Pareto-Optimal Learning-Augmented Algorithms for Online Conversion Problems

Bo Sun, Russell Lee, Mohammad Hajiesmaili, Adam Wierman, Danny H.K. Tsang

HKUST

UMass Amherst

Caltech

Learning-Augmented Online Algorithms

- Online algorithm makes online decisions when inputs are revealed **piece-by-piece** and **future input pieces are unknown**
  - competitive ratio

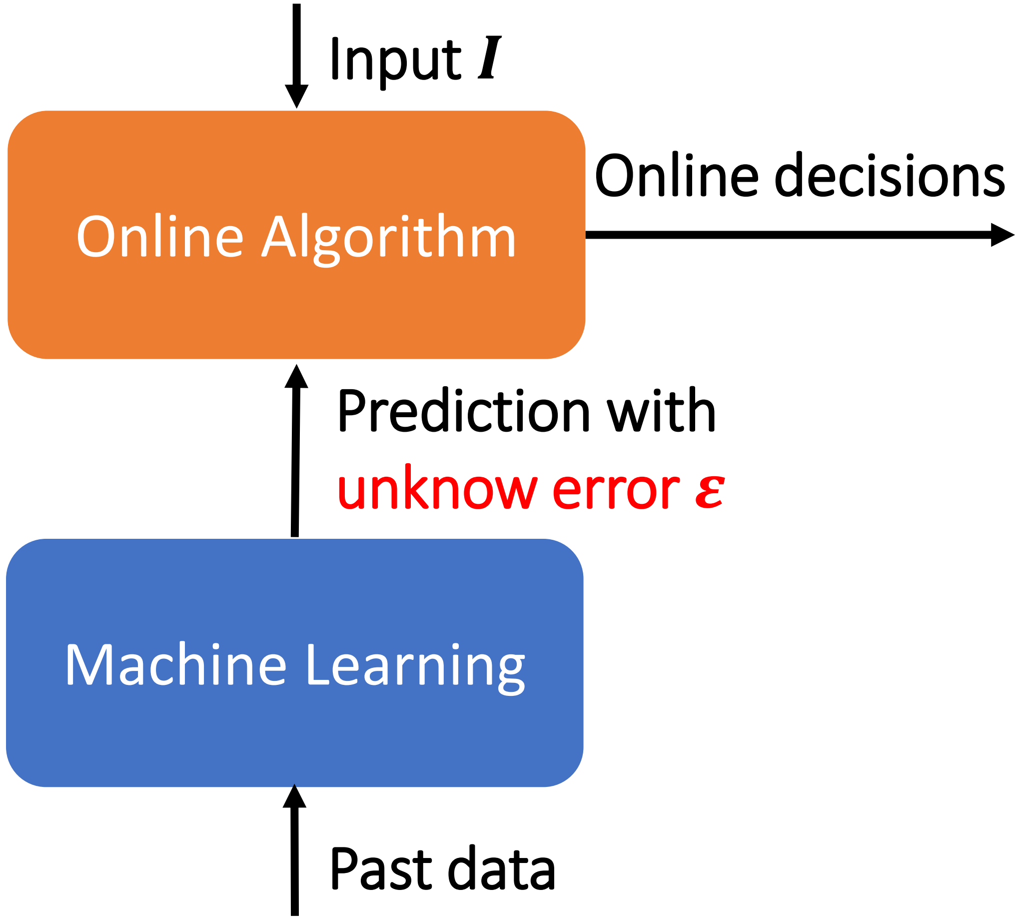
$$\mathrm{CR} = \max_I \frac{\mathrm{OPT}(I)}{\mathrm{ALG}(I)}$$

OPT(I) → offline optimal
ALG(I) → online algorithm

- Learning-augmented algorithm leverages predictions about the future
  - competitive ratio as a function of ε

$$\mathrm{CR}(\varepsilon) = \max_I \frac{\mathrm{OPT}(I)}{\mathrm{ALG}(I,\varepsilon)}$$

ALG(I, ε) → Learning-augmented algorithm when prediction error is ε



Two Important Metrics: Consistency and Robustness

Competitive Caching with Machine Learned Advice
ICML 2018 & Journal of the ACM 2021

Improving Online Algorithms via ML Predictions
NeurIPS 2018

The Primal-Dual method for Learning Augmented Algorithms
NeurIPS 2020

Optimal Robustness-Consistency Trade-offs for Learning-Augmented Online Algorithms
NeurIPS 2020

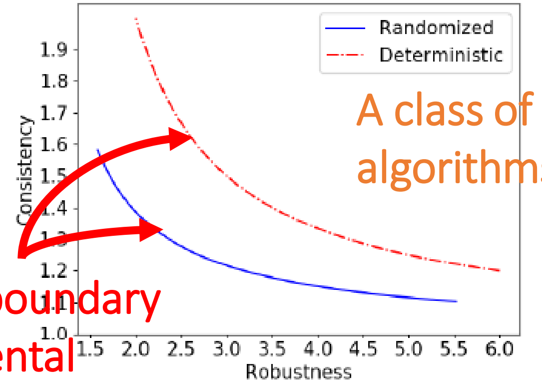
Alexander Wei
UC Berkeley
awei@berkeley.edu

Fred Zhang
UC Berkeley
z0@berkeley.edu

Concepts

- $\mathrm{CR}(0) \leq \eta$ (consistency)
- $\max_{\varepsilon} \mathrm{CR}(\varepsilon) \leq \gamma$ (robustness)

Trade-offs



Pareto-optimality

- for a given robustness, no online algorithms can achieve a better consistency

Can we design learning-augmented algorithms with bounded robustness and consistency for other online problems?

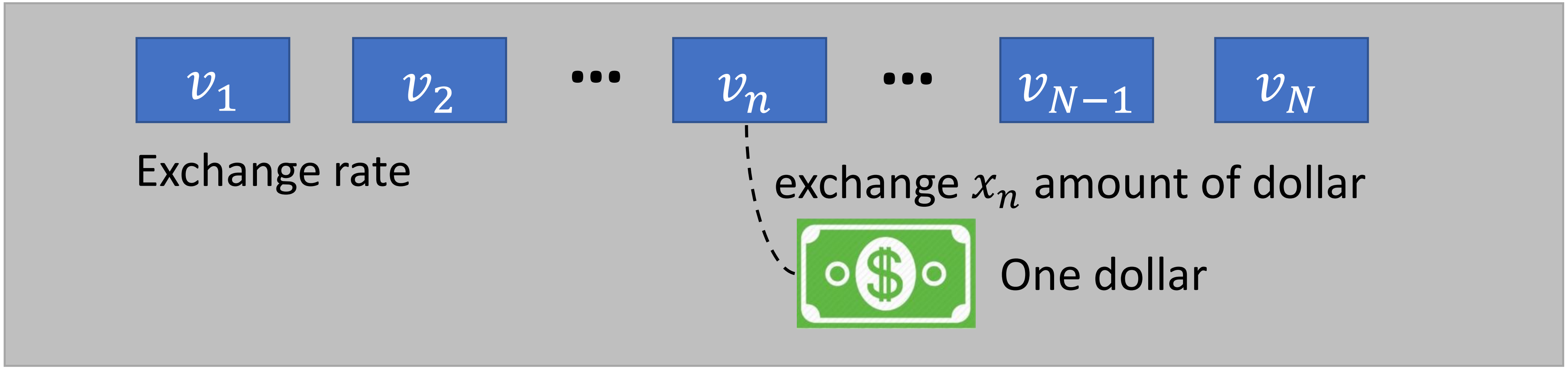
Yes, secretary and online matching [Antoniadis2020, NeurIPS], metrical task systems [Antoniadis2020, ICML], etc.

Can we design Pareto-optimal learning-augmented algorithms for other online problems?

Only known for ski rental problem

Our contribution: we design Pareto-optimal algorithms for online conversion problems

Online Conversion Problem



- Exchange one asset (e.g., one dollar) to another asset over time varying exchange rates
- For $n = 1, \dots, N$
  - observe an exchange rate (or price) v_n
  - decide the amount of dollar to trade, $x_n \in \mathcal{X}_n$, and obtain return $x_n v_n$
- Goal: maximize the total return $\sum_{n\in[N]} x_n v_n$

1-max-search: $\mathcal{X}_n = \{0,1\}$

one-way trading: $\mathcal{X}_n = [0,1]$

Unified Online Threshold-Based Algorithm (OTA)

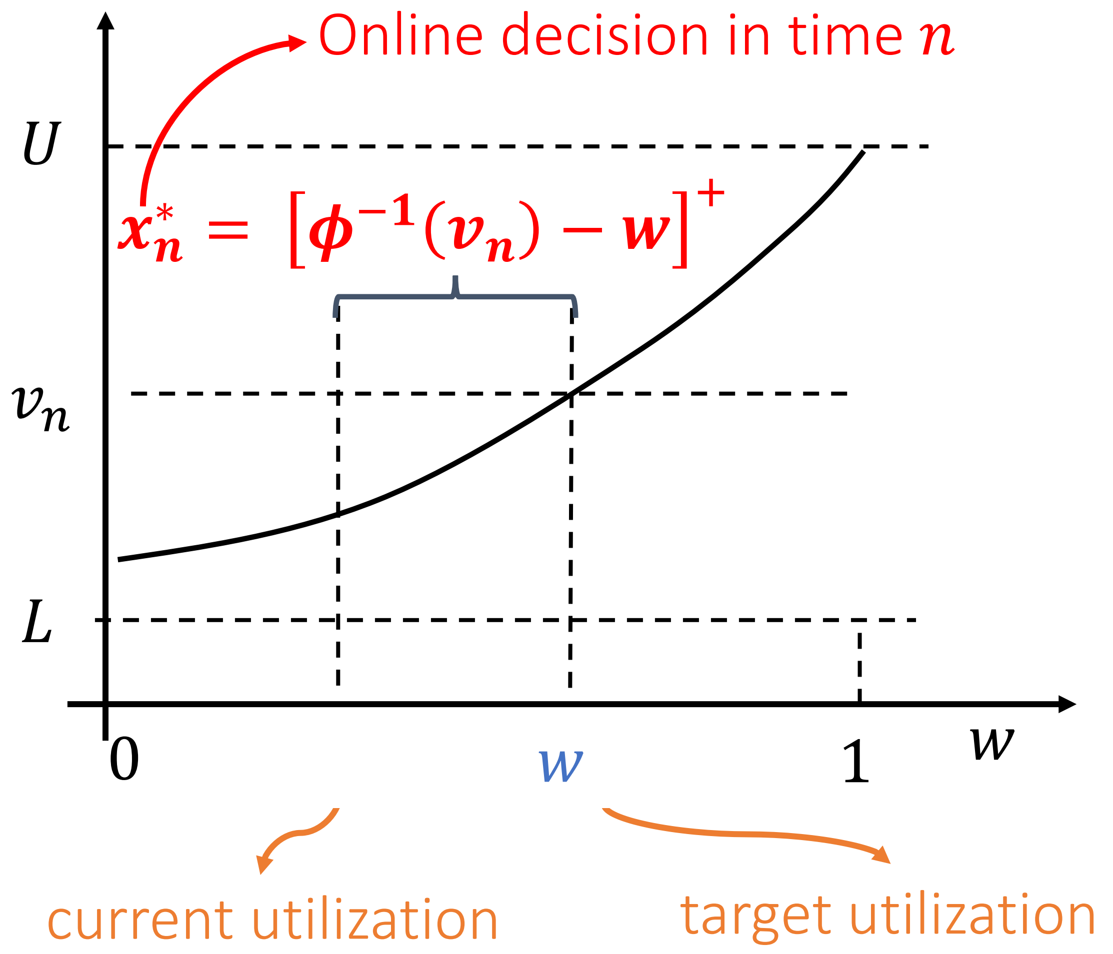
- Initialization
  - threshold $\phi(w)$: $[0,1] \rightarrow [L, U]$

    utilization (cumulative traded dollar)

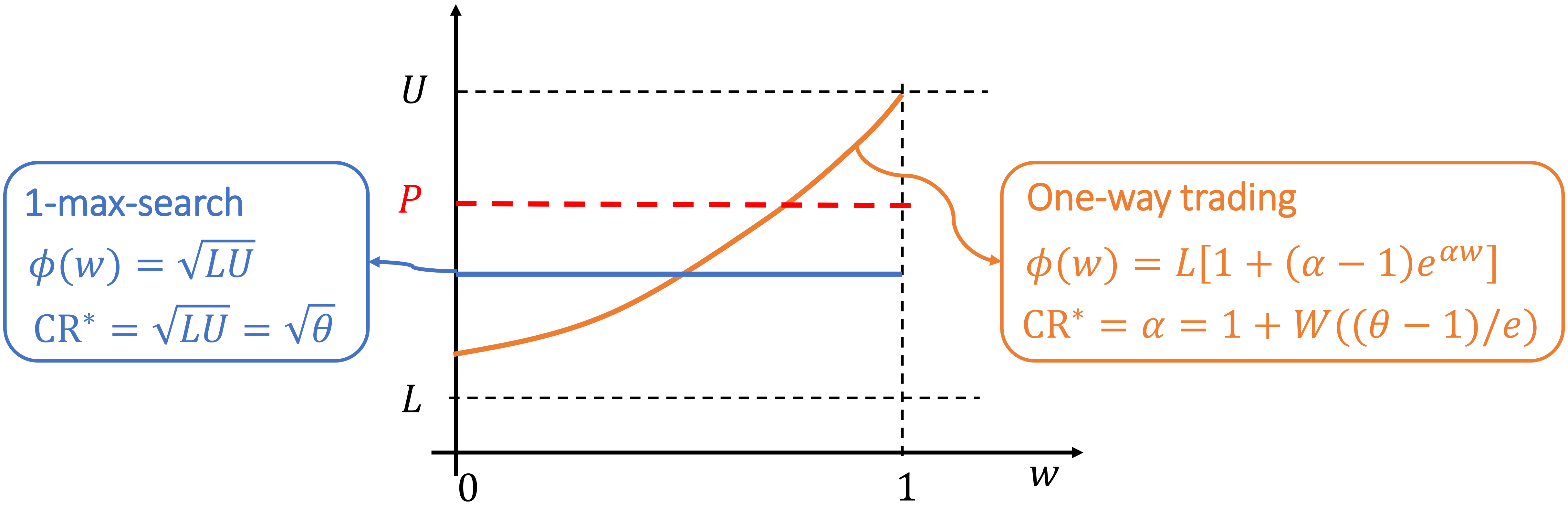
- For each n do
  - observe current utilization w and exchange rate v_n
  - determine x_n^* by solving

$$\max_{x_n \in \mathcal{X}_n} \quad v_n x_n - \int_{w^{(n-1)}}^{w^{(n-1)}+x_n} \phi(u) du$$

  - update $w \leftarrow w + x_n^*$



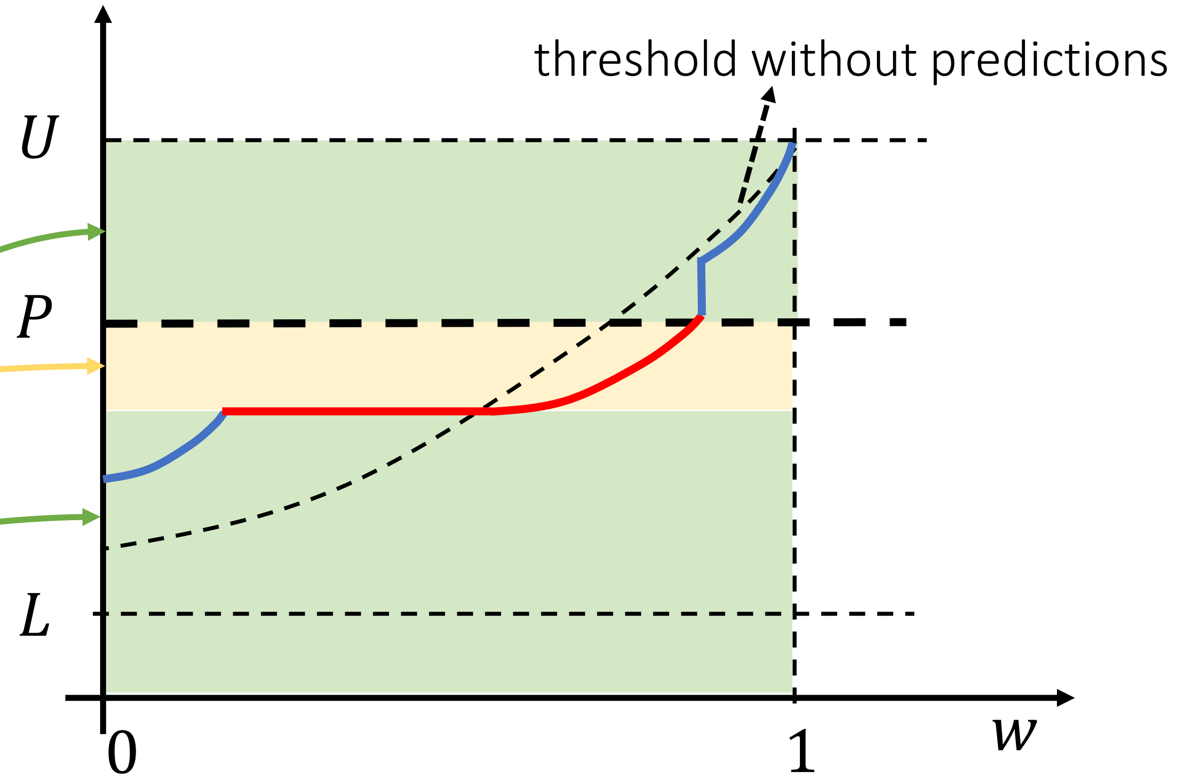
Design of Threshold Function for OTA



Given a prediction of the maximum exchange rate $\boldsymbol{P}$, how to incorporate it into the threshold function to guarantee robustness and consistency?

Threshold Function for Learning-Augmented OTA

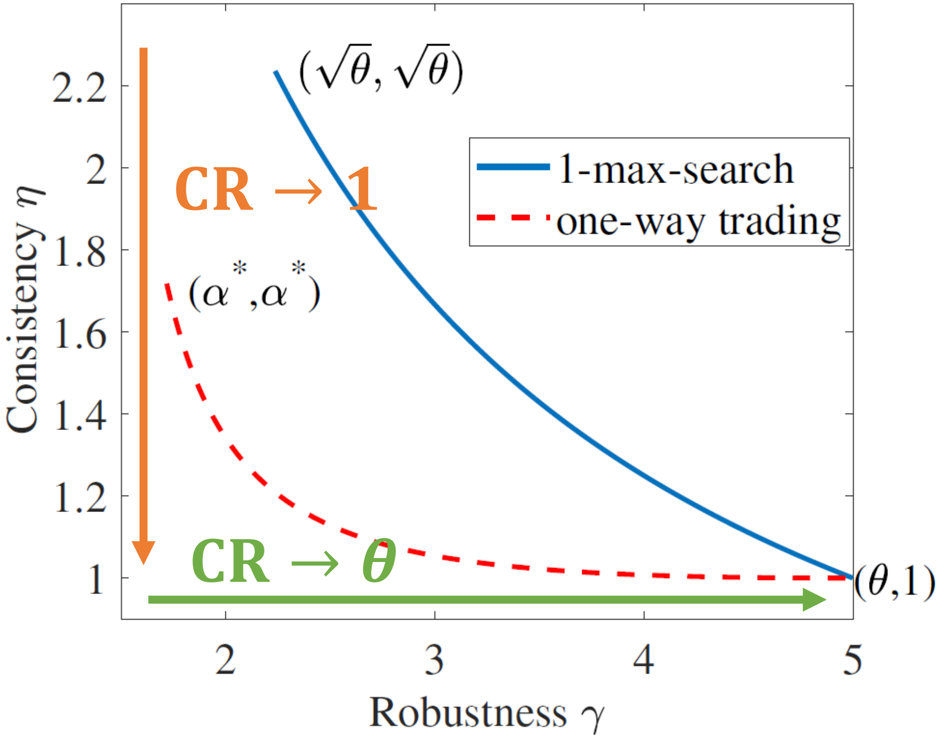
- The threshold function consists of multiple segments
  - price region near prediction P, ensure consistency
  - price region away from prediction P, ensure robustness



Theorem (sufficient conditions):

If each segment satisfies a set of differential equations, learning-augmented OTA is with consistency and robustness upper bounds.

Pareto-Optimal Learning-Augmented OTA



- As consistency improves $\mathrm{CR} \to 1$ (best possible ratio), robustness degrades $\mathrm{CR} \to \theta$ (worst possible ratio)
- Pareto boundary of one-way trading dominates that of 1-max-search

Theorem (Pareto-Optimality):

Learning-augmented OTA is Pareto-optimal, i.e., for a given robustness, no online algorithms can achieve a better consistency

Take-Away Message

We have designed Pareto-optimal learning-augmented threshold-based algorithms for online conversion problems