# Shape As Points

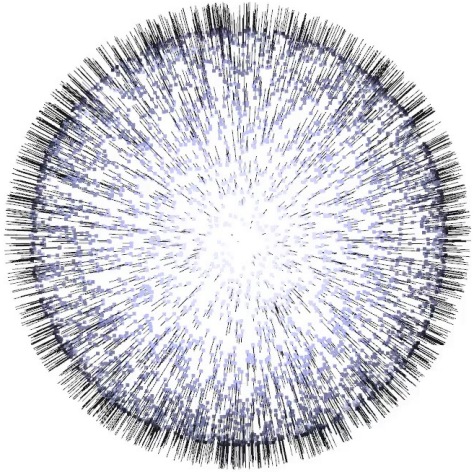
## A Differentiable Poisson Solver


**Songyou Peng** Chiyu “Max” Jiang Yiyi Liao Michael Niemeyer Marc Pollefeys Andreas Geiger

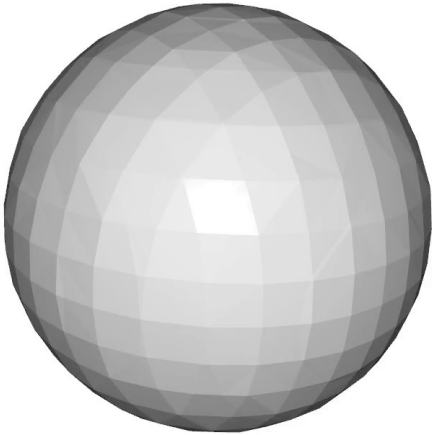








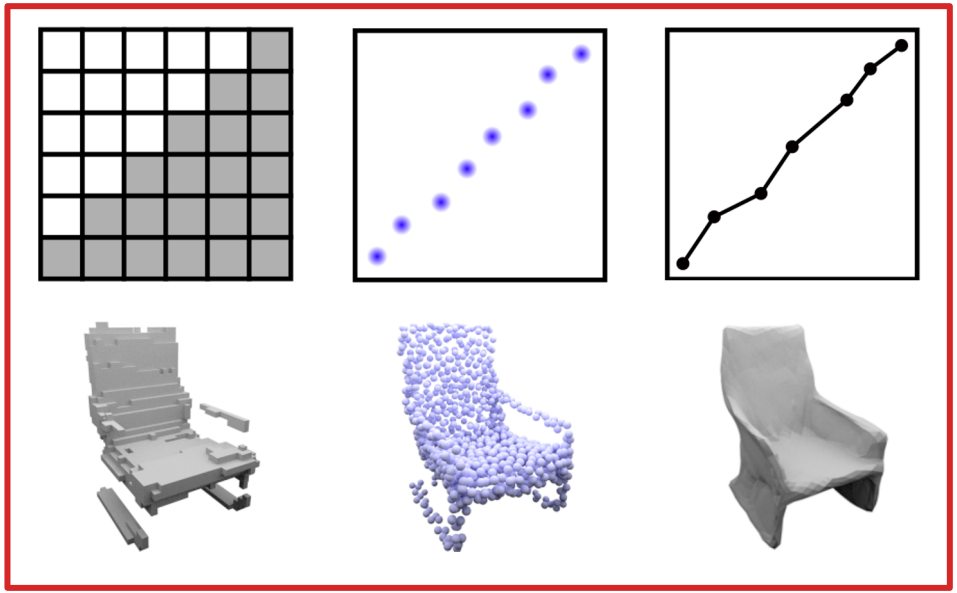
Shape As Points (SAP)



Duality between **oriented point clouds** and **3D dense geometry**

What is a good **3D shape representation**?

# 3D Shape Representations

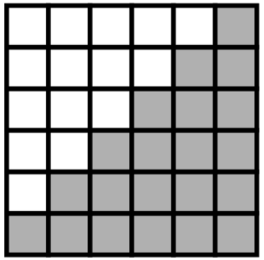


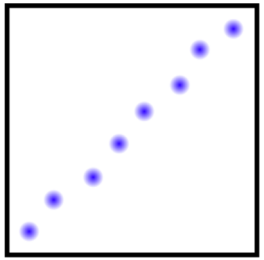
**Traditional Explicit Representations**

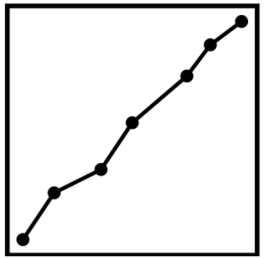
- ✚ Fast inference
- ▬ Discrete

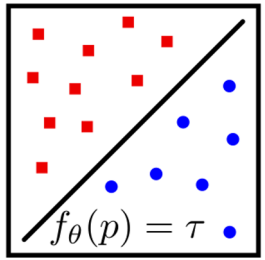
Mescheder, Oechsle, Niemeyer, Nowozin and Geiger: Occupancy Networks: Learning 3D Reconstruction in Function Space. CVPR, 2019

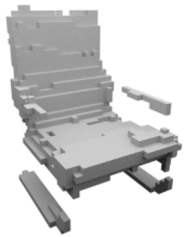
# 3D Shape Representations









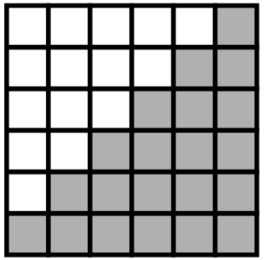



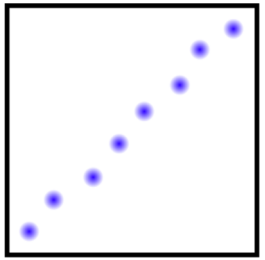
## Neural Implicit Representations

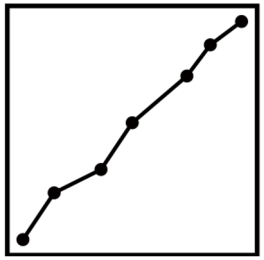
- \+ Continuous, watertight
- − Slow inference
- − Difficult to initialize

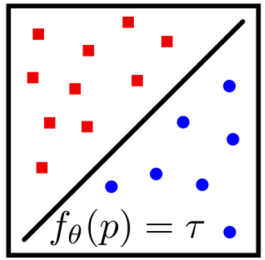
Mescheder, Oechsle, Niemeyer, Nowozin and Geiger: Occupancy Networks: Learning 3D Reconstruction in Function Space. CVPR, 2019

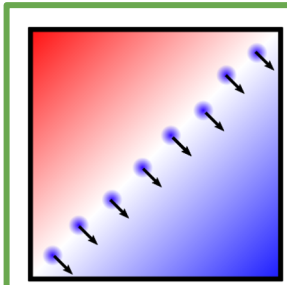
# 3D Shape Representations

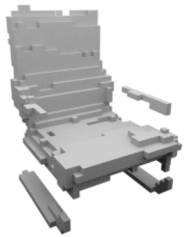


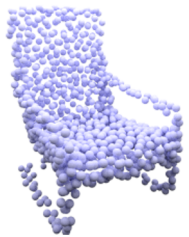










**Shape As Points (SAP) - Hybrid Representation**

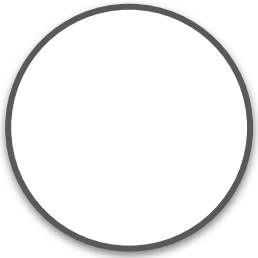
- ✚ Discrete ⇒ Continuous
- ✚ Fast inference
- ✚ Easy initialization

# Method

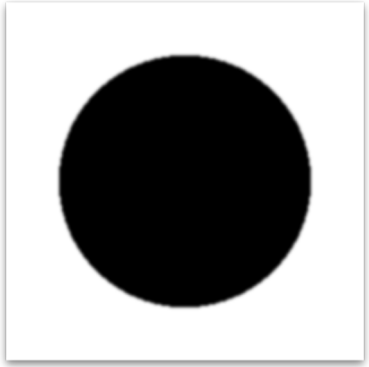
# Differentiable Poisson Solver

# Intuition of Poisson Equation

$$\nabla^2 \chi := \nabla \cdot \nabla \chi = \nabla \cdot \mathbf{v}$$



Shape



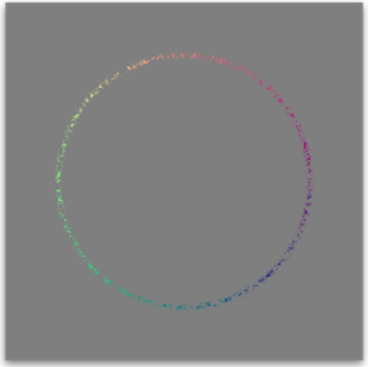
$\chi$

Indicator Function

$\nabla \chi$

Gradient

$\approx$



$\mathbf{v}$

Point Normals

# Our Poisson Solver

$$\nabla^2 \chi := \nabla \cdot \nabla \chi = \nabla \cdot \mathbf{v}$$

- **Discretization** allows to invert the divergence operator

$$\chi = (\nabla^2)^{-1} \nabla \cdot \mathbf{v}$$

- **Spectral methods** to solve the Poisson equation
  - Derivatives of signals in spectral domain are computed analytically
  - Fast Fourier Transform (FFT) are **highly optimized on GPUs/TPUs**
  - Only **25-line code**

$$\tilde{\mathbf{v}} = \mathrm{FFT}(\mathbf{v}) \quad \longrightarrow \quad \tilde{\chi} = \tilde{g}_{\sigma,r}(\mathbf{u}) \odot \frac{i\mathbf{u} \cdot \tilde{\mathbf{v}}}{-2\pi \|\mathbf{u}\|^2} \quad \longrightarrow \quad \chi' = \mathrm{IFFT}(\tilde{\chi})$$

# Surface Reconstruction from Unoriented Point Clouds

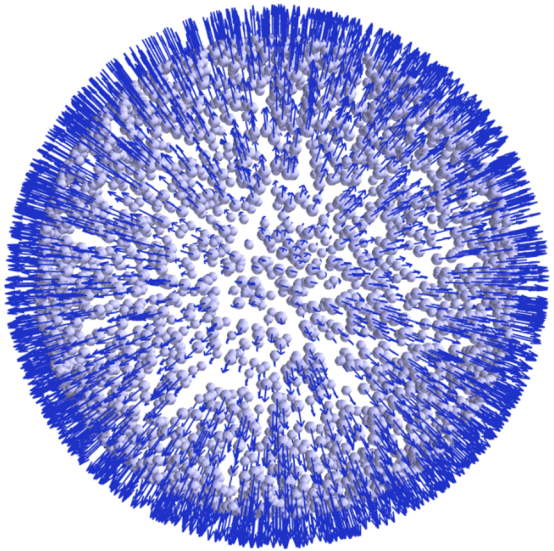
1. SAP for **Optimization-based** 3D Reconstruction
2. SAP for **Learning-based** 3D Reconstruction

# SAP for Optimization-based 3D Reconstruction

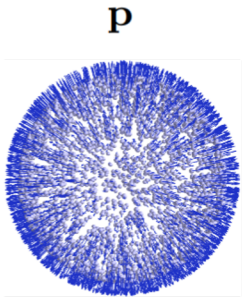
# Pipeline - Forward Pass

**Input an initial oriented point cloud**

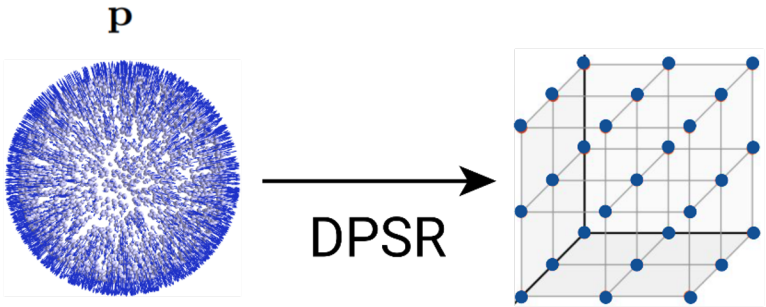
(noisy / incomplete observations)



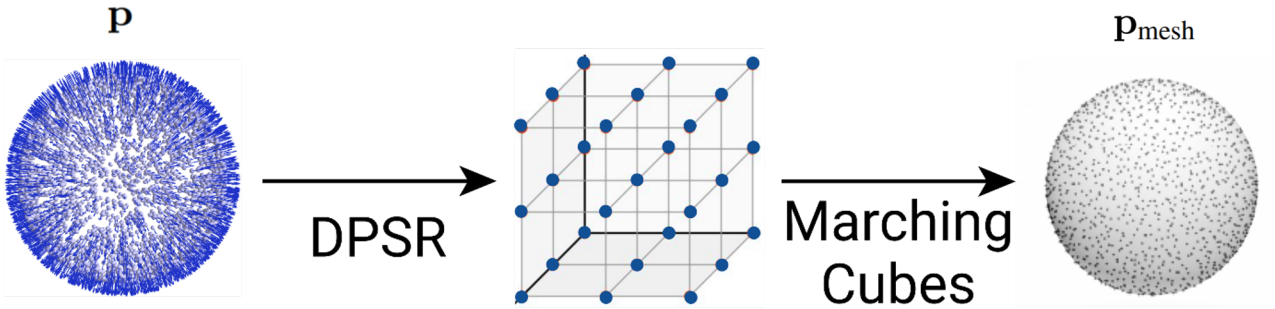
# Pipeline - Forward Pass



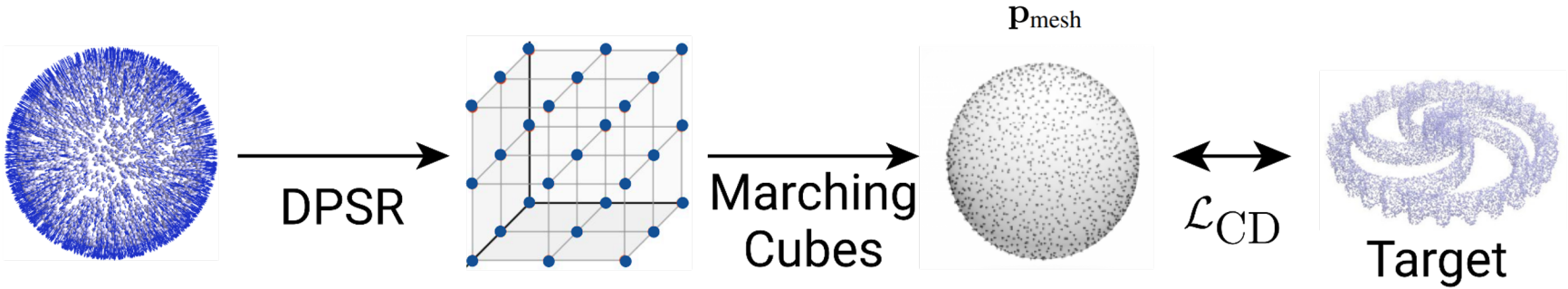
# Pipeline - Forward Pass



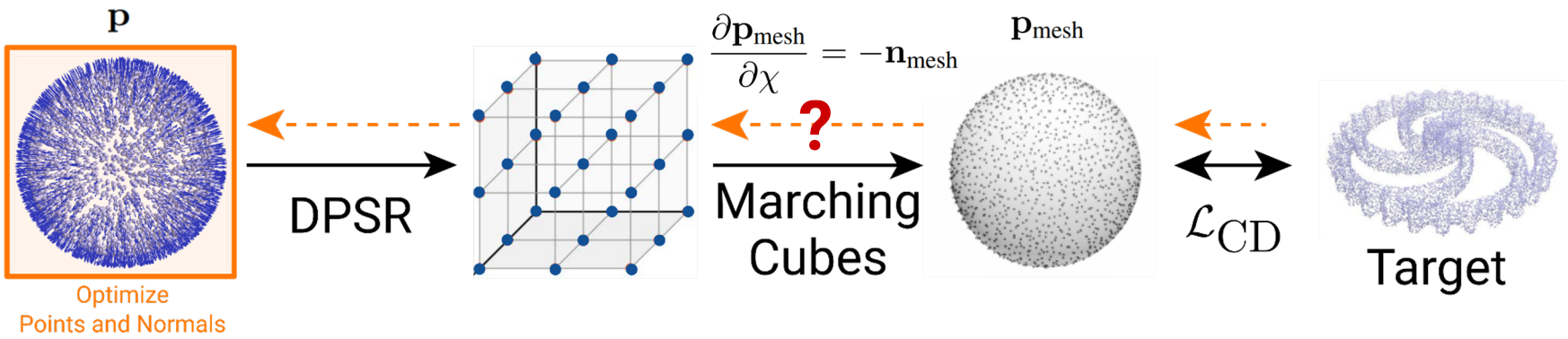
# Pipeline - Forward Pass



# Pipeline - Forward Pass



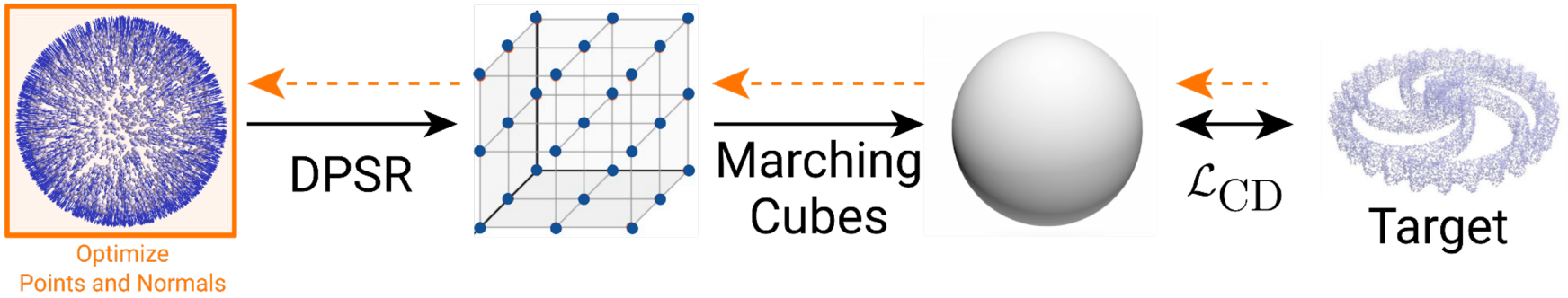
# Pipeline - Backward Pass



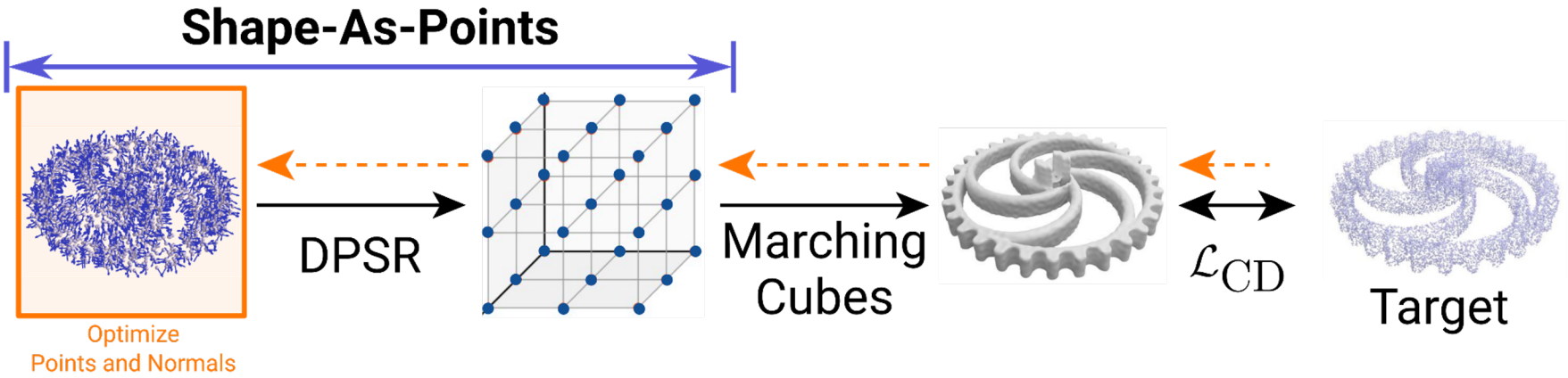

$$\frac{\partial \mathcal{L}_{\text{CD}}}{\partial \mathbf{p}} = \frac{\partial \mathcal{L}_{\text{CD}}}{\partial \mathbf{p}_{\text{mesh}}} \frac{\partial \mathbf{p}_{\text{mesh}}}{\partial \chi} \frac{\partial \chi}{\partial \mathbf{p}}$$

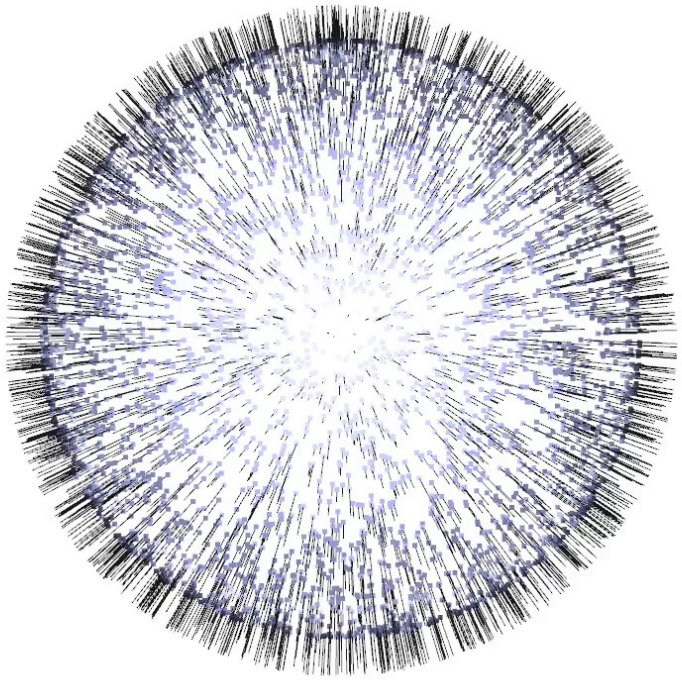
?

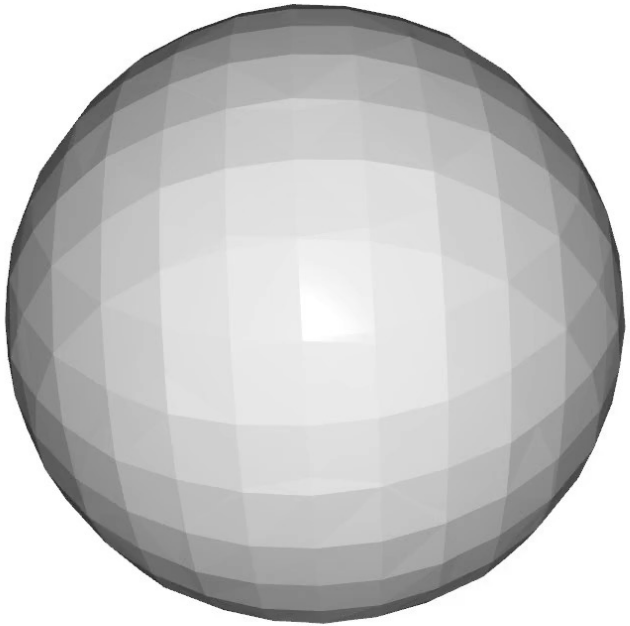
# Pipeline


Optimize
Points and Normals
DPSR
Marching
Cubes
$\mathcal{L}_{\mathrm{CD}}$
Target

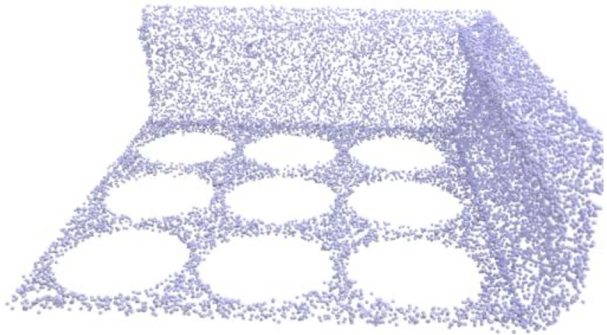
# Pipeline


Shape-As-Points
Optimize
Points and Normals
DPSR
Marching
Cubes
$\mathcal{L}_{\mathrm{CD}}$
Target





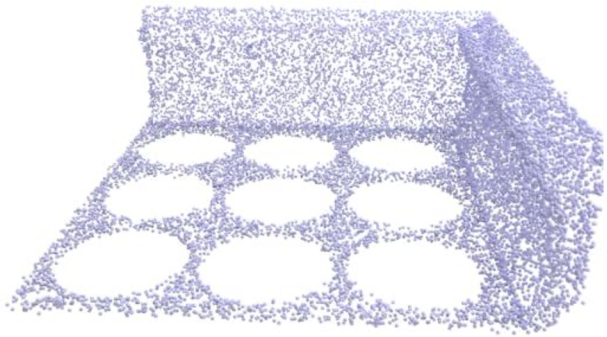
# Comparison



Unoriented Point Clouds

GT Mesh

# Comparison



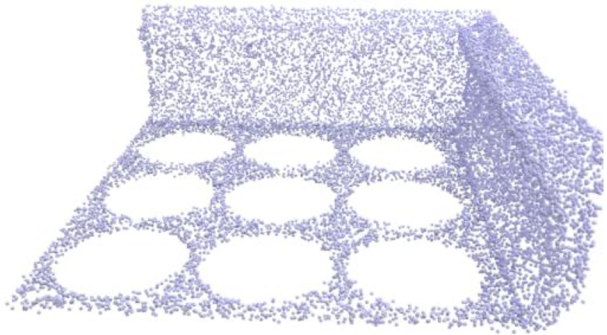
Unoriented Point Clouds

Point2Mesh

**Runtime**: 62 mins

Hanocka, Metzer, Giryes, Cohen-Or: Point2Mesh: A Self-Prior for Deformable Meshes. SIGGRAPH, 2020

# Comparison



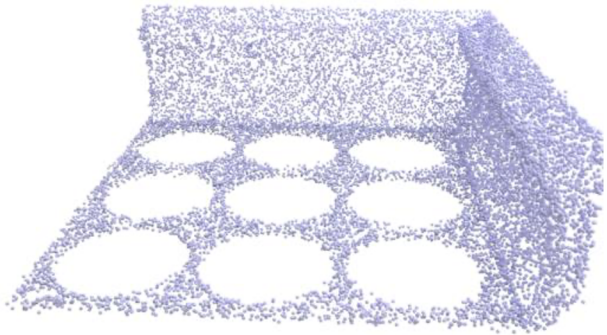
Unoriented Point Clouds

IGR

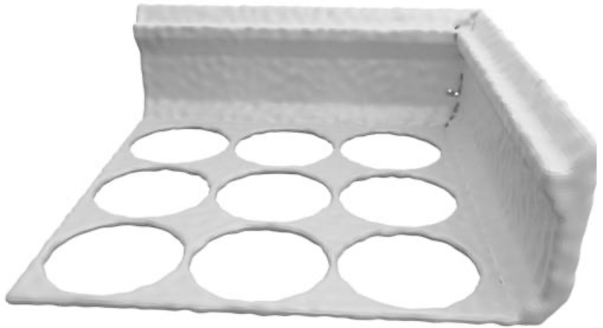
**Runtime**: 30 mins

Gropp, Yariv, Haim, Atzmon and Lipman: Implicit Geometric Regularization for Learning Shapes. ICML, 2020

# Comparison



Unoriented Point Clouds



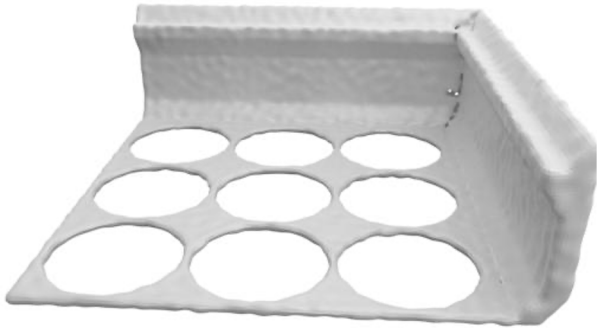
**SAP**

**Runtime**: ~6 mins

# Comparison

SPSR

**Runtime**: ~9 sec



**SAP**

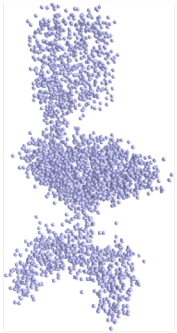
**Runtime**: ~6 mins

Kazhdan and Hoppe: Screened Poisson Surface Reconstruction. SIGGRAPH, 2013

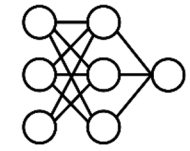
Can we further leverage the **differentiability** of the Poisson solver for **deep neural networks**?

## SAP for Learning-based 3D Reconstruction

# Learning-based Pipeline



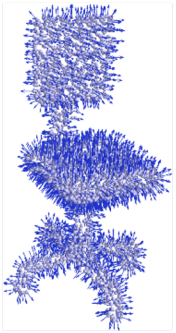
Noisy Input



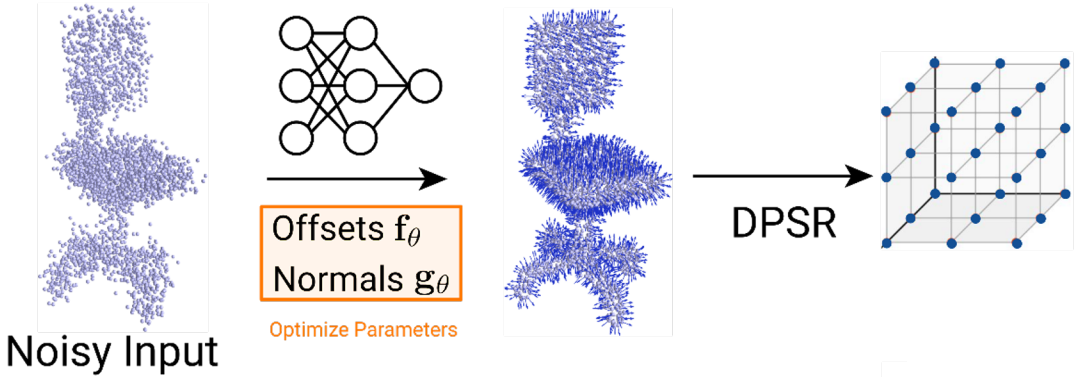
Offsets $\mathbf{f}_\theta$

Normals $\mathbf{g}_\theta$

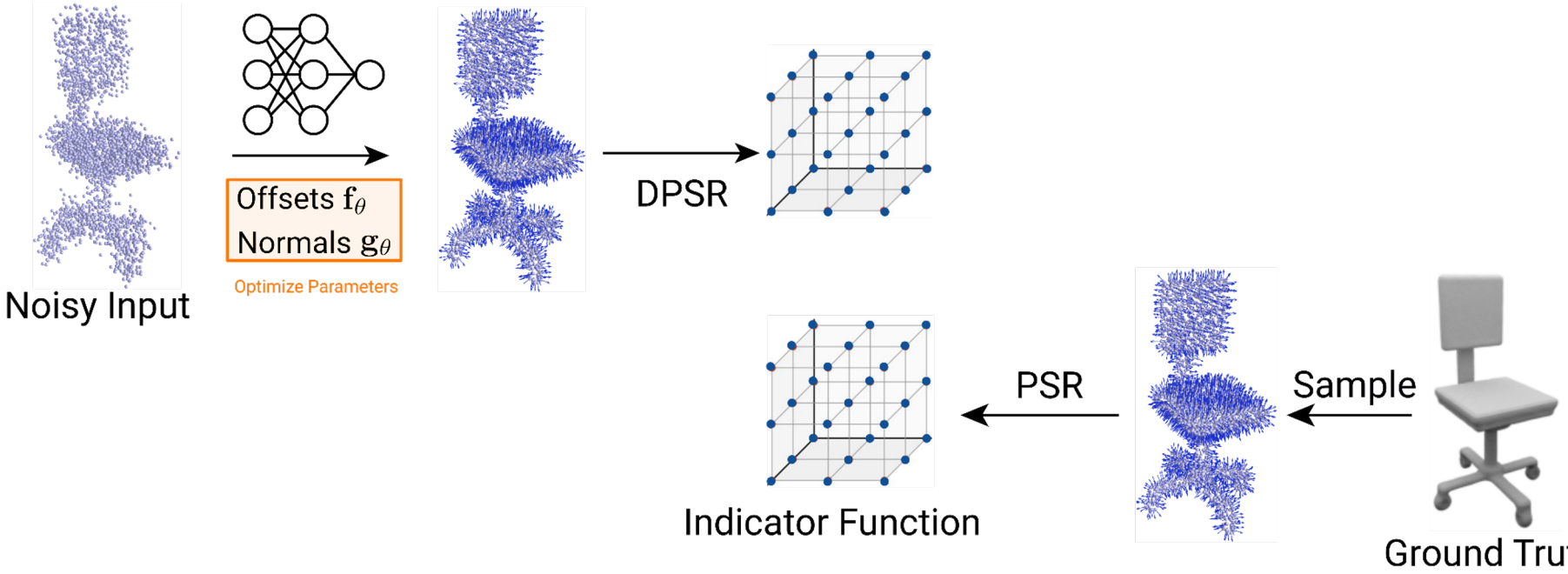
Optimize Parameters



# Learning-based Pipeline



# Learning-based Pipeline



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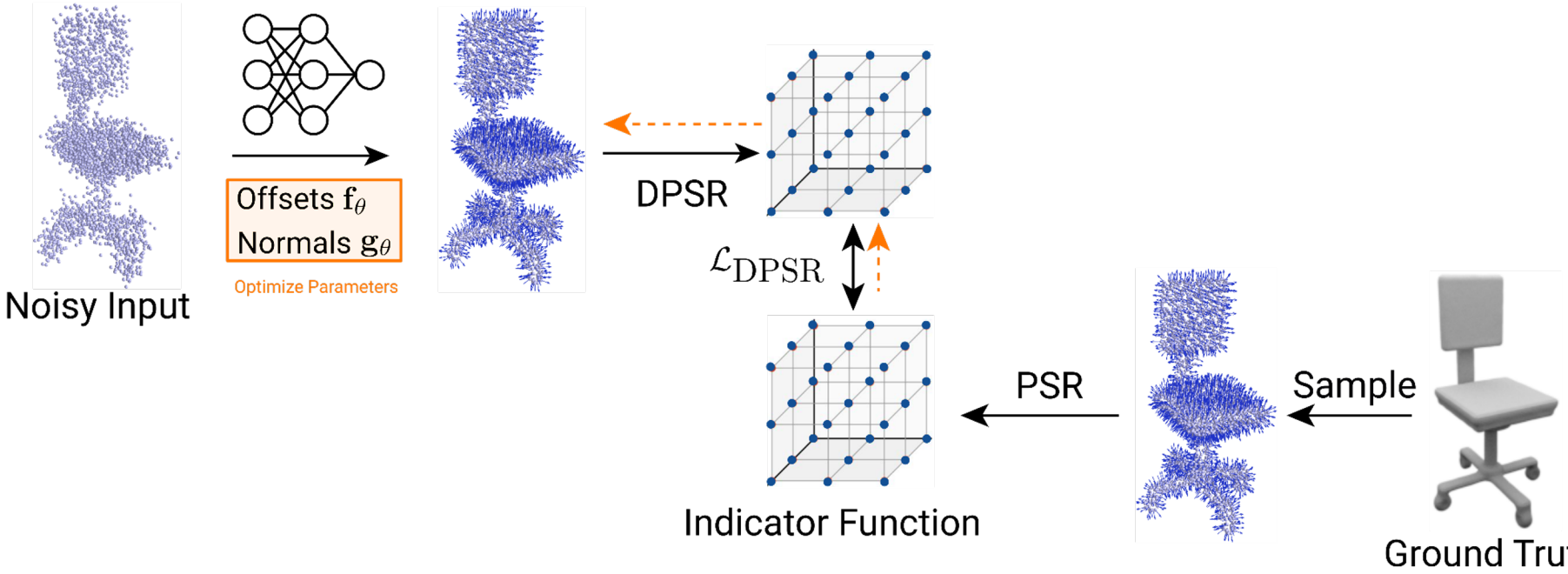

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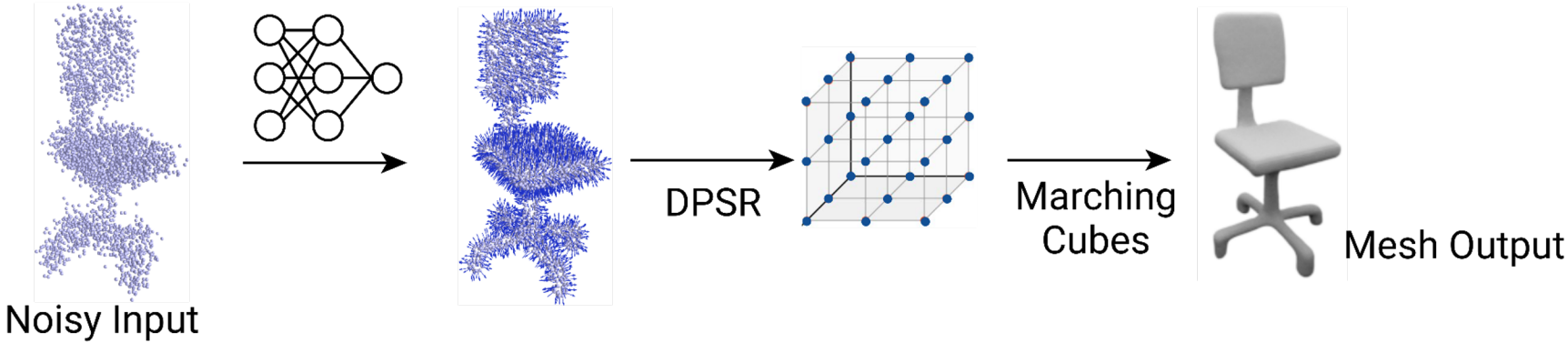
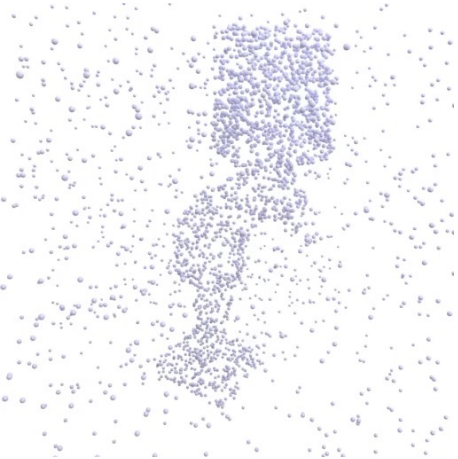
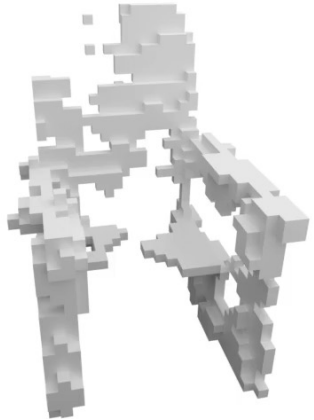


# Results



Inputs

GT Mesh





Inputs

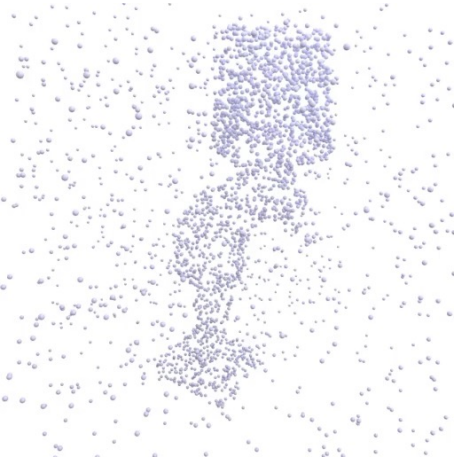
GT Mesh

R2N2

**15 ms**

AtlasNet

**25 ms**



Inputs

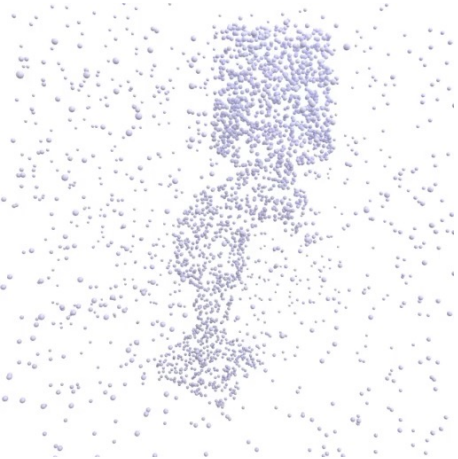
GT Mesh

ConvONet

327 ms








Inputs

GT Mesh

ConvONet

327 ms

**Ours**

64 ms

# Benefit of Geometric Initialization

Chamfer distance over the training process

| **Iterations** | 10K | 50K | 100K | 200K | Best |
|---|---|---|---|---|---|
| ConvONet | 0.082 | 0.058 | 0.055 | 0.050 | 0.044 |
| **Ours** | **0.041** | **0.036** | **0.035** | **0.034** | **0.034** |

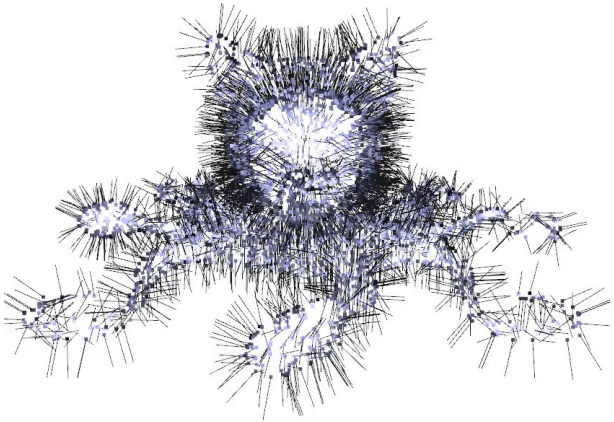
**SAP converges much faster!**

# Conclusions

- SAP is **interpretable**, **lightweight** and guarantees **HQ watertight meshes**
- SAP is also **topology agnostic**, enables **fast inference**
- Our Poisson solver is **differentiable** and **GPU-accelerated**

**Limitation**: Cubic memory requirements limits SAP for small scenes

# Thank You!



https://pengsongyou.github.io/sap