

# On the value of Interaction and Function approximation in Imitation Learning

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# Challenges in RL

Rewards for practical RL problems are often hard to specify.

$$r(b_z^{(1)}, s^P, s^{B1}, s^{B2}) = \begin{cases} 1 & \text{if stack}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

$$r(b_z^{(1)}, s^P, s^{B1}, s^{B2}) = \begin{cases} 1 & \text{if stack}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \\ 0.25 & \text{if } \neg \text{stack}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \wedge \text{grasp}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

$$r(b_z^{(1)}, s^P, s^{B1}, s^{B2}) = \begin{cases} 1 & \text{if stack}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \\ 0.25 & \text{if } \neg \text{stack}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \wedge \text{grasp}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \\ 0.125 & \text{if } \neg(\text{stack}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \vee \text{grasp}(b_z^{(1)}, s^P, s^{B1}, s^{B2})) \wedge \text{reach}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$r(b_z^{(1)}, s^P, s^{B1}, s^{B2}) = \begin{cases} 1 & \text{if stack}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \\ 0.25 + 0.25r_{S2}(s^{B1}, s^P) & \text{if } \neg \text{stack}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \wedge \text{grasp}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \\ 0.125 & \text{if } \neg(\text{stack}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \vee \text{grasp}(b_z^{(1)}, s^P, s^{B1}, s^{B2})) \wedge \text{reach}(b_z^{(1)}, s^P, s^{B1}, s^{B2}) \\ 0 + 0.125r_{S1}(s^{B1}, s^P) & \text{otherwise} \end{cases}$$

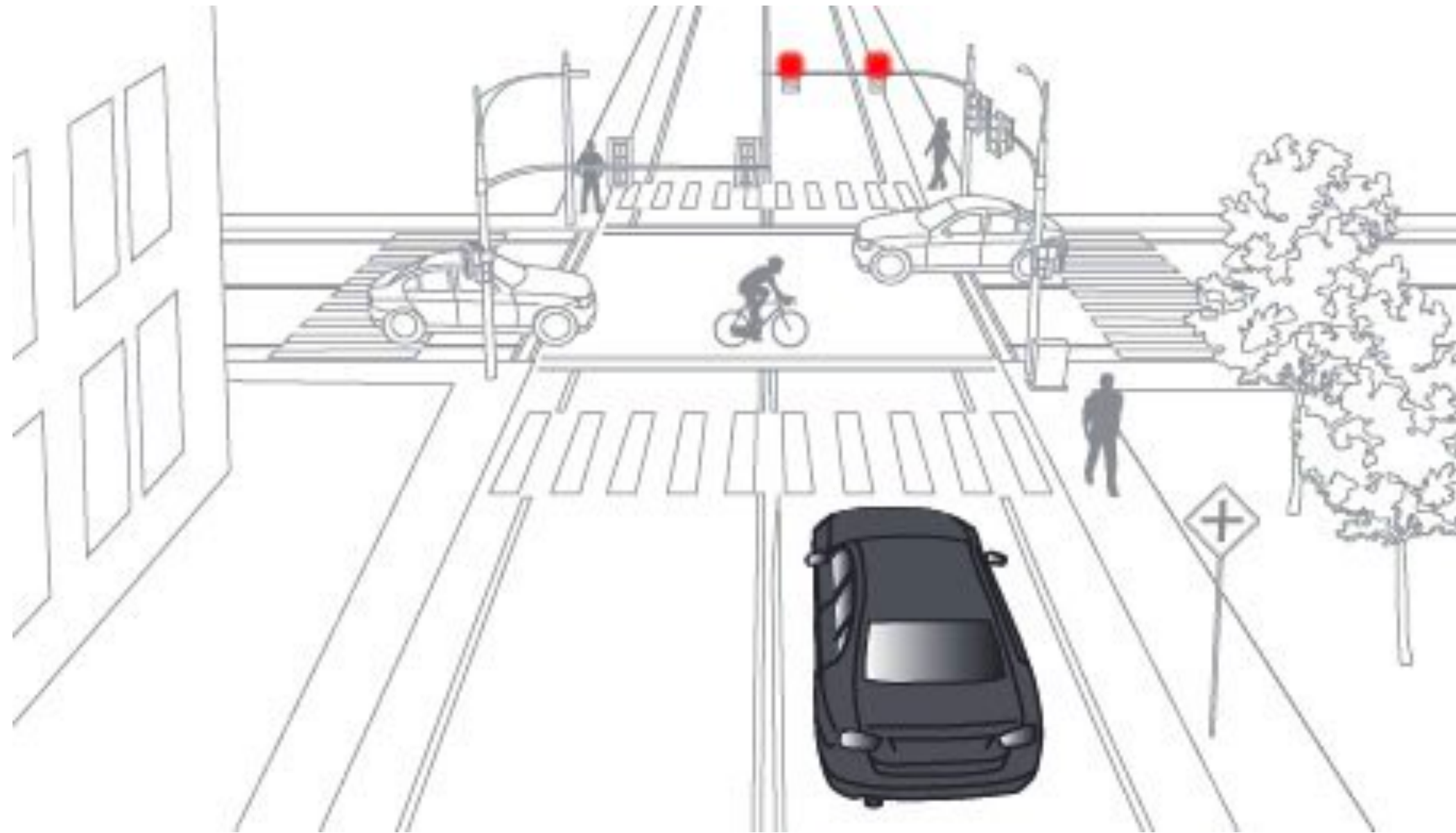
Popov et al. 2017

Reward design must be consistent with counterfactual questions:

***“What would an expert have done?”***

Need to correctly balance **interpretability** and **sparsity**.

# Imitation learning over reward engineering



Expert demonstrations



Learner

“Learning from demonstrations in the absence of reward feedback”

# Motivation

*What are the theoretical limits of Imitation Learning (i) with interaction and (ii) in the presence of function approximation?*

## Notation:

$J(\pi)$ : Expected total reward of policy  $\pi$  in an episode of length  $H$ .

Learner  $\hat{\pi}$  tries to minimize Suboptimality  $\triangleq \mathbb{E} [J(\pi^*) - J(\hat{\pi})]$ ,  $\pi^*$  is expert's policy

- Difference in expected reward of the expert and the learner policy.

# Theoretical understanding of IL: Prior work

**No interaction:** *Learner is only provided a dataset of  $N$  expert demonstrations;  
Cannot interact with the MDP*

## **Theorem [RYJR20]**

*In the no-interaction and tabular setting, Behavior Cloning achieves,*

$$\text{Suboptimality} \lesssim \frac{SH^2 \log(N)}{N}$$

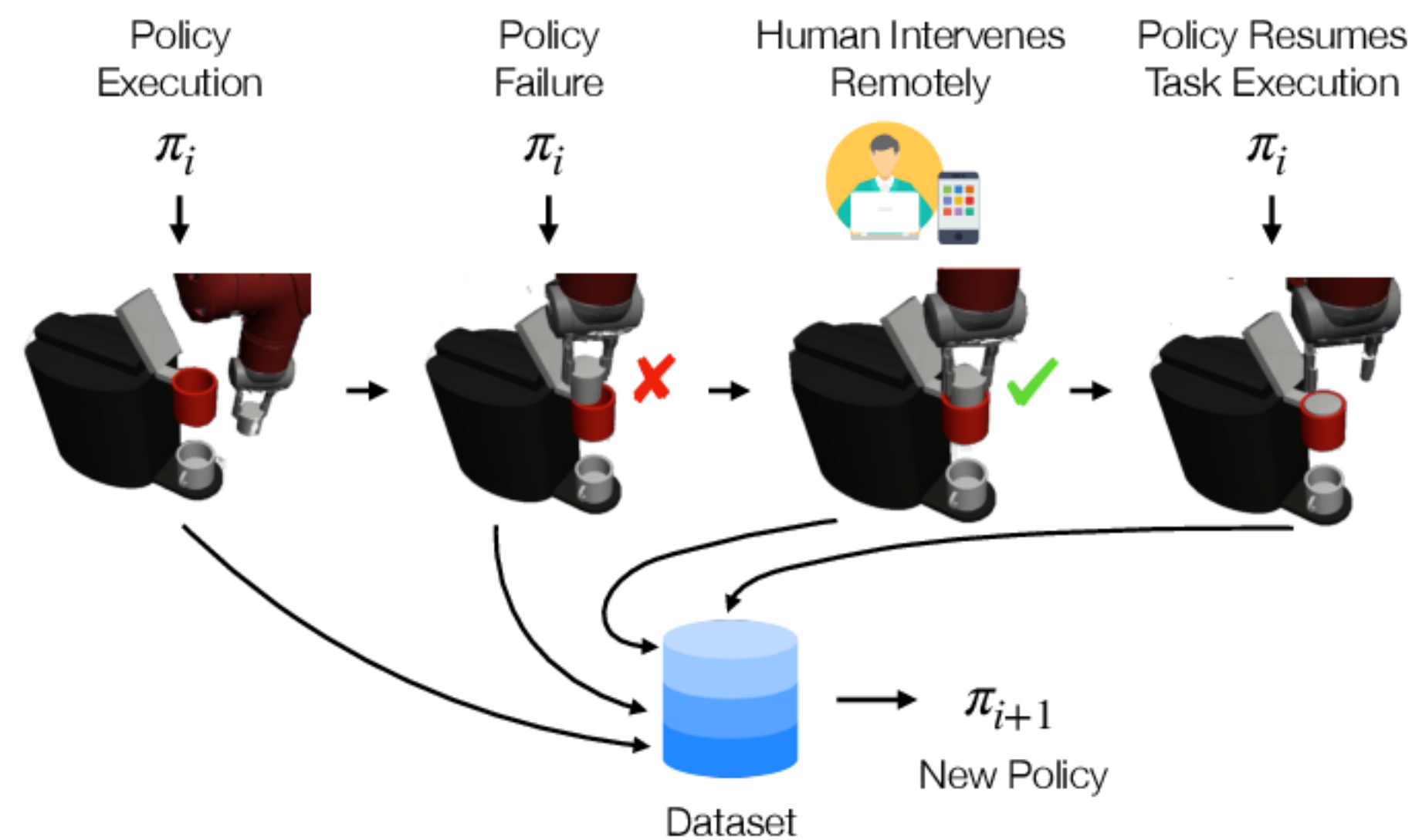
*Best achievable (up to log-factors) by any algorithm.*



# Going beyond the no-interaction setting

**Interactive expert:** *Learner can interact with the environment  $N$  times and query the expert policy at visited states*

Setting is closely related to human-in-the-loop RL

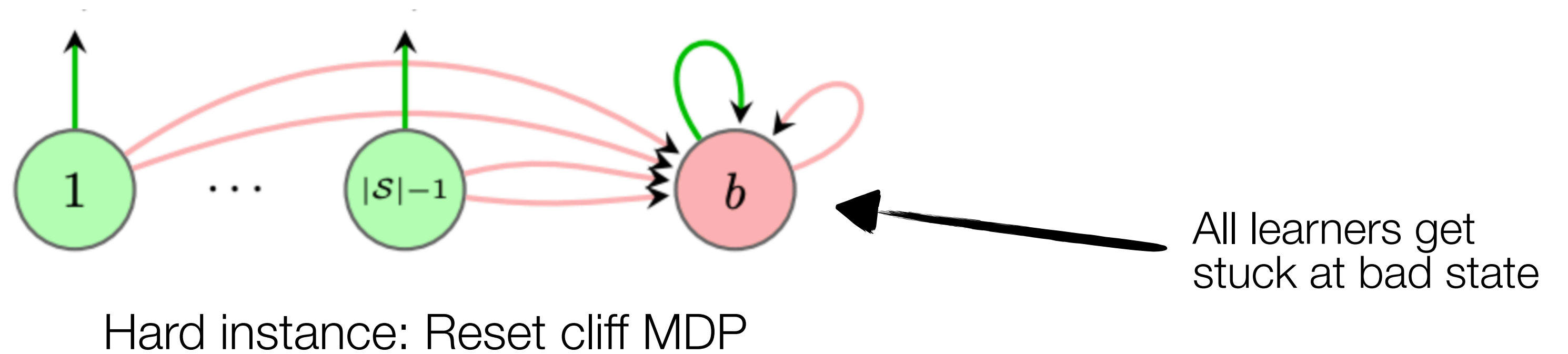


# IL with an interactive expert

Is it possible to improve the suboptimality of behavior cloning if the expert is **interactive**?

In the worst case, **no**.

For all algorithms even with an interactive expert, in the worst case,  
Suboptimality  $\gtrsim SH^2/N$  [RYJR20]



# IL with an interactive expert

Is it possible to improve the suboptimality of behavior cloning if the expert is **interactive**?

$\mu$ -recoverability assumption [RB11]: For any state  $s$ , action  $a'$ ,

$$\max_a Q_t^*(s, a) - Q_t^*(s, a') \leq \mu$$

**Interpretation:** *Expert knows how to “recover” after making a mistake at some time  $t$  and pays an expected cost of at most  $\mu$ .*



# IL with an interactive expert

Is it possible to improve the suboptimality of behavior cloning if the expert is **interactive**?

## Theorem 1 [RHYLJR21]

Under  $\mu$ -recoverability, in the *interactive* and *tabular* setting, **DAGGER** (FTRL) achieves,

$$\text{Suboptimality} \lesssim \frac{\mu SH \log(N)}{N}$$

*Best achievable (up to log-factors) by any algorithm.*

# IL with function approximation

*How do approaches such as BC and Mimic-MD [RYJR20] perform in the presence of function approximation?*

# IL with linear function approximation

**Linear expert:** For every state  $s$ , the deterministic expert plays an action

$$\pi_t^*(s) \in \operatorname{argmax}_a \langle \theta_t, \phi_t(s, a) \rangle$$

$\phi_t(s, a) \in \mathbb{R}^d$  is a known representation of state-actions

**Interpretation:** Expert policy is realized by a linear multi-class classifier

# Linear expert with no MDP interaction

## **Theorem 2 [RHYLJR21]:**

*In the **no-interaction** and **linear expert** setting, **Behavior Cloning** achieves,*

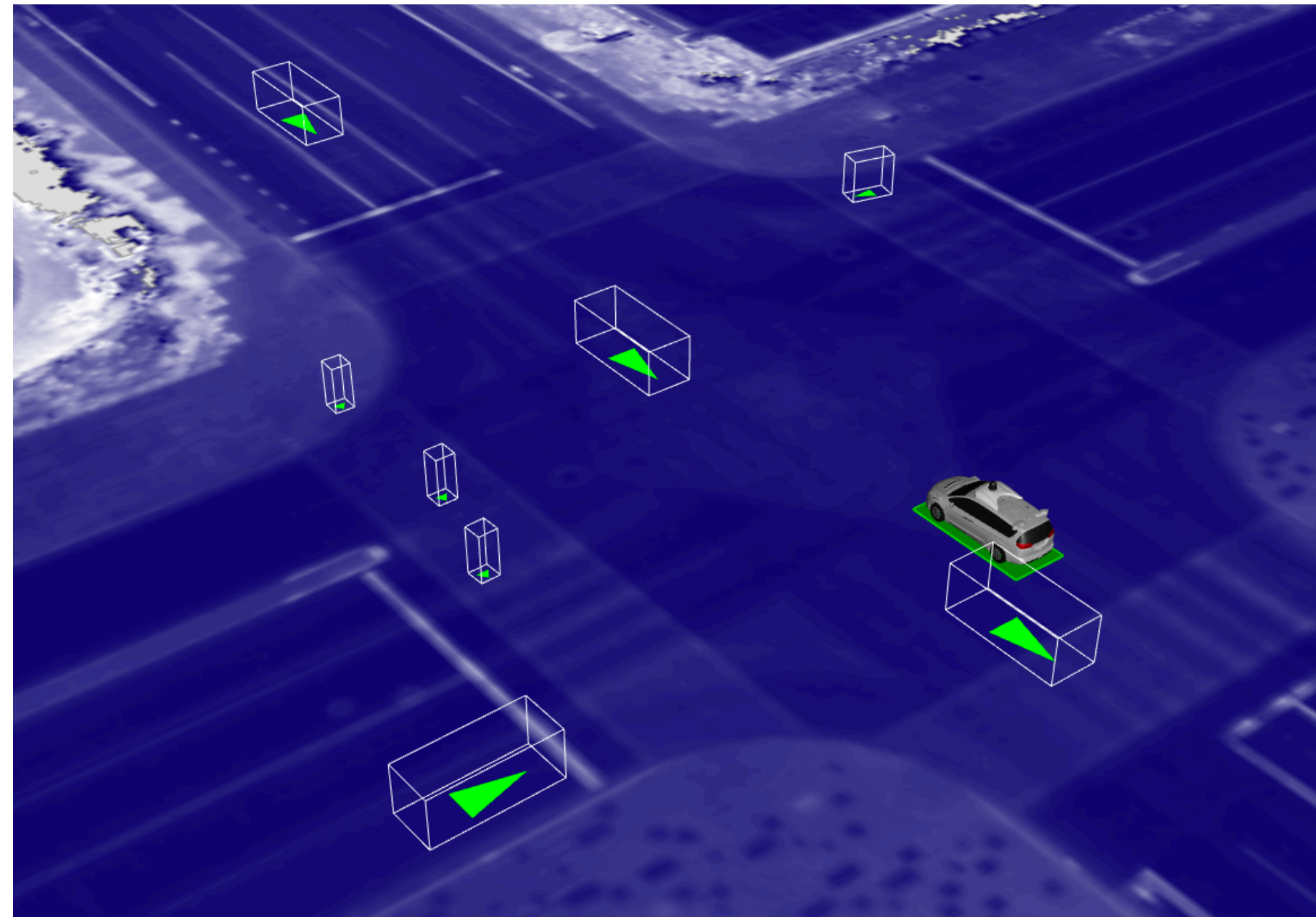
$$\text{Suboptimality} \lesssim \frac{dH^2 \log(N)}{N}$$

*With  $d = S$  recovers bounds in the tabular setting.*



# Linear expert with known transition

**Known transition:** *Learner is provided a dataset of  $N$  expert demonstrations;*  
***Knows the MDP transition***



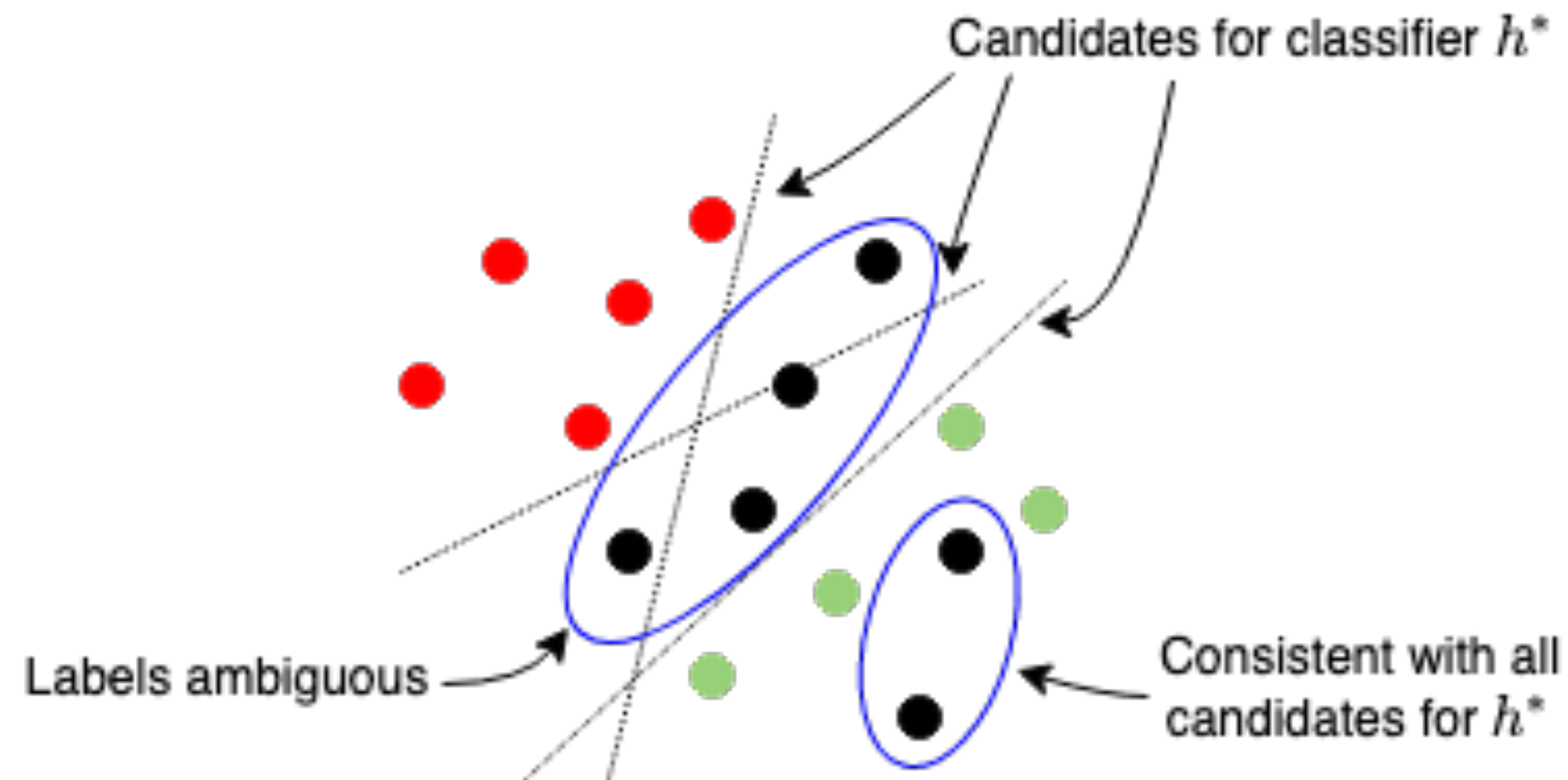
**Interpretation:** carrying out Imitation Learning in a simulation environment.

# Linear expert with known transition

## Confidence set classification:

Consider classification over family of hypotheses,  $\mathcal{H}$  from  $\mathcal{X} \rightarrow \mathcal{Y}$ .

From a dataset of examples  $D$  from a classifier  $h^*$  return the largest measure of points where  $h^*(x)$  is known without ambiguity.



# Linear expert with known transition

## Theorem 3 [RHYLJR21]:

For each  $t$ , consider the linear classifier  $\pi_t^* : S \rightarrow A$ .

Given a confidence set classifier with expected loss  $\ell_t$ , there exists an IL algorithm such that,

$$\text{Suboptimality} \lesssim H^{3/2} \sqrt{\frac{d}{N} \frac{\sum_{t=1}^H \ell_t}{H}}$$

**Message:** *Error compounding ( $H^2$  dependence) can be broken if confidence set linear classification is possible to expected loss of  $o_N(1)$ .*



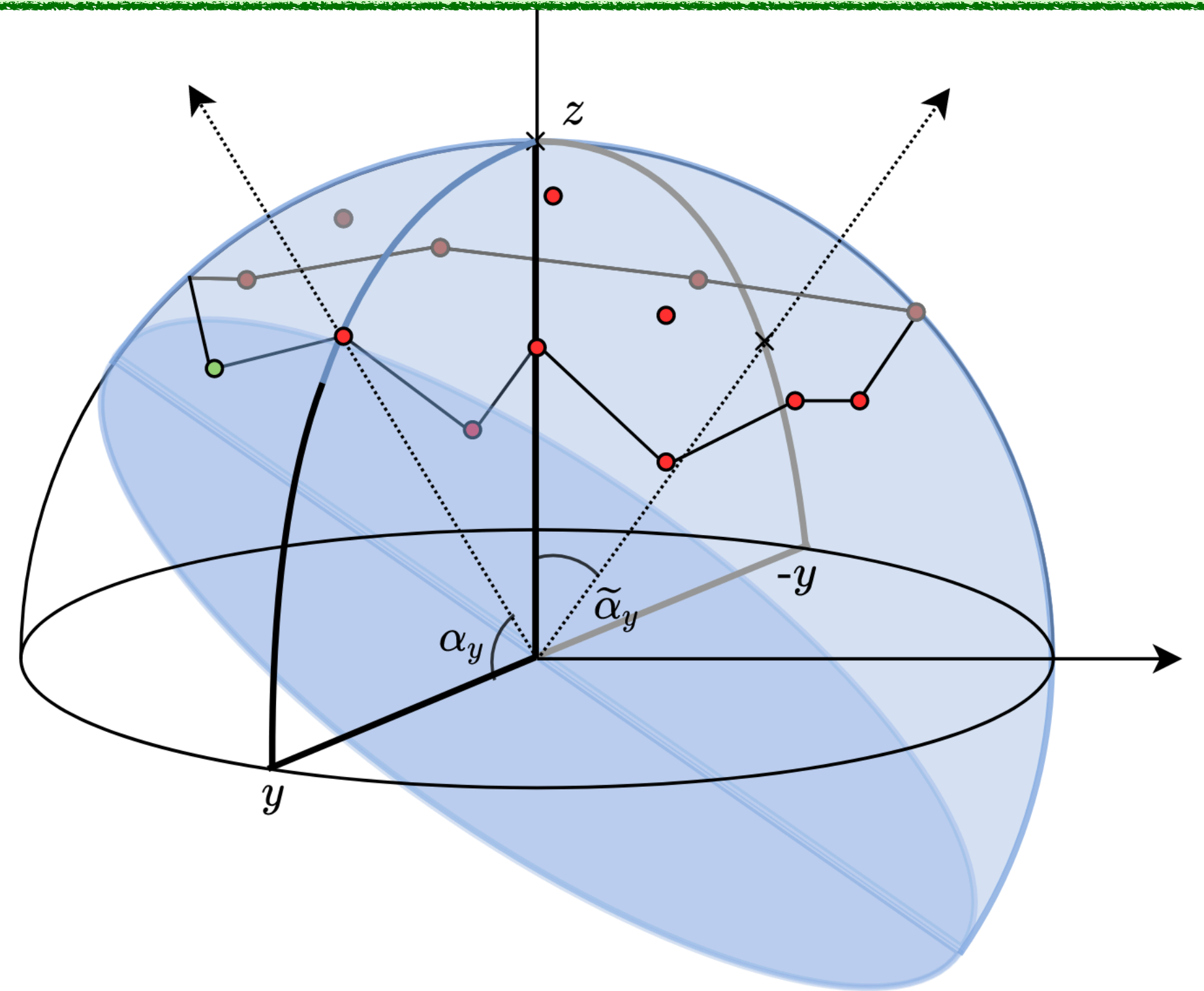
# Linear expert with known transition

## Theorem 4 [RHYLJR21]:

If distribution over inputs is uniform over the unit sphere  $\mathbb{S}^{d-1}$ , the minimax loss of confidence set linear classification is  $\Theta(d^{3/2}/N)$ .

*Confidence set linear classification is sample efficient for the uniform distribution*

**Extending to general distributions?**





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