

Joint Inference for Neural Network Depth and Dropout Regularization

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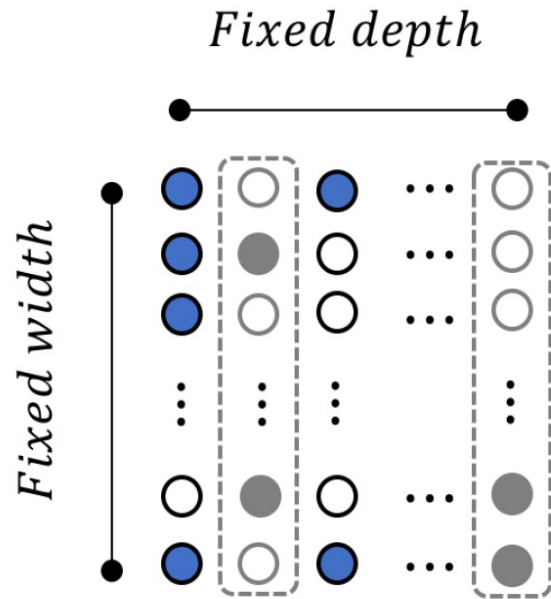
35th Conference on Neural Information Processing Systems (NeurIPS 2021)

Motivation

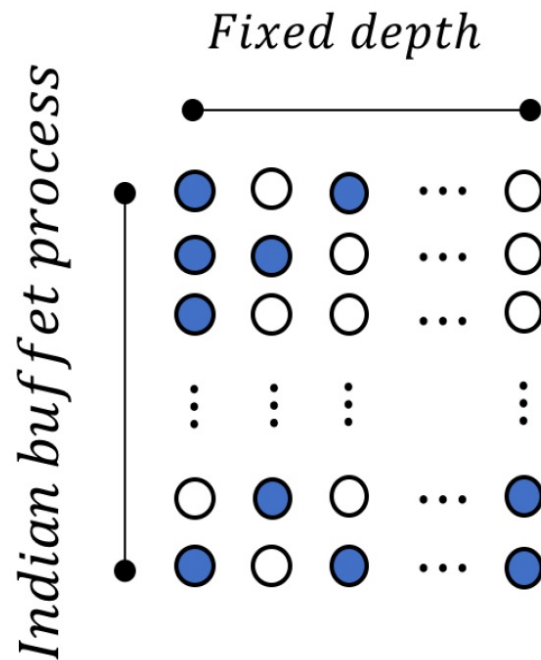
- Pre-determined backbone structures are the key
- Deep networks tend to be
 - Overfitted
 - Poorly calibrated with high confidence on incorrect predictions (Nguyen et al. 2015, Antorán et al. 2020)
- Current solutions
 - Dropout and its variants (Srivastava et al. 2014, Gal et al. 2017, Lee et al. 2019)
 - Structure selection methods (Srinivas et al. 2016, Dikov et al. 2019, Antorán et al. 2020)
- However,
 - Cannot scale the network beyond the pre-determined structure
 - Cannot achieve a balance between network depth and dropout regularization for uncertainty calibration

Our proposed solution

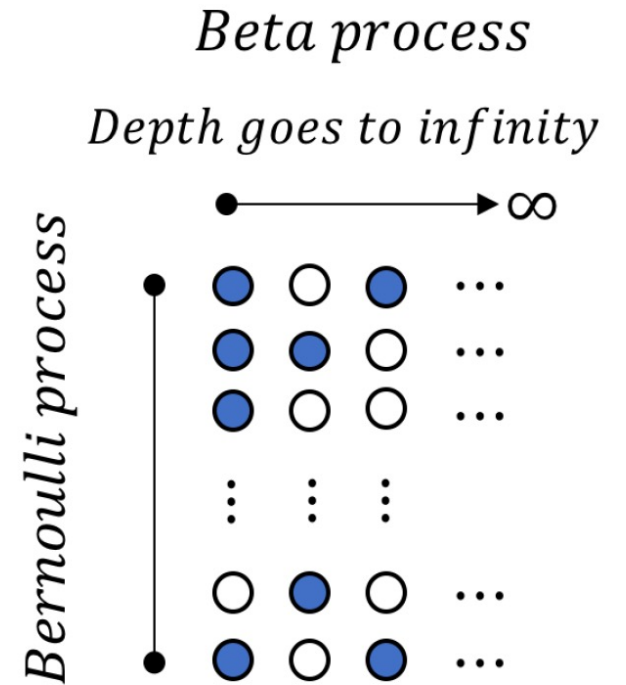
- Model the depth (number of hidden layers) as a Beta Process
- Modulate neuron activations with a conjugate Bernoulli Process
- Joint inference of network depth and neuron activations



A typical structure selection method



A dropout variants



Our proposed solution

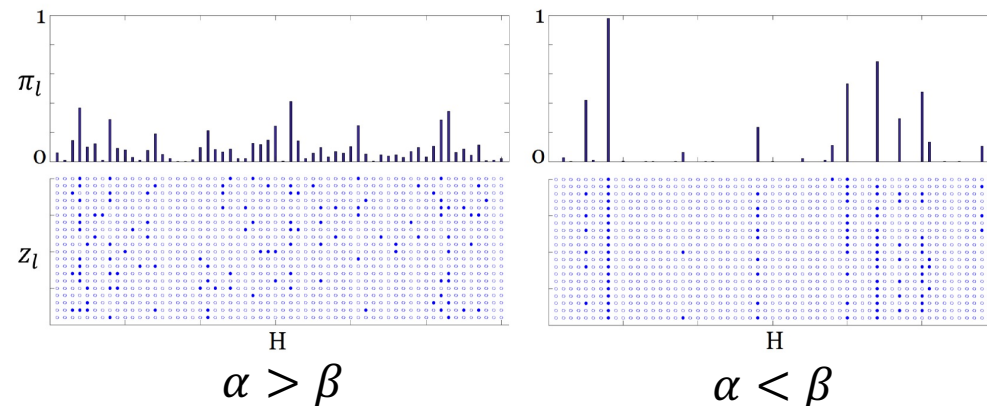
Beta-Bernoulli process over network structures

- Model the depth of a neural network as a Beta Process
 - Stick breaking construction of beta-Bernoulli Process (Paisley et al. 2010, Broderick et al. 2012)

$$v_l \sim \text{Beta}(\alpha, \beta), \quad \pi_l = \prod_{j=1}^l v_j, \quad z_{ml} \sim \text{Bernoulli}(\pi_l)$$

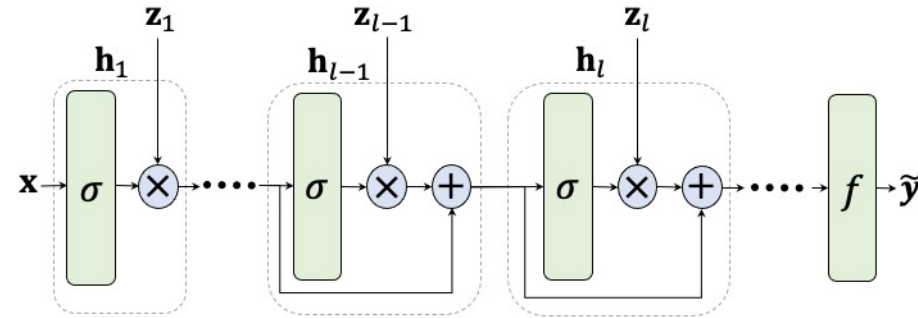
- The prior over the network structures \mathbf{Z} :

$$p(\mathbf{Z}, \mathbf{v} | \alpha, \beta) = p(\mathbf{v} | \alpha, \beta) p(\mathbf{Z} | \mathbf{v}) = \prod_{l=1}^{\infty} \text{Beta}(v_l | \alpha, \beta) \prod_{m=1}^M \text{Bernoulli}(z_{ml} | \pi_l)$$



Network structure with infinite layers

- A neural network has the form



$$\mathbf{h}_l = \sigma(\mathbf{W}_l \mathbf{h}_{l-1}) \otimes \mathbf{z}_l + \mathbf{h}_{l-1} \quad l \in \{1, 2, \dots, \infty\}$$

- A Gaussian likelihood of the neural network for regression task

$$p(D|\mathbf{Z}, \mathbf{W}) = \prod_{n=1}^N \mathcal{N}(\mathbf{y}_n | f(\mathbf{x}_n; \mathbf{Z}, \mathbf{W}), s^2 \mathbf{I})$$

Efficient inference

- The marginal Likelihood over network structures \mathbf{Z} is

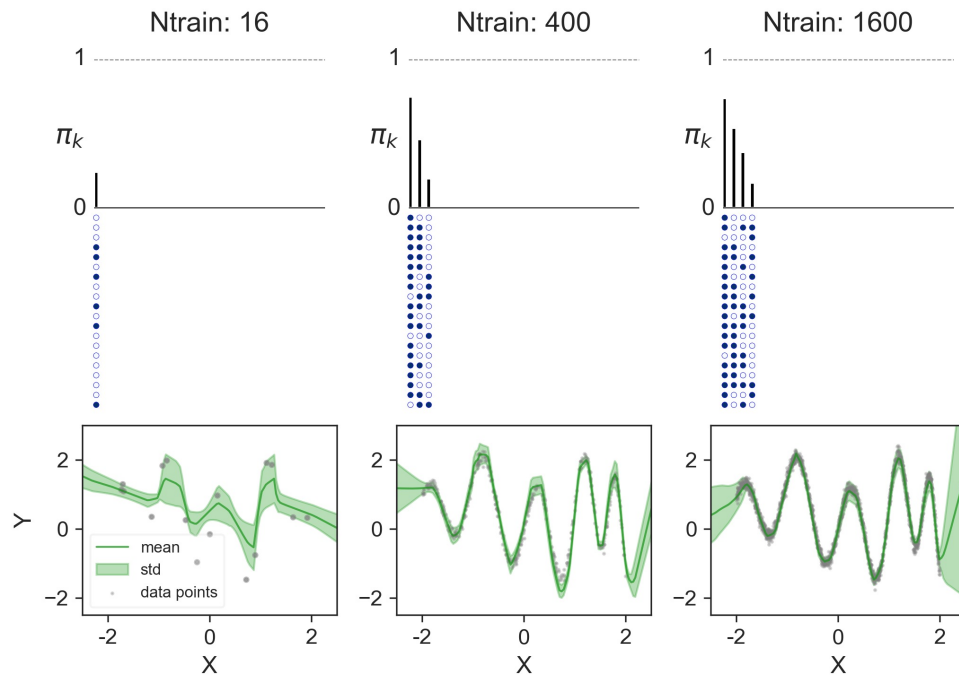
$$p(D|\mathbf{W}, L, \alpha, \beta) = \int p(D|\mathbf{Z}, \mathbf{W}) p(\mathbf{Z}, \mathbf{v}|\alpha, \beta) d\mathbf{Z}d\mathbf{v}$$

- Approximation with structured stochastic variational inference (Hoffman et al. 2013, 2015)

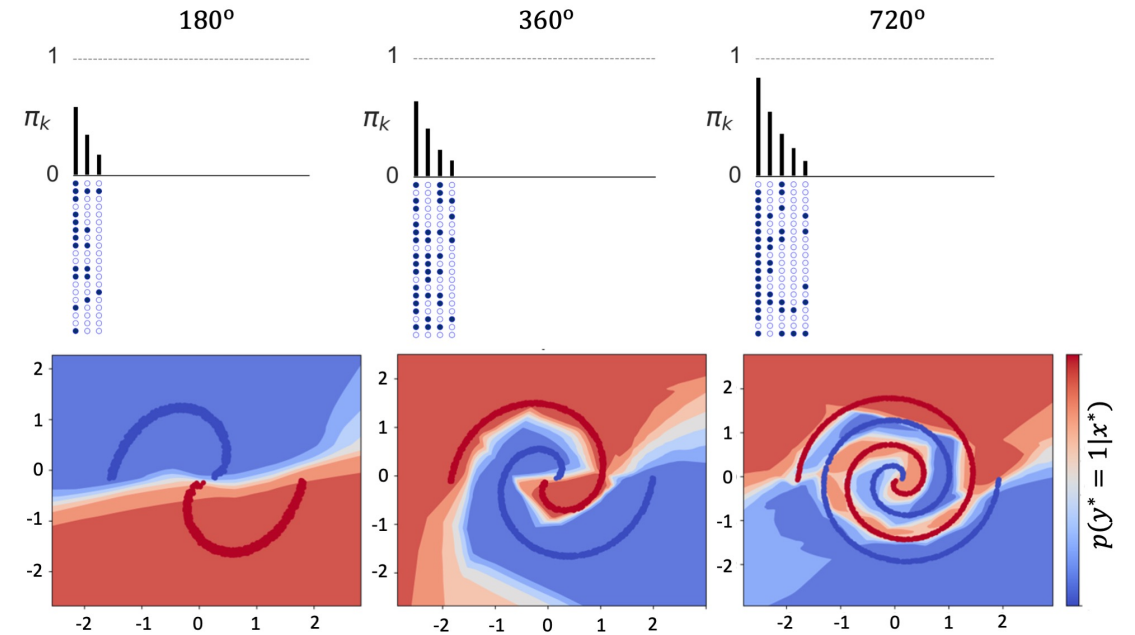
$$\log p(D|\mathbf{W}, L, \alpha, \beta) \geq \mathbb{E}_{q(\mathbf{Z}, \mathbf{v})} [\log p(D|\mathbf{Z}, \mathbf{W})] - \text{KL}[q(\mathbf{Z}|\mathbf{v})||p(\mathbf{Z}|\mathbf{v})] - \text{KL}[q(\mathbf{v})||p(\mathbf{v})]$$

- We use truncation level K for the variational distribution
- Reparameterization of Beta and Bernoulli distribution (Jang et al. 2017, Maddison et al. 2017, Jankowiak et al. 2018)
- We prove that optimizing ELBO is equivalent to Bayesian Information Criterion over the structure \mathbf{Z}

Performance evaluation on synthetic data



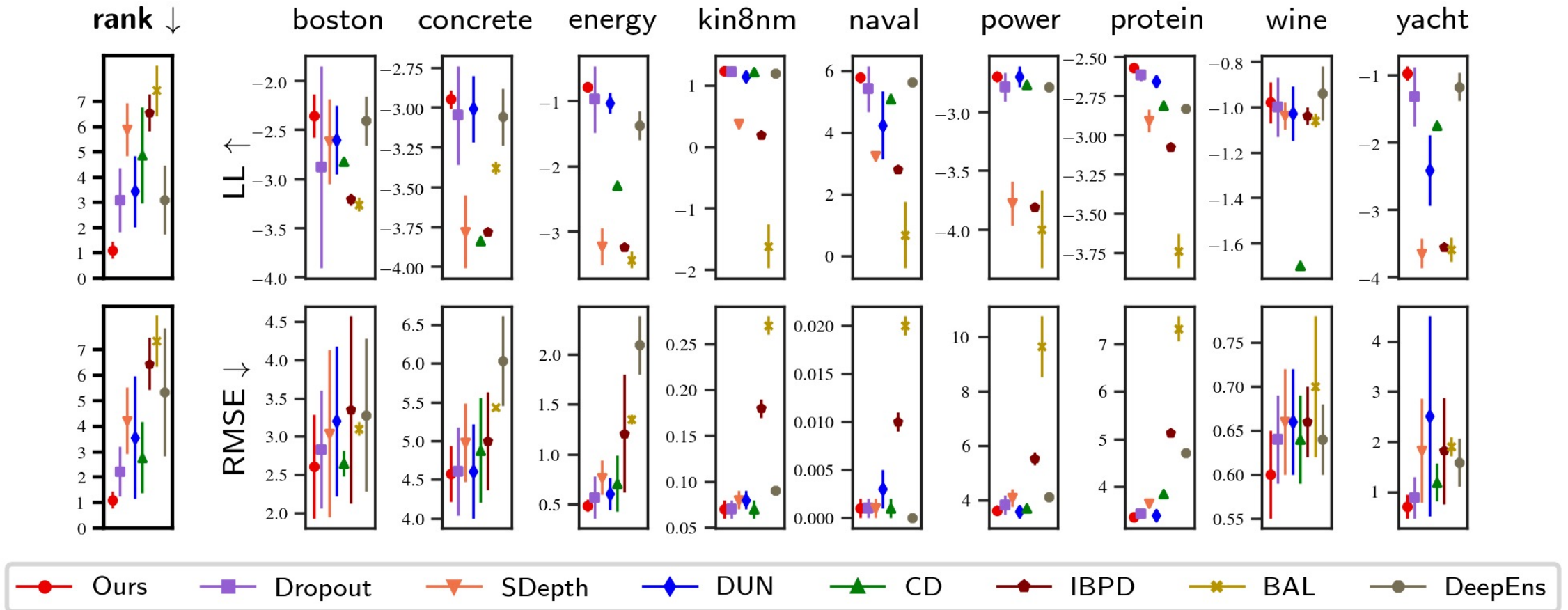
Periodic dataset
(Increasing data sizes)



Spirals dataset
(Increasing complexity)

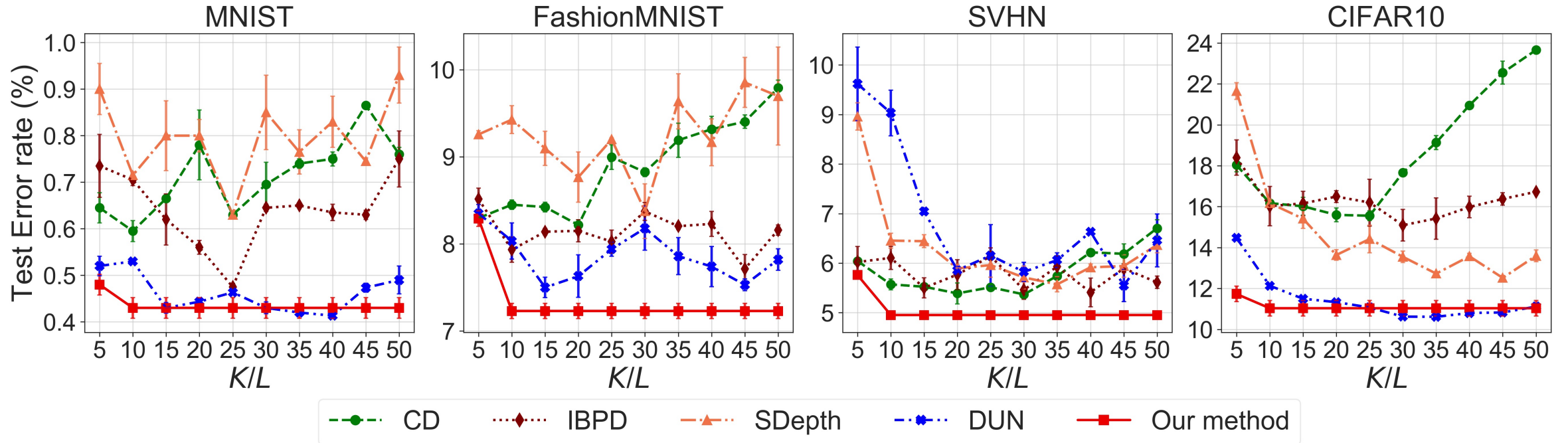
- If the training data size or its complexity increases, network structure grows to accommodate more information.

Performance comparison on UCI datasets



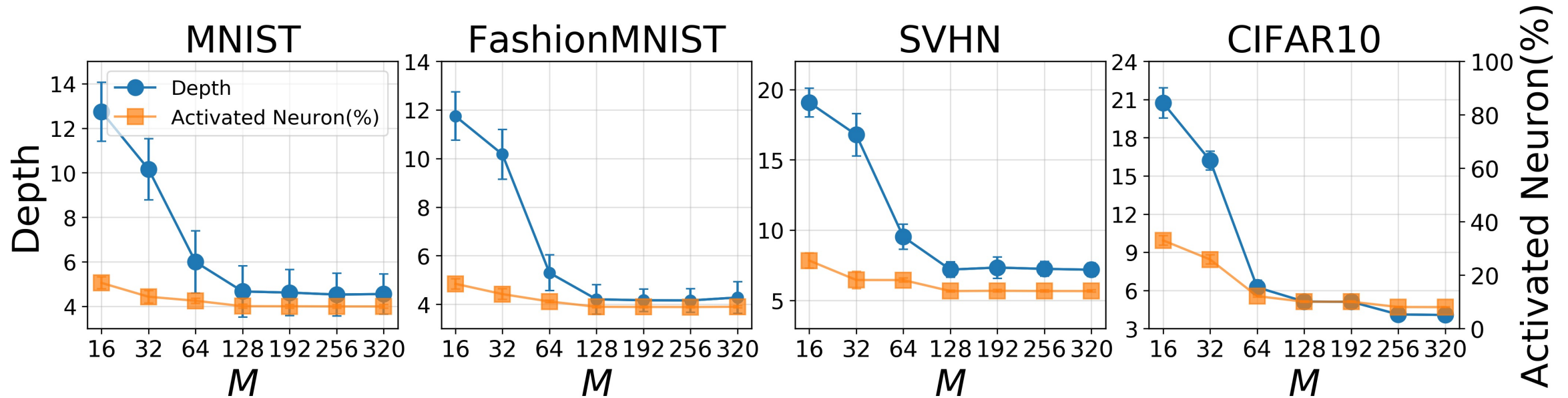
- Our method achieves the overall highest rank for both uncertainty calibration and prediction accuracy.

Effect of truncation level K



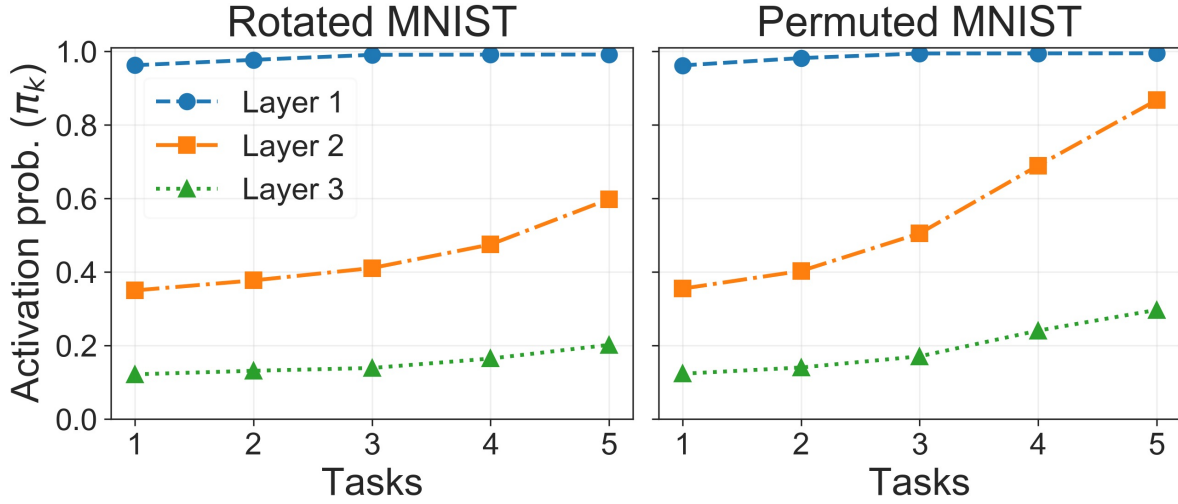
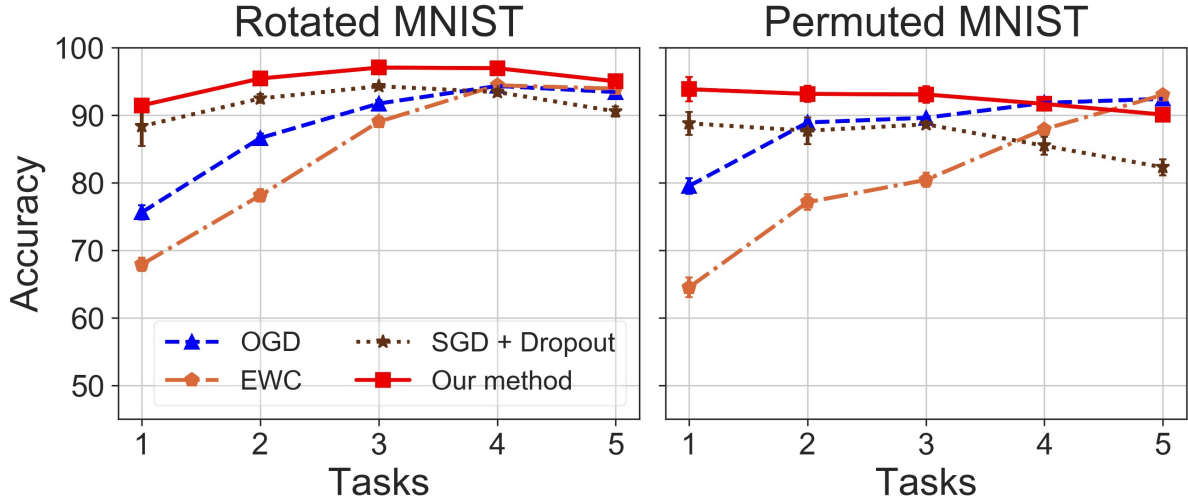
- The truncation level (K) of our method does not affect the performance.
- The depth (L) of other methods significantly affects the performance and should be set carefully.

Effect of maximum width M



- With smaller width M , our method results in deeper network structures to compensate for the relatively narrow layers.
- As M increases, the structures become shallower.

Case study on Continual learning



- Our method alleviates catastrophic forgetting by enabling network depth to dynamically augment to accommodate incrementally available information.

Conclusion

- General joint inference framework applicable for various neural networks
- Experimental results on MLPs and CNNs show that our method achieves superior performance by adapting network depth and neuron activations
- Our model can accommodate incrementally available information by enabling neural network structures to dynamically evolve

Thank you!