\frac{\partial \Psi}{\partial \mathbf{F}} = \boldsymbol{\tau}(\mathcal{Z}(\mathbf{F}))\mathbf{F}^{-\top}$$

Without Plasticity ($\mathcal{Z} = \mathrm{id}$) : ✓

With Plasticity : ✗

⇓

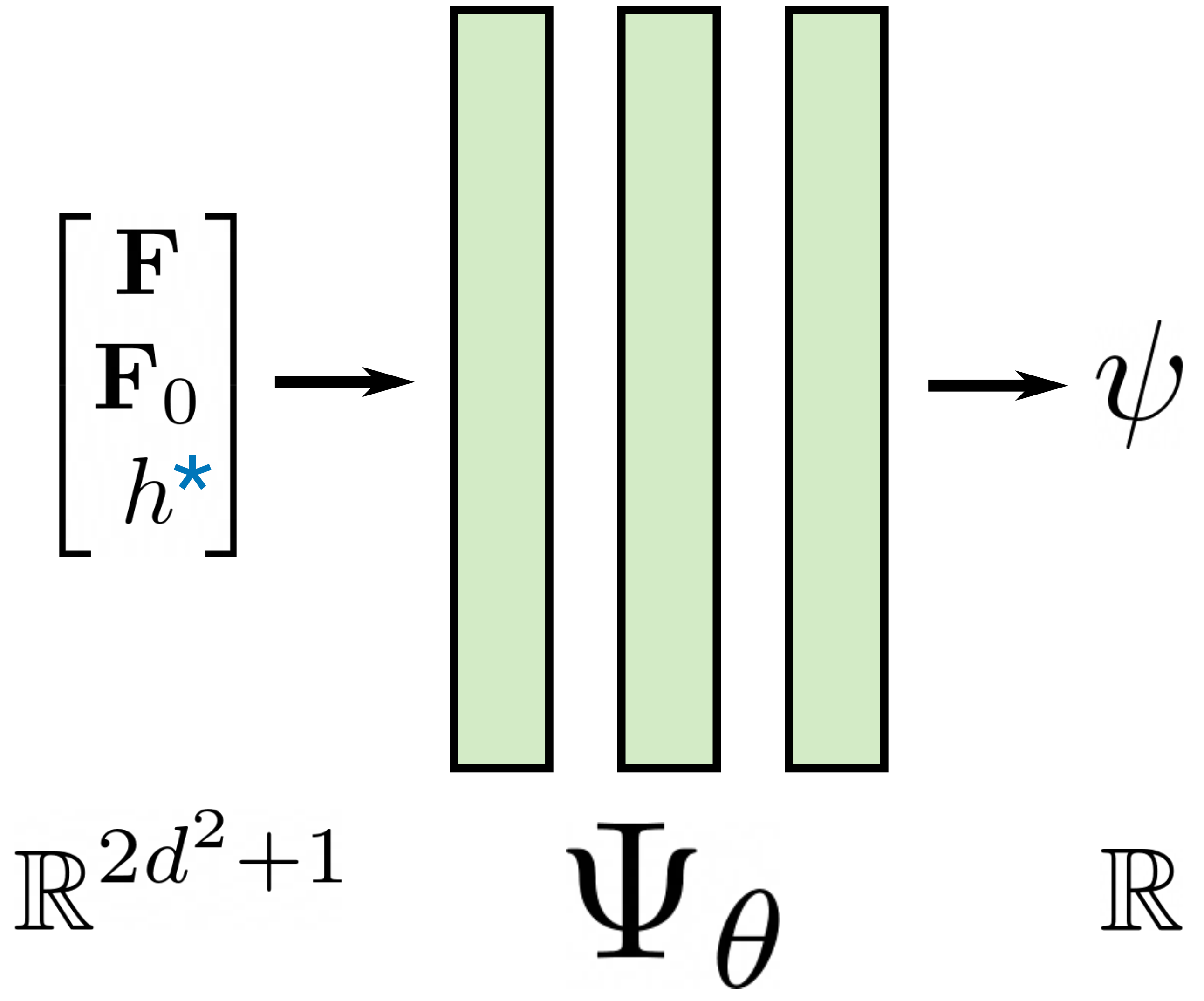
$$\mathbf{v}^{n+1} = \mathrm{argmin}_{\mathbf{v}} \frac{1}{2}\|\mathbf{v} - (\mathbf{v}^n + \mathbf{g}\Delta t)\|^2_{\mathbf{M}} + \sum_q V_q^0 \Psi(\mathbf{F}_q)$$

PlasticityNet

$$\frac{\partial \Psi}{\partial \mathbf{F}} = \boldsymbol{\tau}(\mathcal{Z}(\mathbf{F}))\mathbf{F}^{-\top}$$

Relaxation

$$\begin{cases} \frac{\partial \Psi_\theta}{\partial \mathbf{F}}(\mathbf{F}, \mathbf{F}_0)\big|_{\mathbf{F}=\mathbf{F}_0} = \boldsymbol{\tau}(\mathcal{Z}(\mathbf{F}_0))\mathbf{F}_0^{-\top} \\ \frac{\partial \Psi_\theta}{\partial \mathbf{F}}(\mathbf{F}, \mathbf{F}_0) \approx \boldsymbol{\tau}(\mathcal{Z}(\mathbf{F}))\mathbf{F}^{-\top} \end{cases}$$



* Please see the description of the hardening state in the paper.

$$\Psi_\theta(\mathbf{F}, \mathbf{F}_0) = \mathcal{NN}_\theta(\mathbf{F}, \mathbf{F}_0) - (\nabla_{\mathbf{F}} \mathcal{NN}_\theta(\mathbf{F}_0, \mathbf{F}_0) - \boldsymbol{\tau}(\mathcal{Z}(\mathbf{F}_0))\mathbf{F}_0^{-\top}) \odot \mathbf{F}$$

Training: $\mathcal{L}(\theta) = \mathbb{E}_{\mathbf{F}_0}\mathbb{E}_{\mathbf{F}} \left\| \frac{\partial \Psi_\theta}{\partial \mathbf{F}}(\mathbf{F}, \mathbf{F}_0) - \boldsymbol{\tau}(\mathcal{Z}(\mathbf{F}))\mathbf{F}^{-\top} \right\|_F^2$

Optimization Time Integration with PlasticityNet

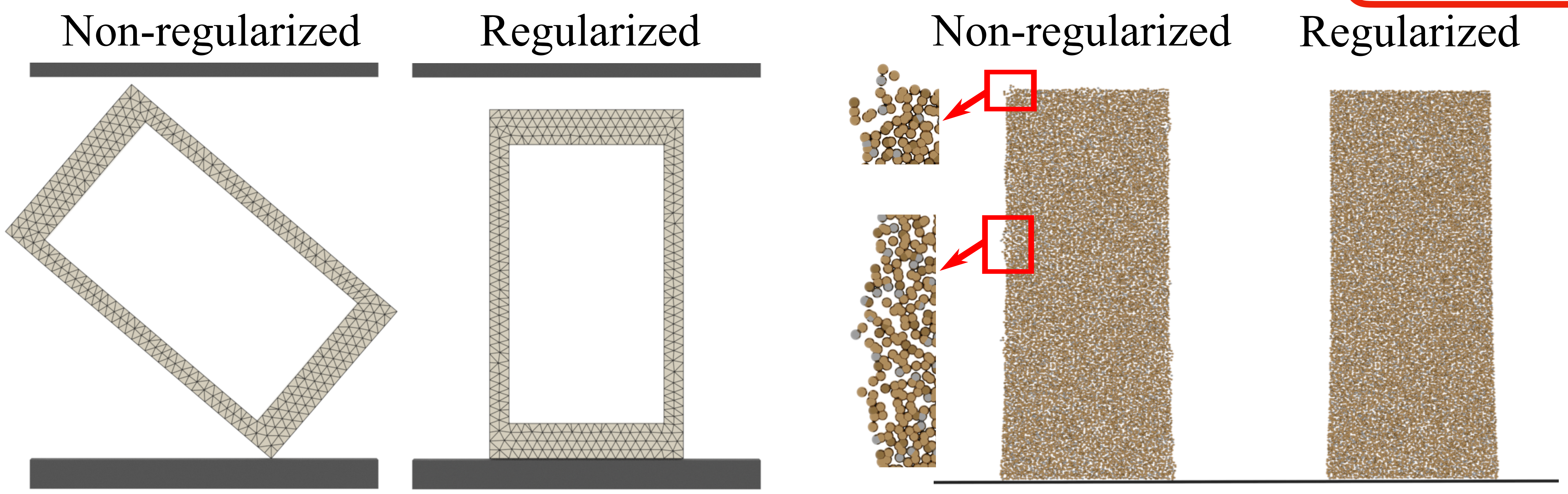
Fixed-point Iteration

$$\mathbf{v}^{n+1,j+1} = \text{argmin}_{\mathbf{v}} \frac{1}{2}\|\mathbf{v} - (\mathbf{v}^n + \mathbf{g}\Delta t)\|_{\mathbf{M}} + \sum_q V_q^0 \Psi_\theta(\mathbf{F}_q, \mathbf{F}_{0,q}^j), \quad \text{for } j = 0, 1, 2, \ldots,$$

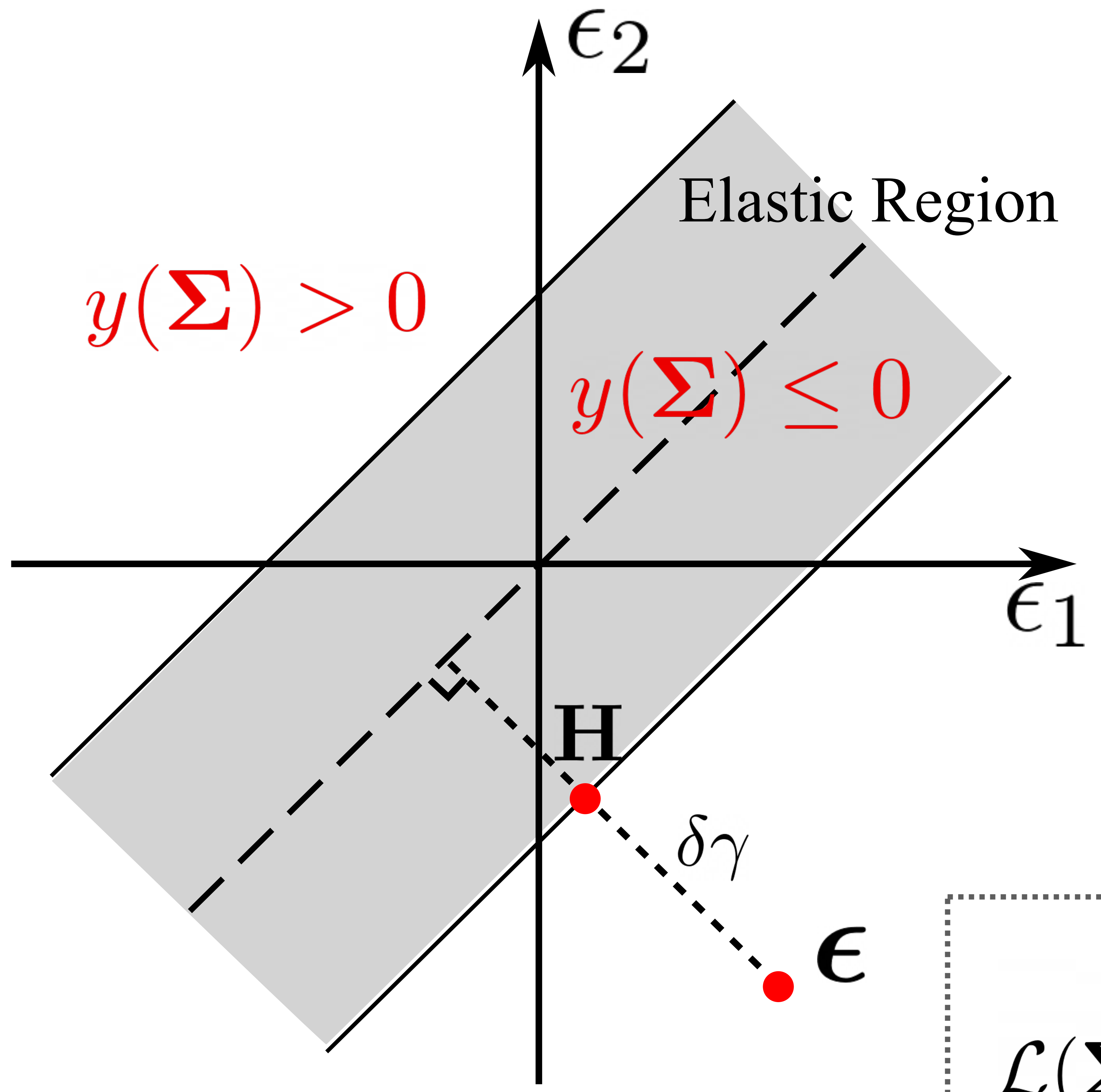
$$\mathbf{F}_0^j = \mathbf{F}(\mathbf{v}^{n+1,j})$$

Stability Regularizer

$$\Psi_\theta(\mathbf{F}, \mathbf{F}_0) = \mathcal{NN}_\theta(\mathbf{F}, \mathbf{F}_0) - (\nabla_{\mathbf{F}} \mathcal{NN}_\theta(\mathbf{F}_0, \mathbf{F}_0) - \boldsymbol{\tau}(\mathcal{Z}(\mathbf{F}_0))\mathbf{F}_0^{-\top}) \odot \mathbf{F} + \frac{1}{2}\mu\|\mathbf{F} - \mathbf{F}_0\|_F^2$$



Learning Return Mapping



Neural Return Mapping:

$$\delta\gamma_\theta(\mathbf{\Sigma}) = \min\{\mathcal{NN}_\theta(\Sigma), \|\hat{\boldsymbol{\epsilon}}\|\}$$

$$\mathcal{Z}_\theta^\Sigma(\mathbf{\Sigma}) = \begin{cases} \exp(\boldsymbol{\epsilon} - \delta\gamma_\theta \frac{\hat{\boldsymbol{\epsilon}}}{\|\hat{\boldsymbol{\epsilon}}\|}), & y(\mathbf{\Sigma}) > 0, \\ \mathbf{\Sigma}, & y(\mathbf{\Sigma}) \leq 0 \end{cases}$$

$$\mathcal{L}(\mathbf{\Sigma};\theta) = \begin{cases} y(\mathcal{Z}_\theta^\Sigma(\mathbf{\Sigma}))^2 + \max\{\delta\gamma_\theta(\mathbf{\Sigma}) - \|\hat{\boldsymbol{\epsilon}}\|, 0\}, & y(\mathbf{\Sigma}) > 0 \\ 0, & y(\mathbf{\Sigma}) \leq 0 \end{cases}$$

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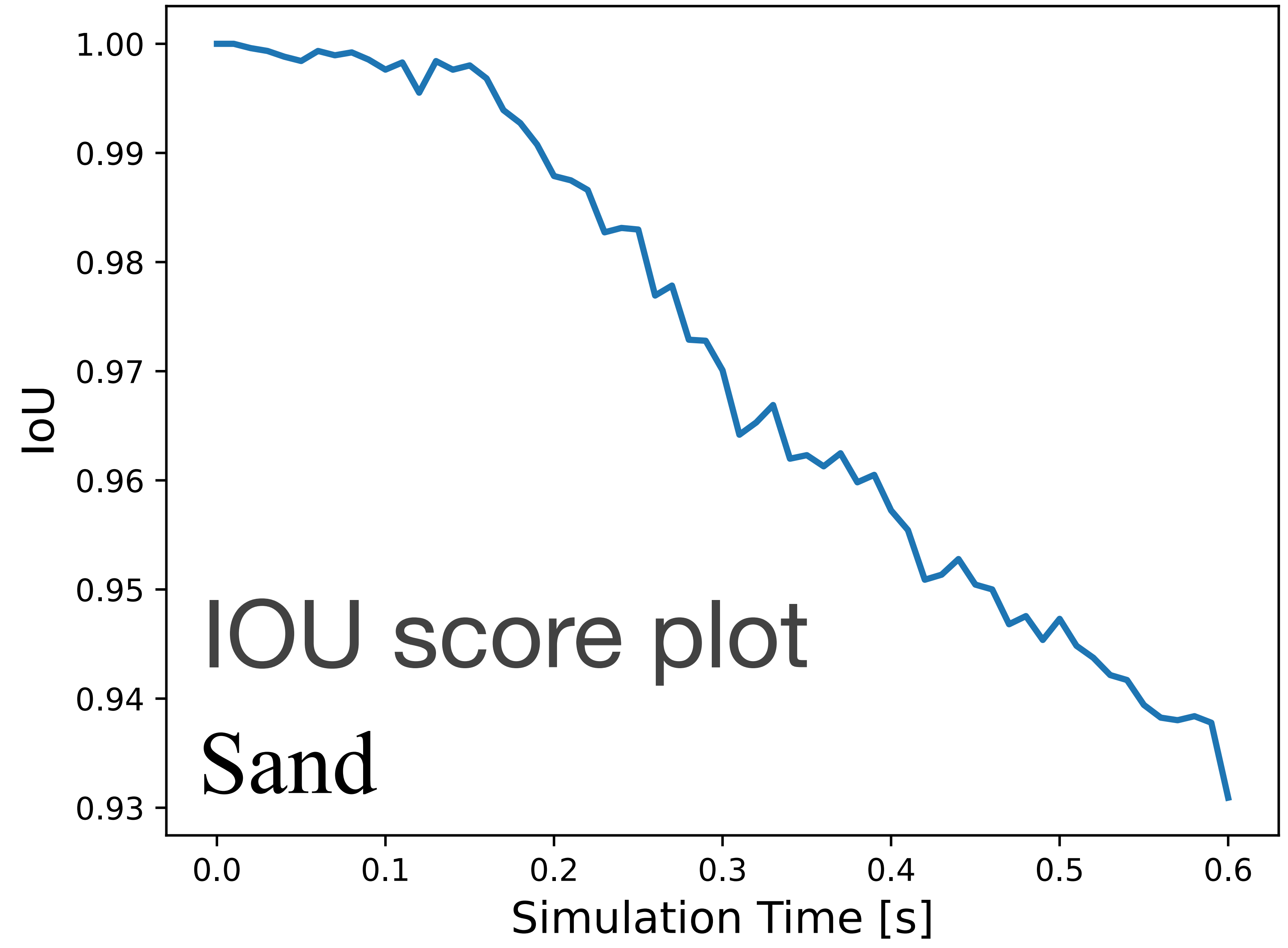
2D Sand



Ground Truth
dt = 1e-5

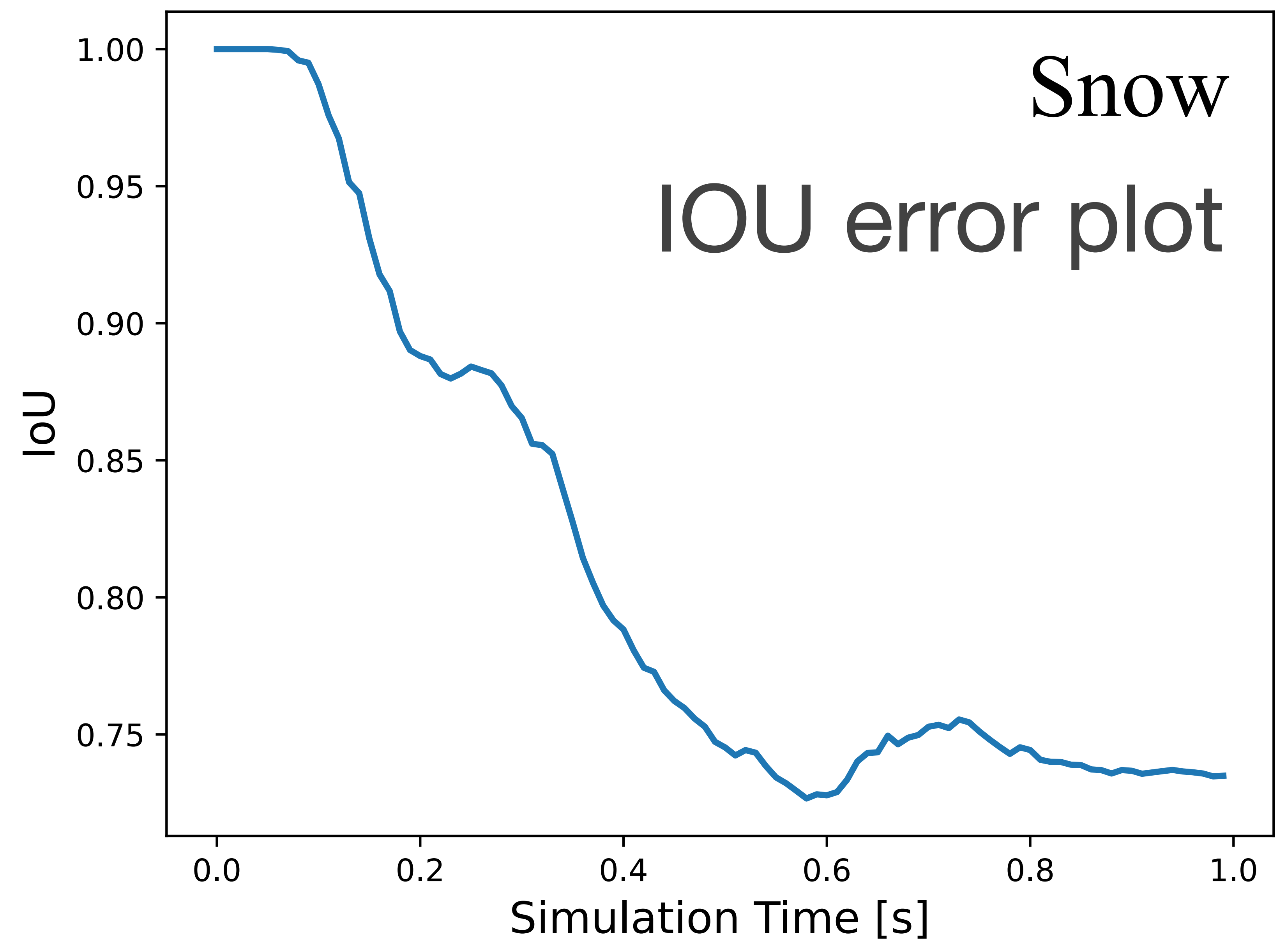
StVK Elasticity + Drucker-Prager Plasticity

Ours
dt = 1e-5

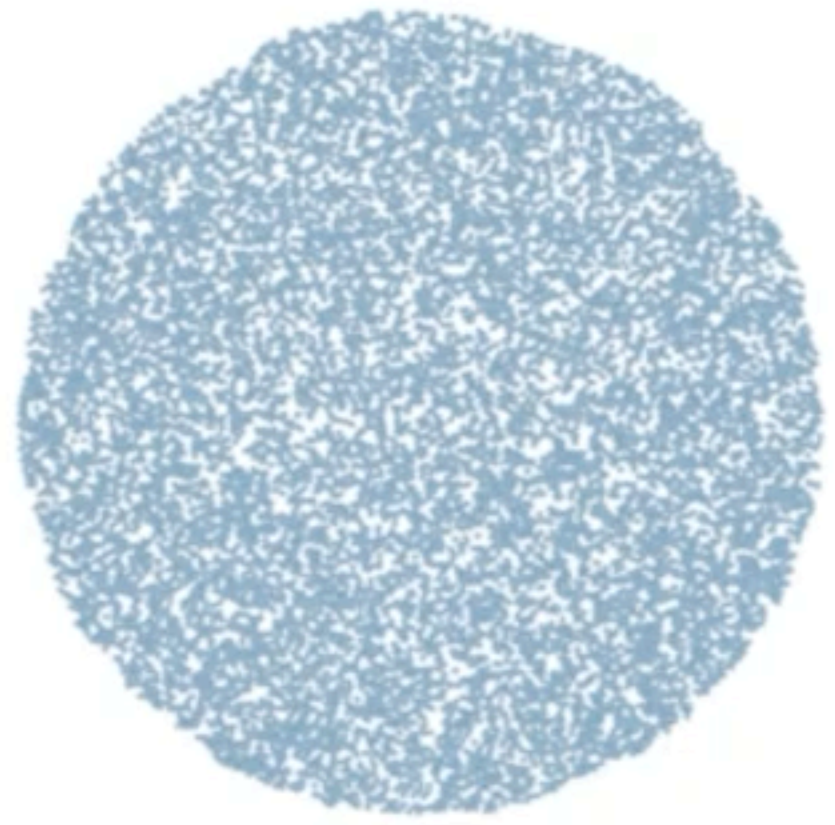
2D Sand

Ours
dt = 1e-3

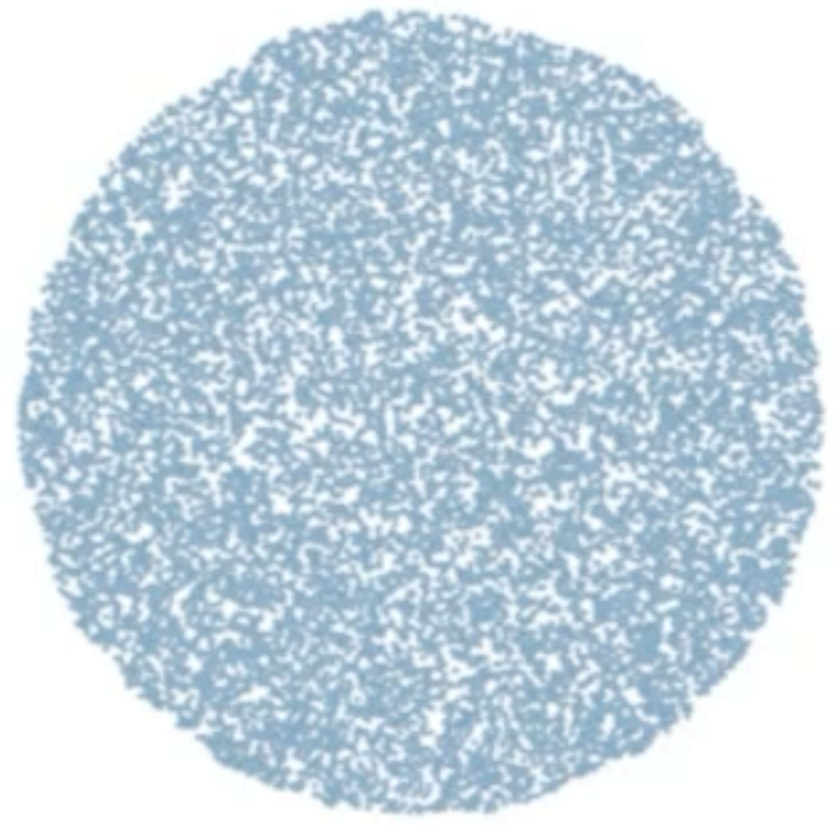
2D Snow



Ground Truth
dt = 1e-5



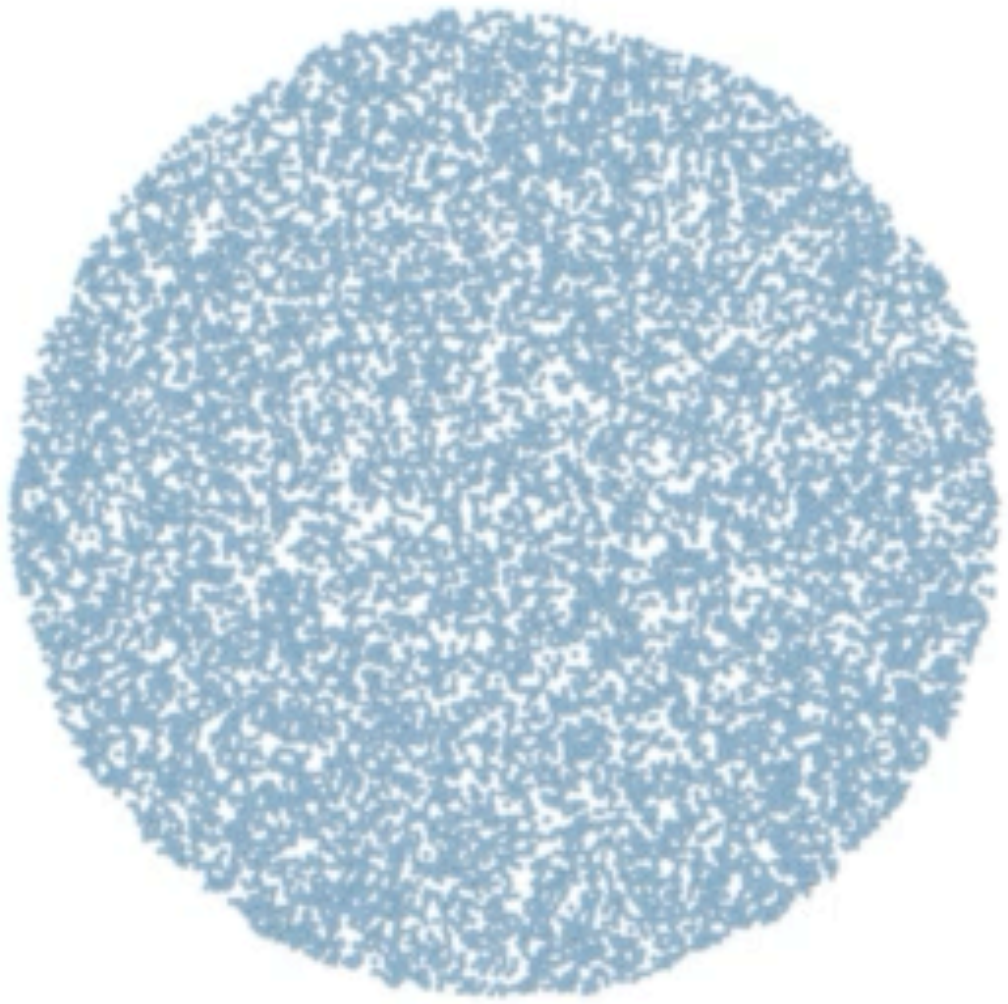
Ours
dt = 1e-5



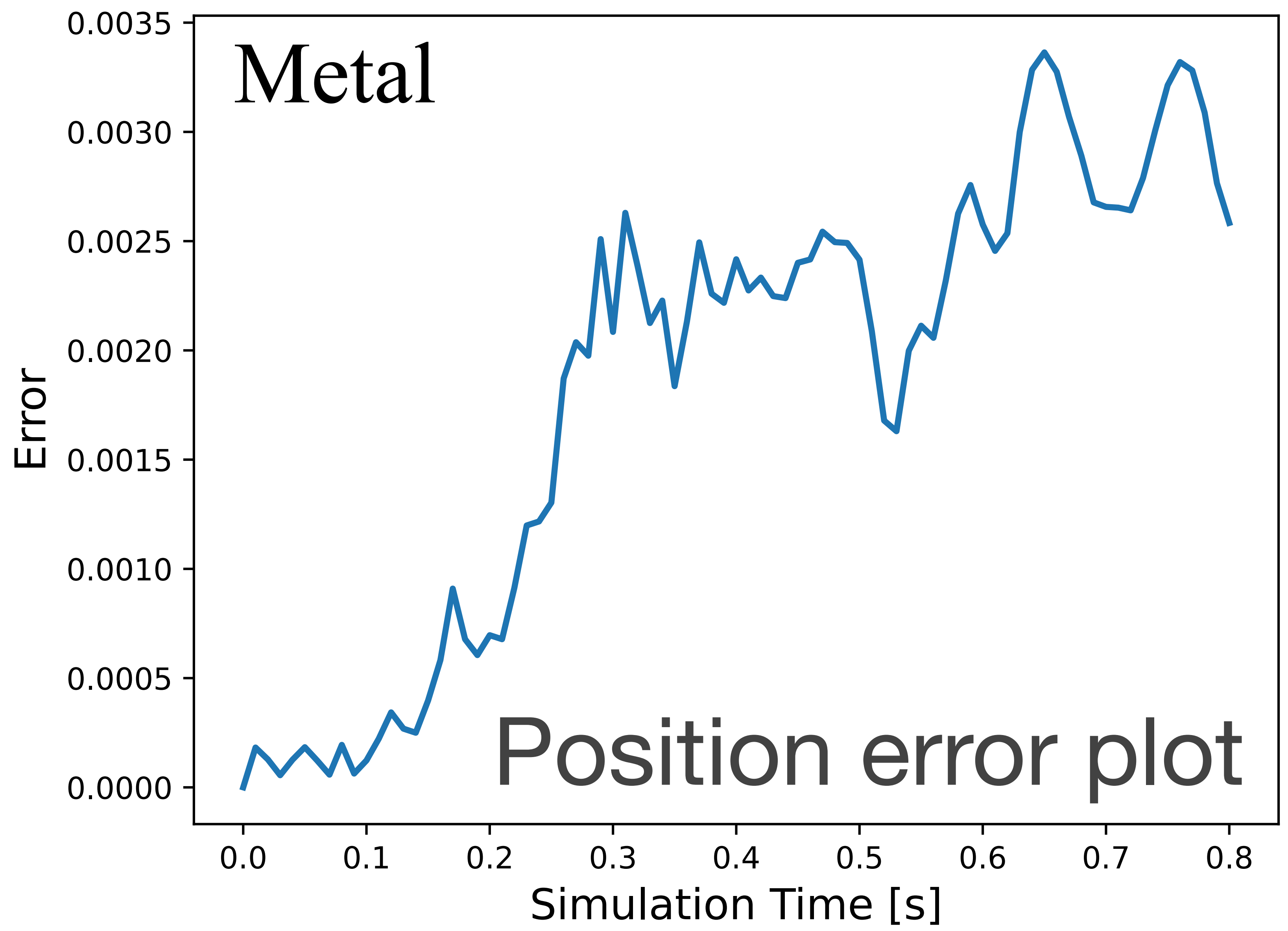
Neo-Hookean Elasticity + Cam-Clay Plasticity

2D Snow

Ours
dt = 1e-3

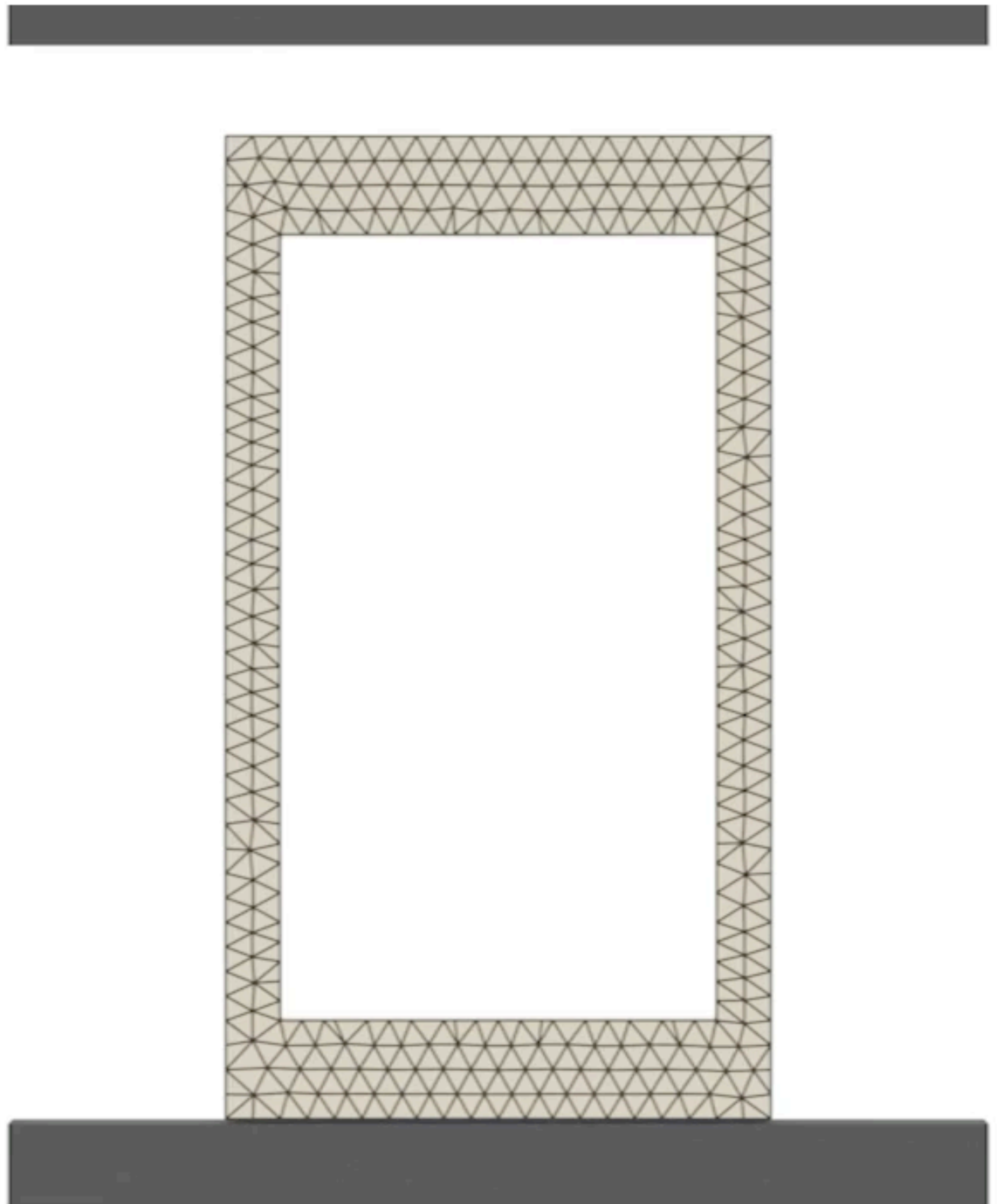


2D Metal



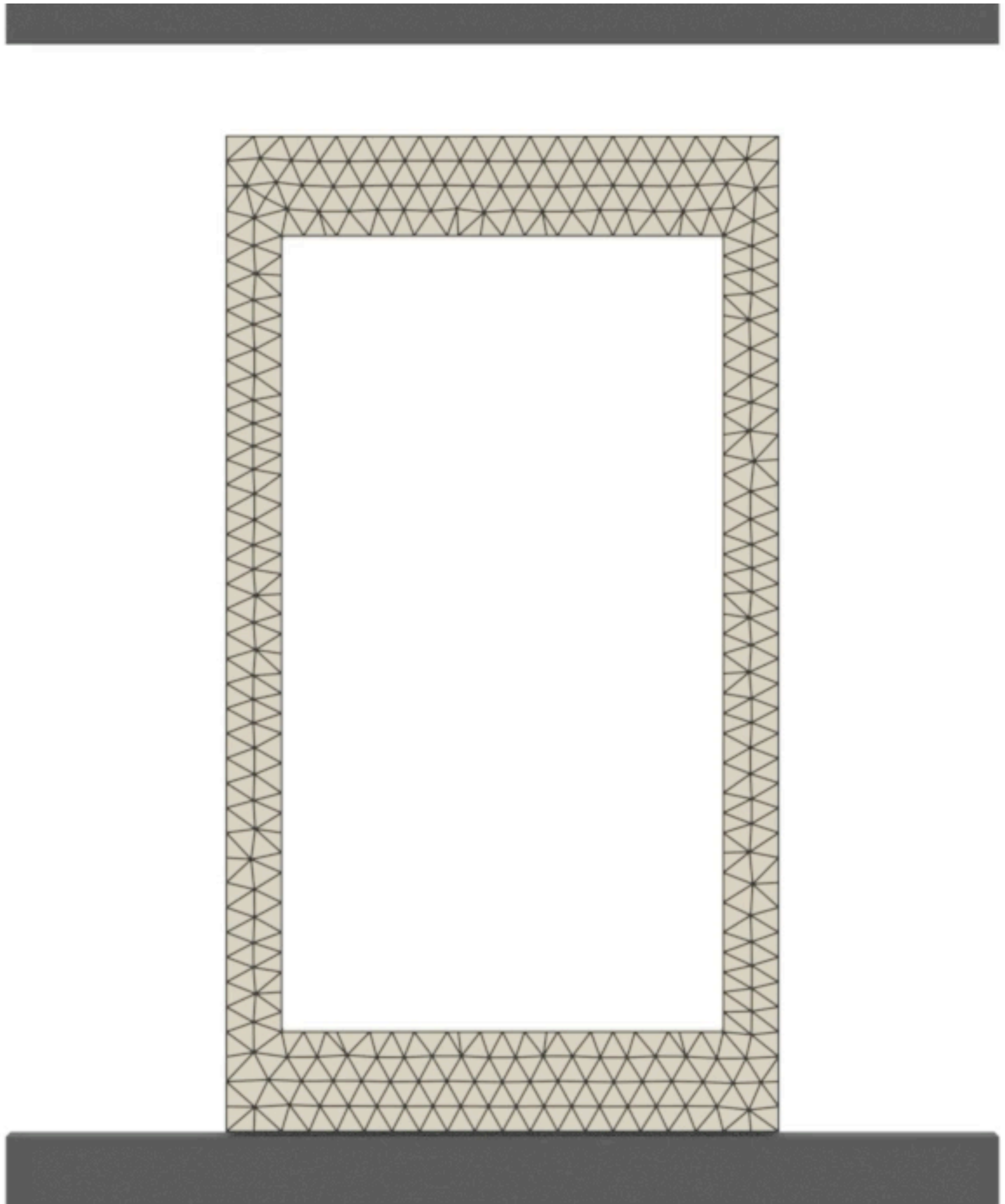
Ground Truth
E = 1e7
dt = 1e-5

Ours
E = 1e7
dt = 1e-5



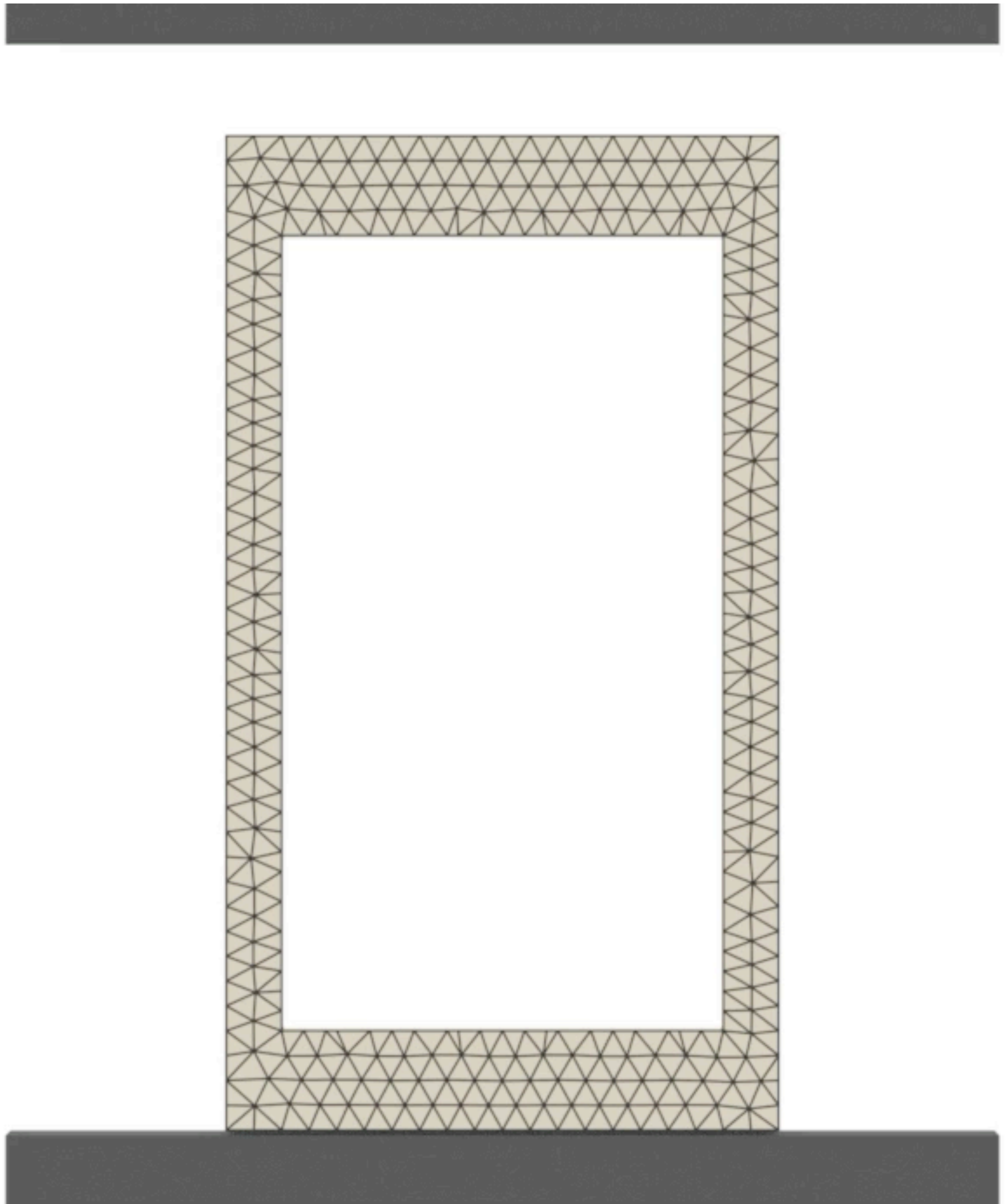
StVK Elasticity
Von-Mises Plasticity

2D Metal



Ours
E = 1e10
dt = 1e-2

Learning Metal Plasticity Return Mapping



E = 1e10
dt = 1e-2

Neo-Hookean Elasticity
Learned Von-Mises Plasticity

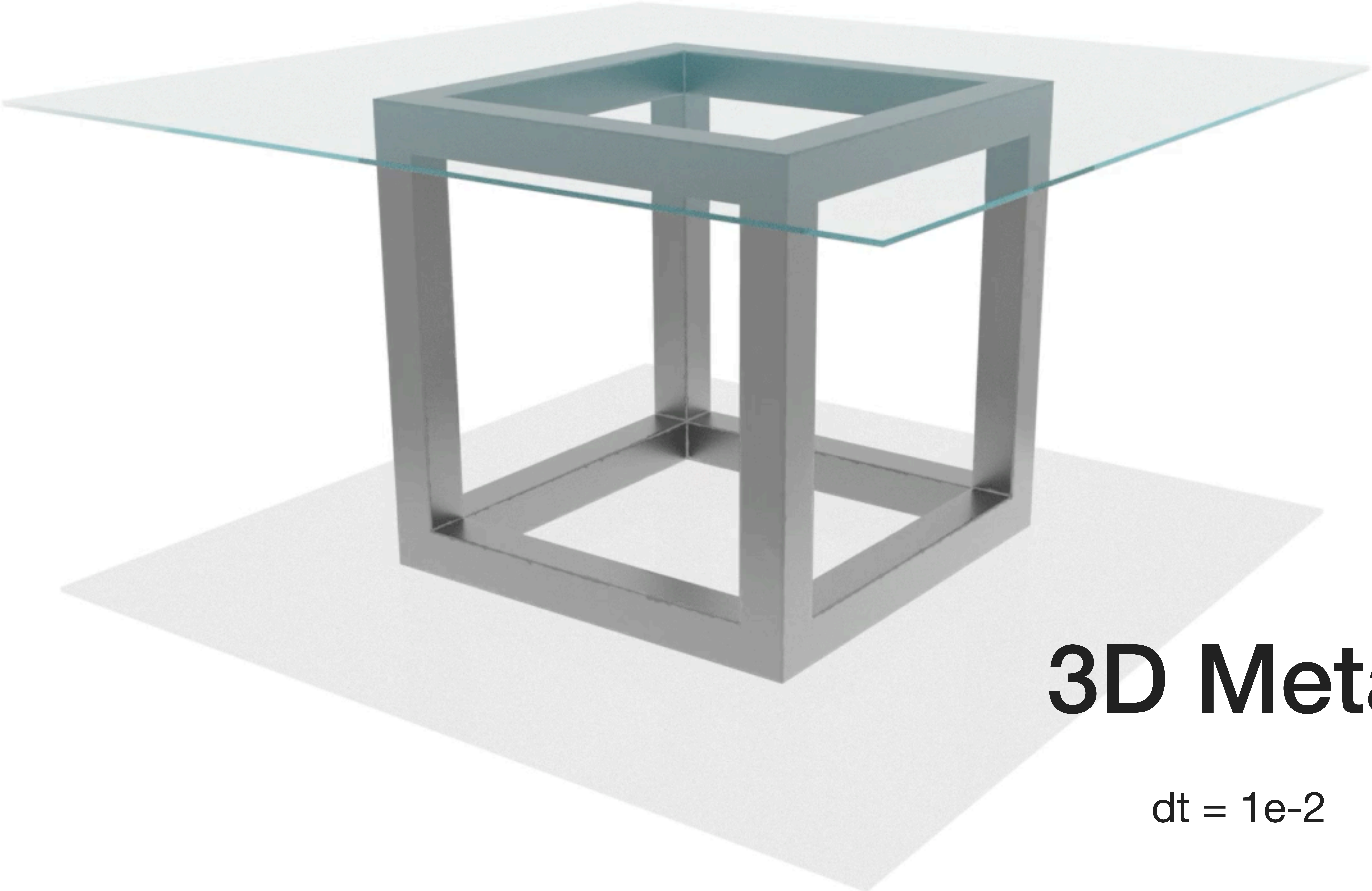
3D Examples

3D Sand

dt = 1e-3

3D Snow

dt = 1e-3



3D Metal

dt = 1e-2

Thanks for Watching!