Directed Probabilistic Watershed

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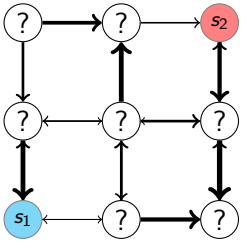
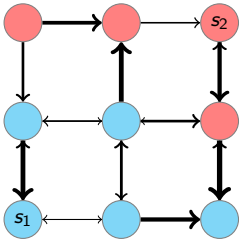
Transductive semi-supervised learning algorithm on **directed** graphs

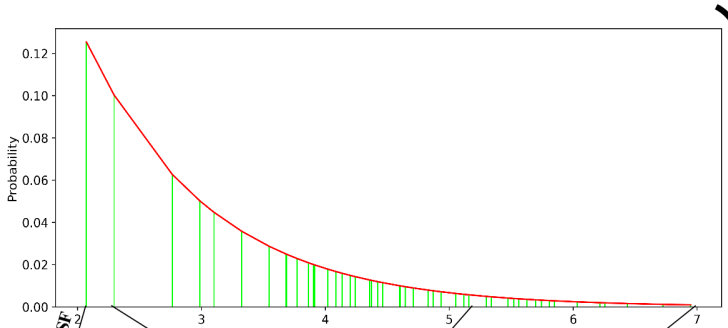


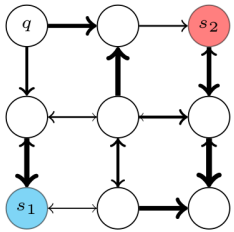
$\Longrightarrow$

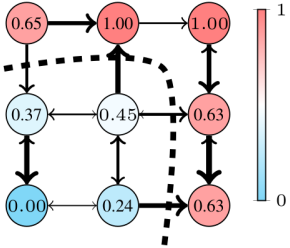
Directed Probabilistic Watershed



- Web graphs
- Citation graphs


Probability
0.12
0.10
0.08
0.06
0.04
0.02
0.00
2
3
4
5
6
7
Cost
mSF
q
s_2
s_1


q
s_2
s_1


0.65
1.00
1.00
0.37
0.45
0.63
0.00
0.24
0.63
1
0

Probabilistic Watershed

Undirected graphs

Fita Sanmartín et al. (2019)

$\Longrightarrow$

Directed Probabilistic Watershed

Directed graphs

Forests

$\Longrightarrow$

In-forests rooted at the seeds

Probabilistic Watershed: Sampling all spanning forests for seeded segmentation and semi-supervised learning, Fita Sanmartín et al. (2019)

Probabilistic Watershed

Undirected graphs

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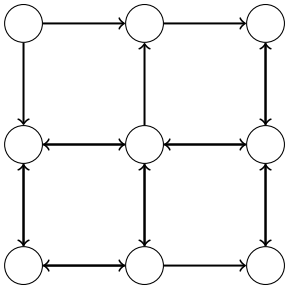
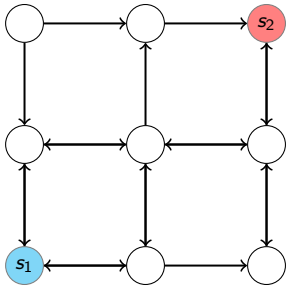
Framework



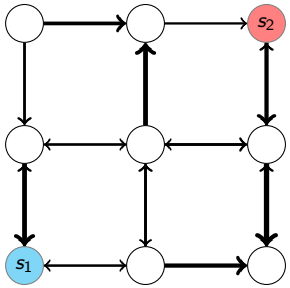
- **Directed Graph**

Framework



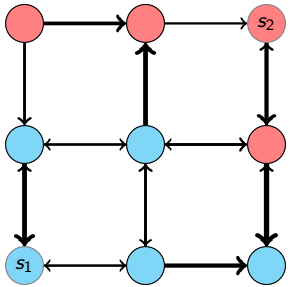
- Directed Graph
- **Seeds (labeled nodes)**

Framework



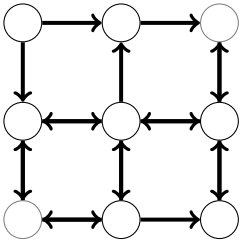
- Directed Graph
- Seeds (labeled nodes)
- **Edge-Costs,** c_e
  (∼affinity between nodes)

Framework



- Directed Graph
- Seeds (labeled nodes)
- Edge-Costs, c_e
  ($\sim$affinity between nodes)
- **Classification**

Definition in-tree & in-forest

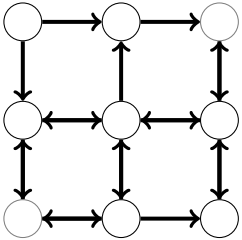


(a) Directed graph

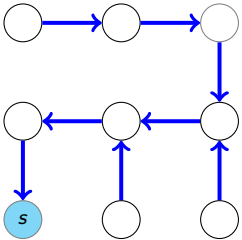
(b) In-tree rooted at s

(c) In-forest rooted at s_1 and s_2

Definition in-tree & in-forest



(a) Directed graph



(b) In-tree rooted at s

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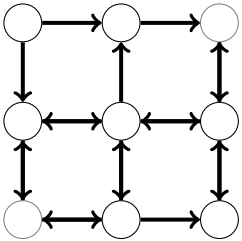
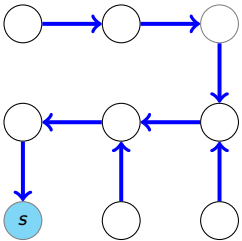
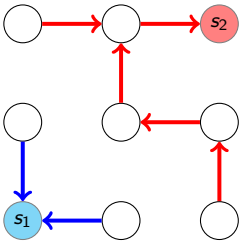
Definition in-tree & in-forest



(a) Directed graph



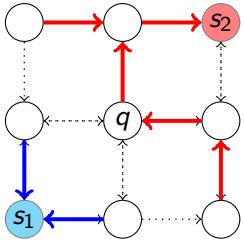
(b) In-tree rooted at s



(c) In-forest rooted at s_1 and s_2

Main question

What is the probability of sampling an in-forest such that a node of interest, q, belongs to a tree rooted at a certain seed?



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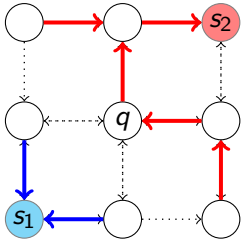
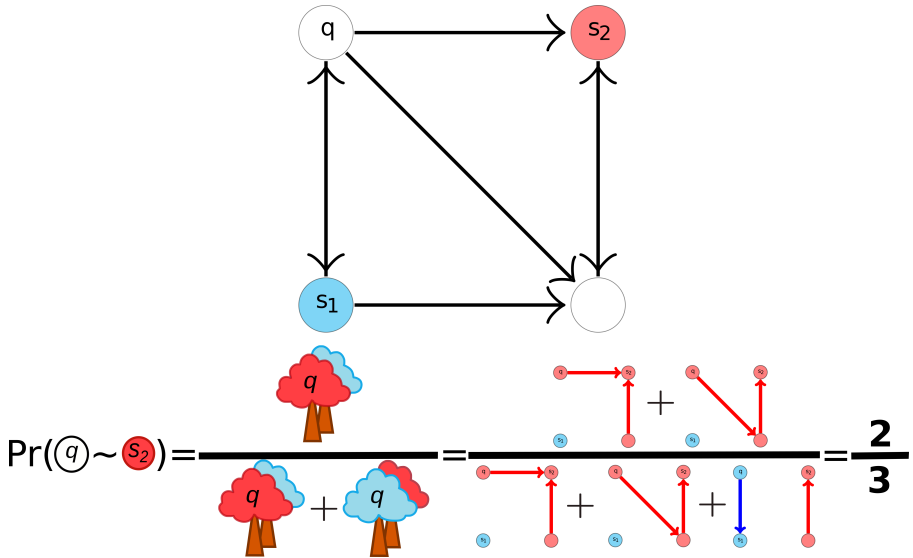
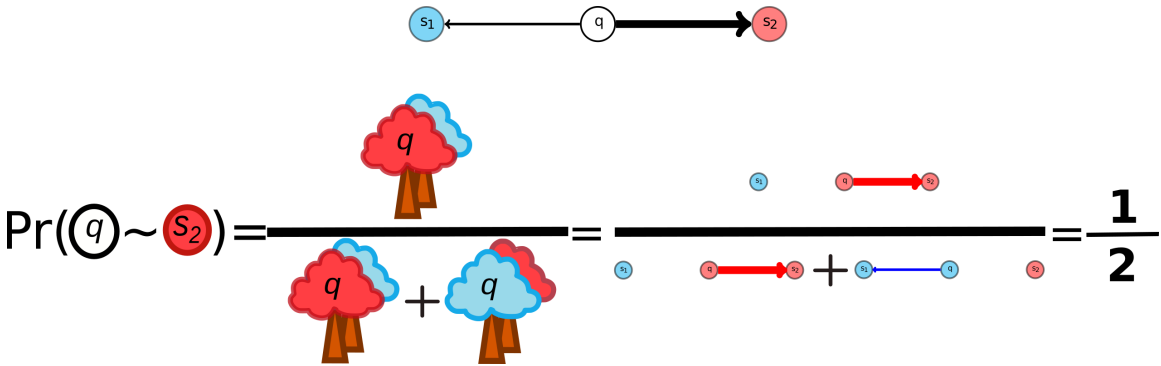


$$\Pr(q \sim s_2) = \frac{\text{q}}{\text{q} + \text{q}}$$

Directed Probabilistic Watershed Probabilities



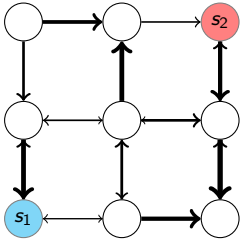
Directed Probabilistic Watershed Probabilities

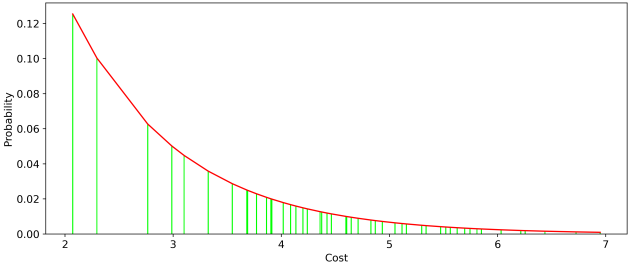


Gibbs distribution

Gibbs distribution

$$\Pr(F) \propto w(F) := \exp\Big(-\mu \underbrace{c(F)}_{\sum_{e \in F} c_e} \Big)$$

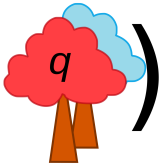




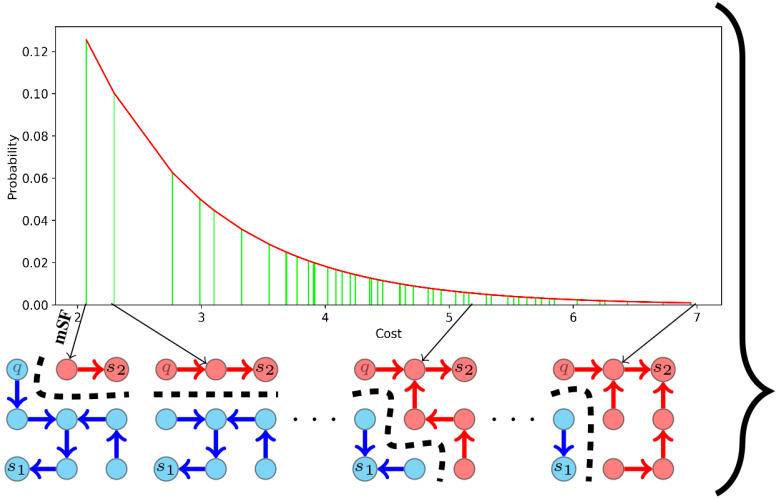
Directed Probabilistic Watershed Probabilities

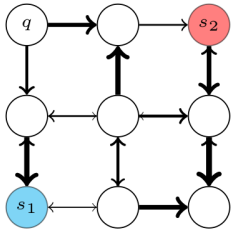
$$w(F) := \exp\left(-\mu \frac{c(F)}{\sum_{e \in F} c_e} \right)$$

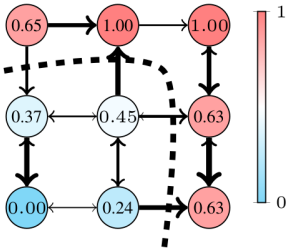
$$\Pr(q \sim s_2) = \frac{w(q)}{w(q) + w(q)}$$


Directed Probabilistic Watershed Probabilities







Computation Probabilities Directed Probabilistic Watershed

Number in-forests increases **exponentially** with the number of nodes and edges $\Longrightarrow$ Naive approach infeasible

An elementary proof of a matrix tree theorem for directed graphs, Leenheer (2019)

Computation Probabilities Directed Probabilistic Watershed

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Directed Matrix Tree Theorem

Leenheer (2019) $\Longrightarrow$ Efficient computation probabilities

Linear System

$$L_U^\top x_U^{s_2} = -[B_1^\top]_{s_2}$$

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Comparison linear systems

Probabilistic Watershed

$$L_U x_U^{s_2} = -[B_1^\top]_{s_2}$$

Directed Probabilistic Watershed

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Equivalence Random Walker

Probabilistic Watershed

Random Walker
(Seed absorption probability)
Grady (2006)

$\sim$

Probabilisic Watershed
Fita (2019)

Directed Probabilistic Watershed

Directed Random Walker
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$\sim$

Directed
Probabilisic Watershed

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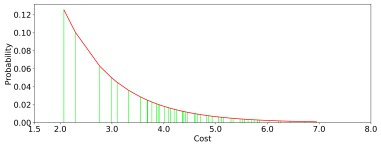
Random walks for image segmentation, Grady (2006)

Minimum entropy case

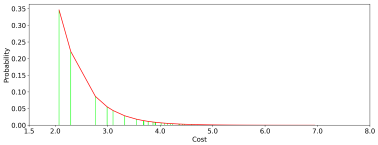
Gibbs distribution

$$\Pr(F) \propto \exp\Big(-\mu c(F)\Big) = \exp\Big(-\mu \sum_{e \in F} c_e\Big)$$

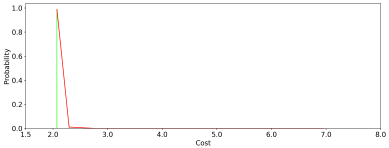
$\mu \to \infty \quad \Longrightarrow \quad$ Minimum entropy $\quad \Longrightarrow \quad$ Count minimum cost in-forests (in-mSF)



(a) $\mu = 1$



(b) $\mu = 2$



(c) $\mu = 20$

Minimum entropy case

Probabilistic Watershed

Power Watershed
Couprie et al. (2011)

$\sim$

Minimum entropy
Probabilisic Watershed
Fita (2019)

Directed Probabilistic Watershed

Directed
Power Watershed

$\sim$

Minimum entropy
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Power Watershed: A Unifying Graph-Based Optimization Framework, Couprie at al. (2011)

Summary

Probabilistic Watershed Fita Sanmartín et al. (2019)	$\Longrightarrow$	Directed Probabilistic Watershed
Forests	$\Longrightarrow$	In-forests rooted at the seeds
Count efficiently number forests	$\Longrightarrow$	Count efficiently number in-forests
Equivalence Random Walker Grady (2006)	$\Longrightarrow$	Equivalence Directed Random Walker
Minimum entropy Power Watershed Couprie et al. (2011)	$\Longrightarrow$	Minimum entropy Directed Power Watershed

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Thank you for your attention