UNIVERSITÄT BERN

Human Brain Project

Latent Equilibrium:

Arbitrarily fast computation with arbitrarily slow neurons

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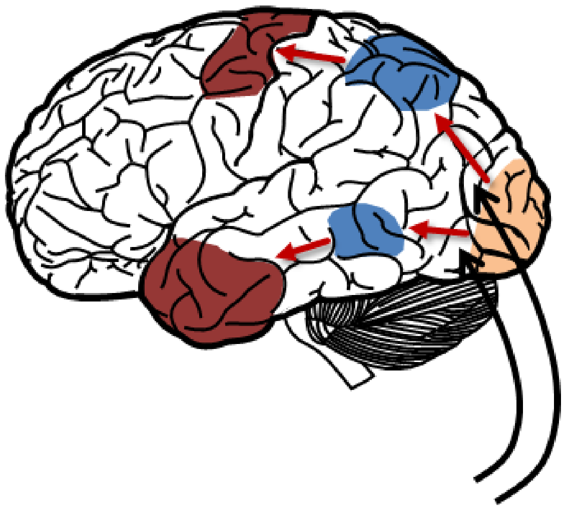
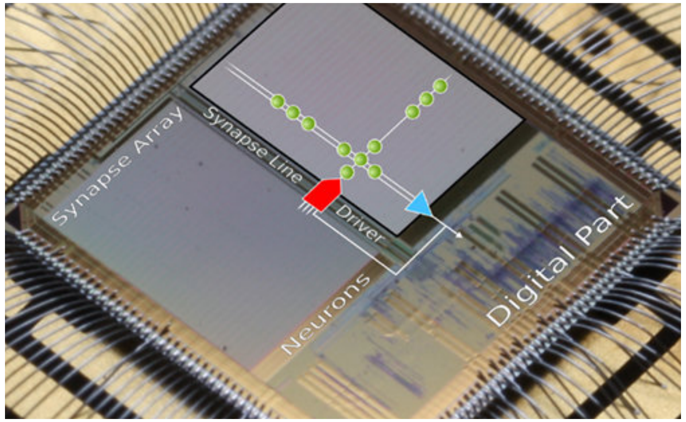
Efficient information processing in dynamical physical systems



continuous learning in
dynamical physical systems

Credit Assignment problem



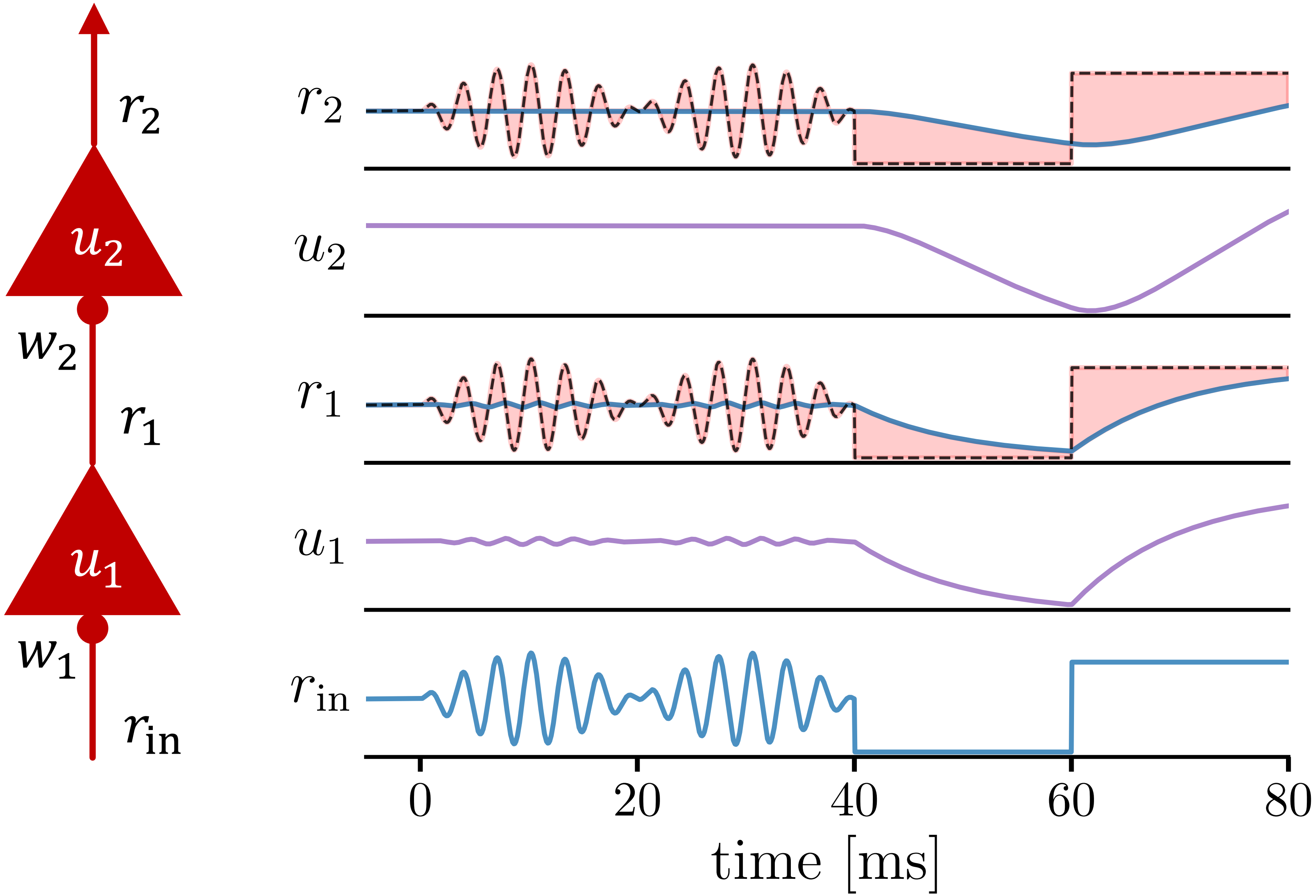
Spikey chip (Pfeil et al., 2013)

solution for AI: error-backpropagation (BP) algorithm

models of approximate, bio-plausible BP with **common problem:**

limited speed of information processing due to **slow** components

The relaxation problem



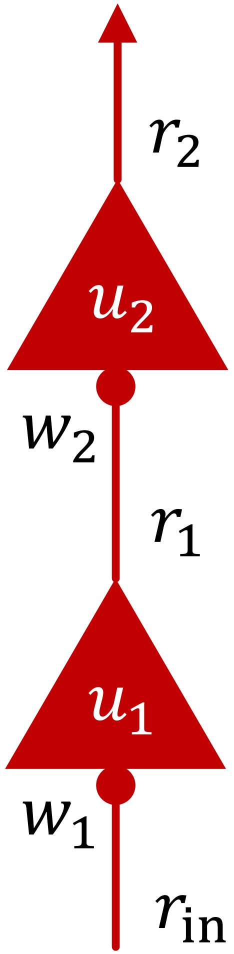
output rate:

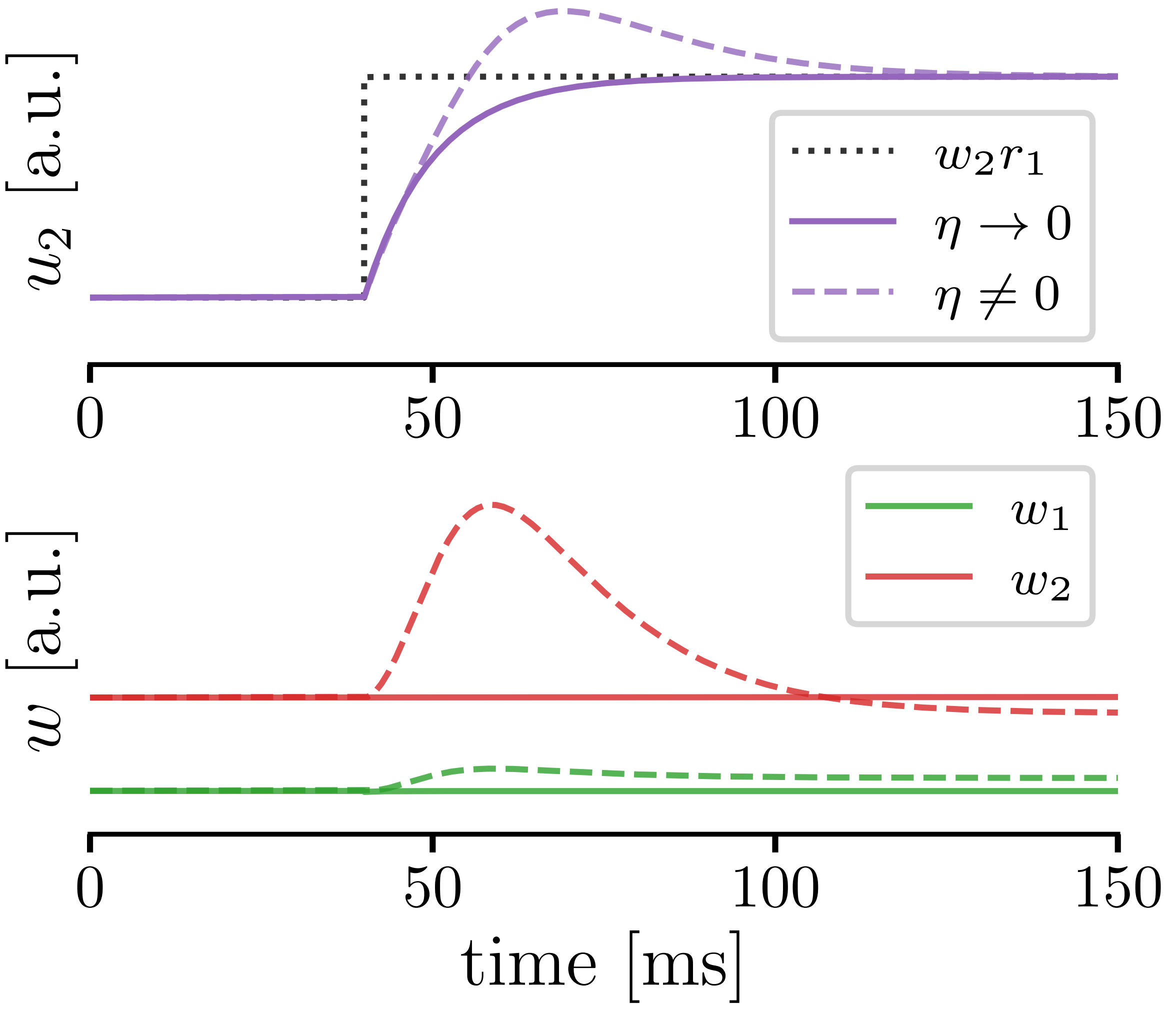
$$r_i = \varphi(u_i)$$

leaky-integrator dynamics:

$$\tau \dot{u}_i = -u_i + w_i r_{i-1}$$

Relaxation disrupts learning





possible solutions:

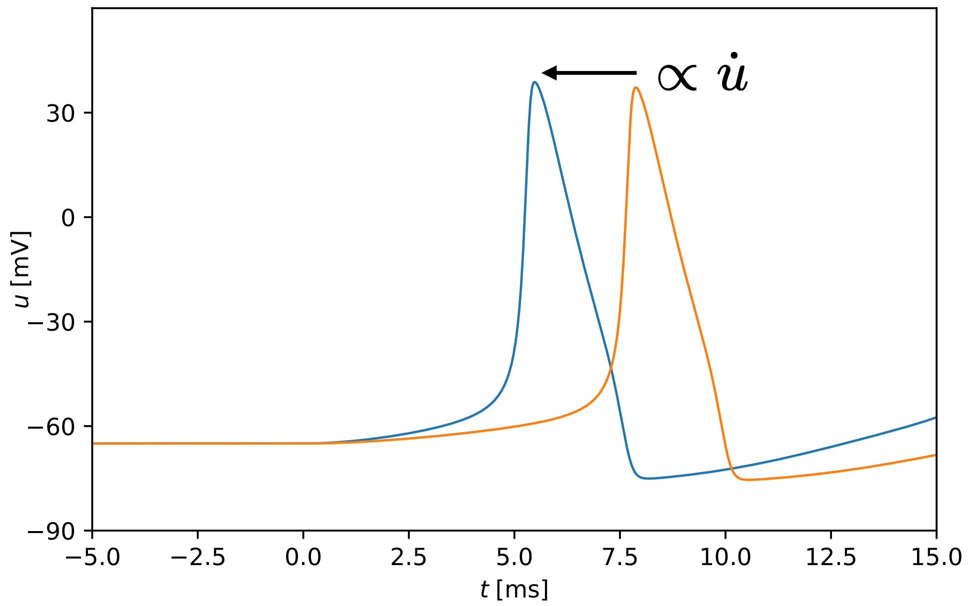
- small learning rates
  - ... but slow
- phased plasticity
  - ... but cumbersome
  - ... and slow

Latent Equilibrium: mathematical framework

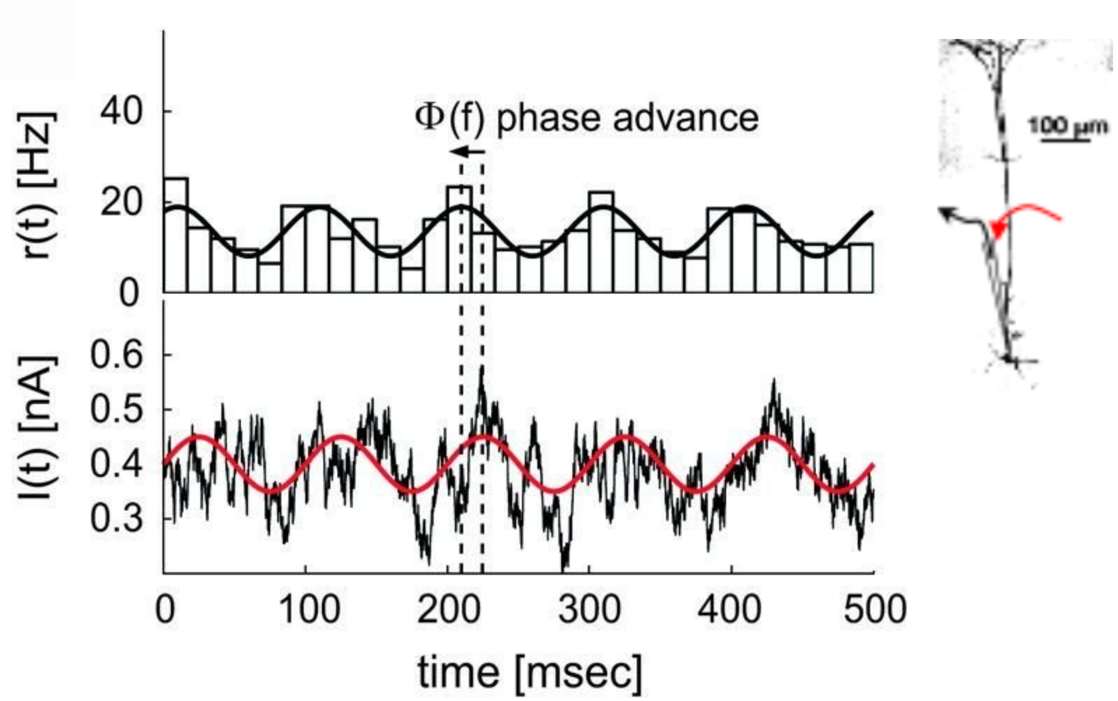
1. Prospective neuronal output

$r_i = \varphi(\breve{u}_i)$ with $\breve{u}_i = u_i + \tau_i \dot{u}_i$

Evidence for prospective coding



Hodgkin & Huxley (1952)



Köndgen et al. (2008)

- spike timing depends on membrane voltage derivative
- firing rate of pyramidal neurons is phase-advanced w.r.t. their input

Latent Equilibrium: mathematical framework

1. Prospective neuronal output

$r_i = \varphi(\breve{u}_i)$ with $\breve{u}_i = u_i + \tau_i \dot{u}_i$

2. Prospective energy function

$$E(\breve{u}) = \underbrace{\frac{1}{2} \sum_{i \in \mathcal{N}} |\breve{u}_i - (Wr)_i|^2}_{\text{mismatch energies}} + \beta \underbrace{\mathcal{L}(\breve{u})}_{\text{loss}}$$

Latent Equilibrium: mathematical framework

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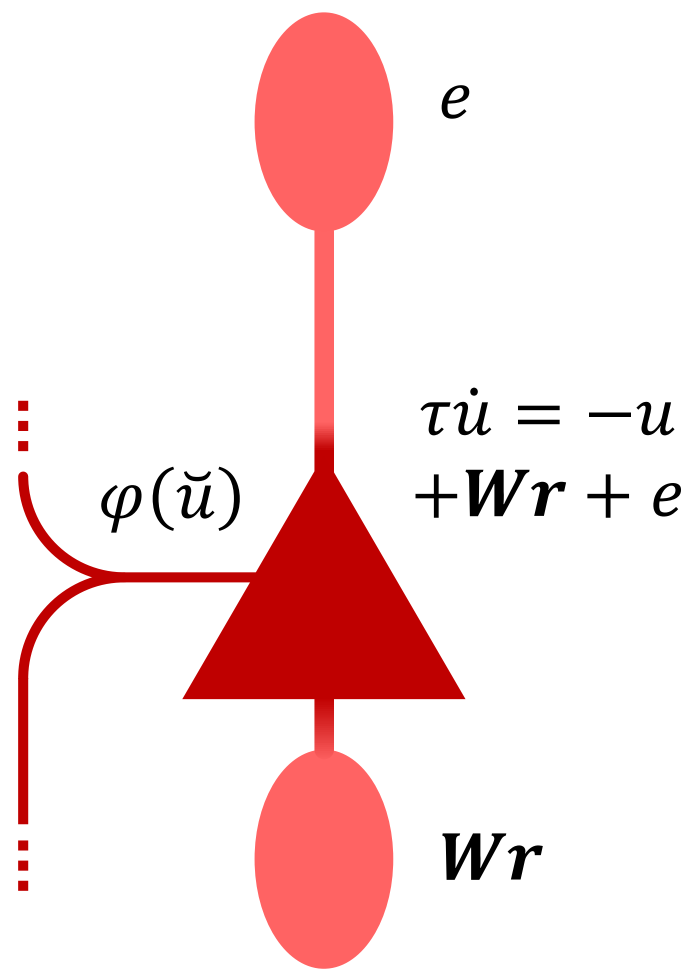
$$E(\breve{u}) = \frac{1}{2} \sum_{i \in \mathcal{N}} \left| \breve{u}_i - (W r)_i \right|^2 + \beta \mathcal{L}(\breve{u})$$

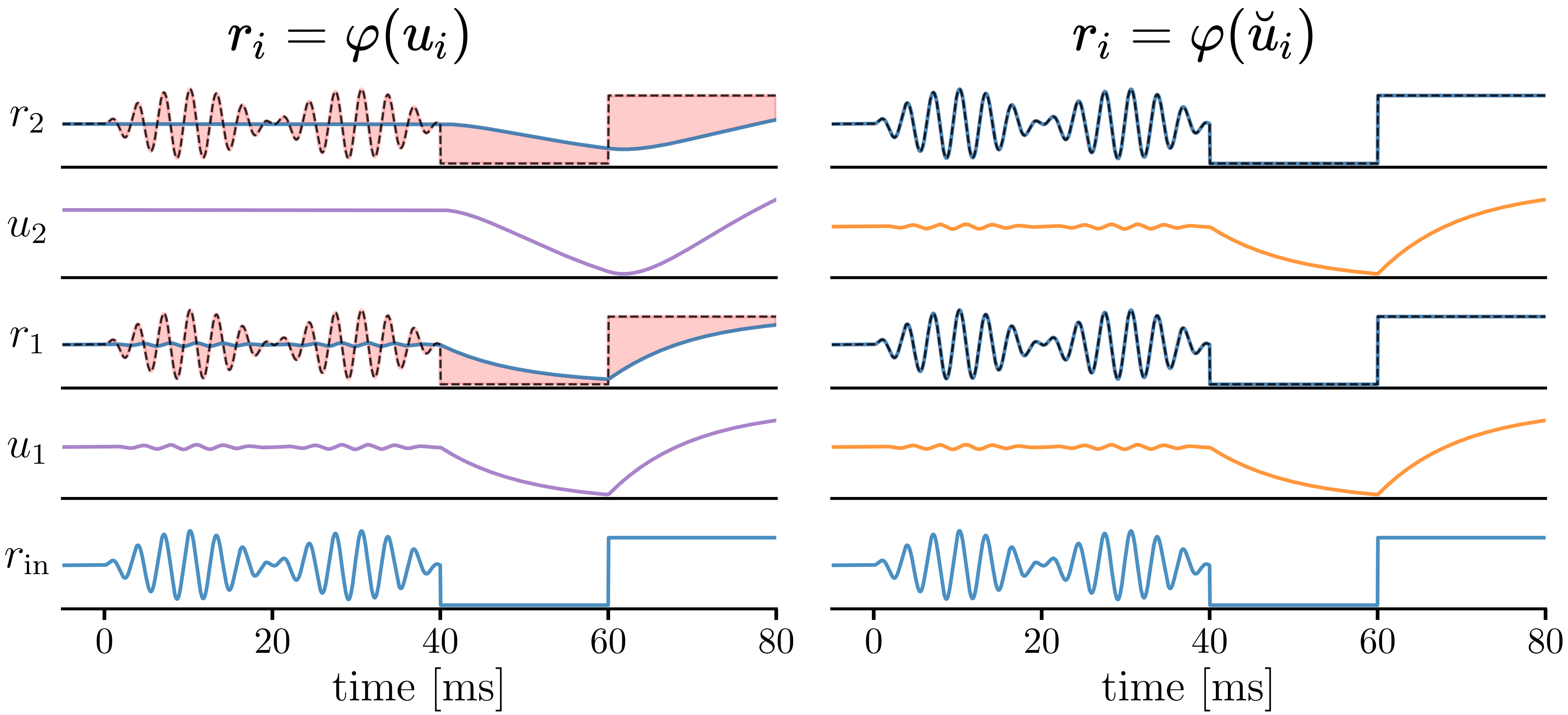
3. Neuronal dynamics and morphology

$$\nabla_{\breve{u}} E = 0 \quad \Longrightarrow \quad \tau \dot{u}_i = -u_i + (W r)_i + e_i$$



Latent Equilibrium solves the relaxation problem

$$\tau \dot{u}_i = -u_i + (Wr)_i + e_i$$



Latent Equilibrium: mathematical framework

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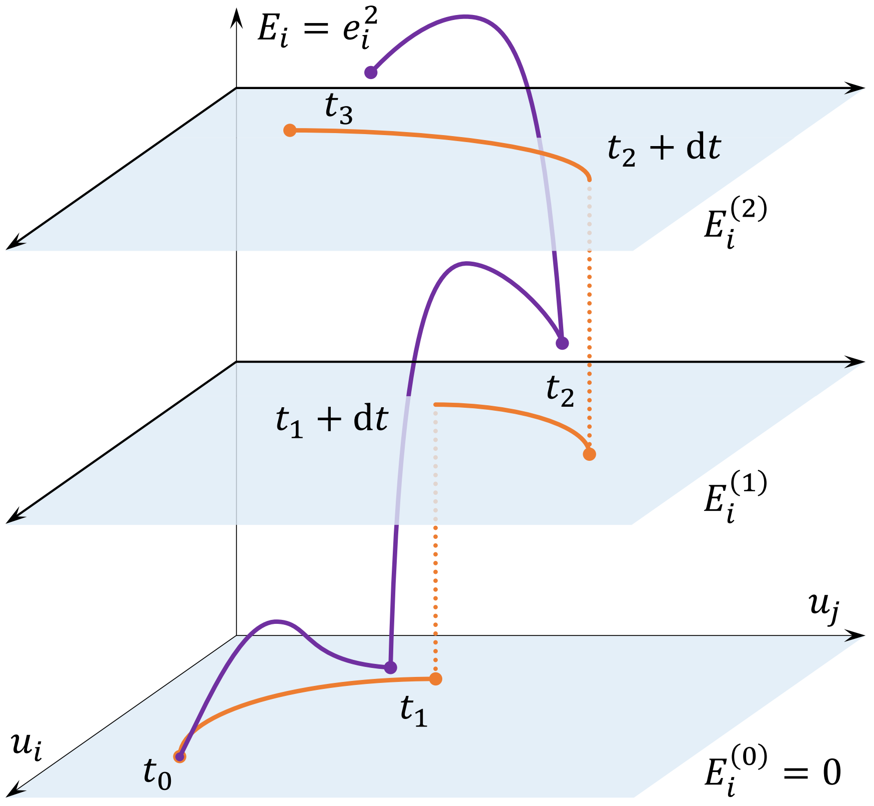
3. Neuronal dynamics and morphology

$$\nabla_{\breve{u}} E = 0 \quad \Longrightarrow \quad \tau \dot{u}_i = -u_i + (\boldsymbol{W}\boldsymbol{r})_i + e_i$$

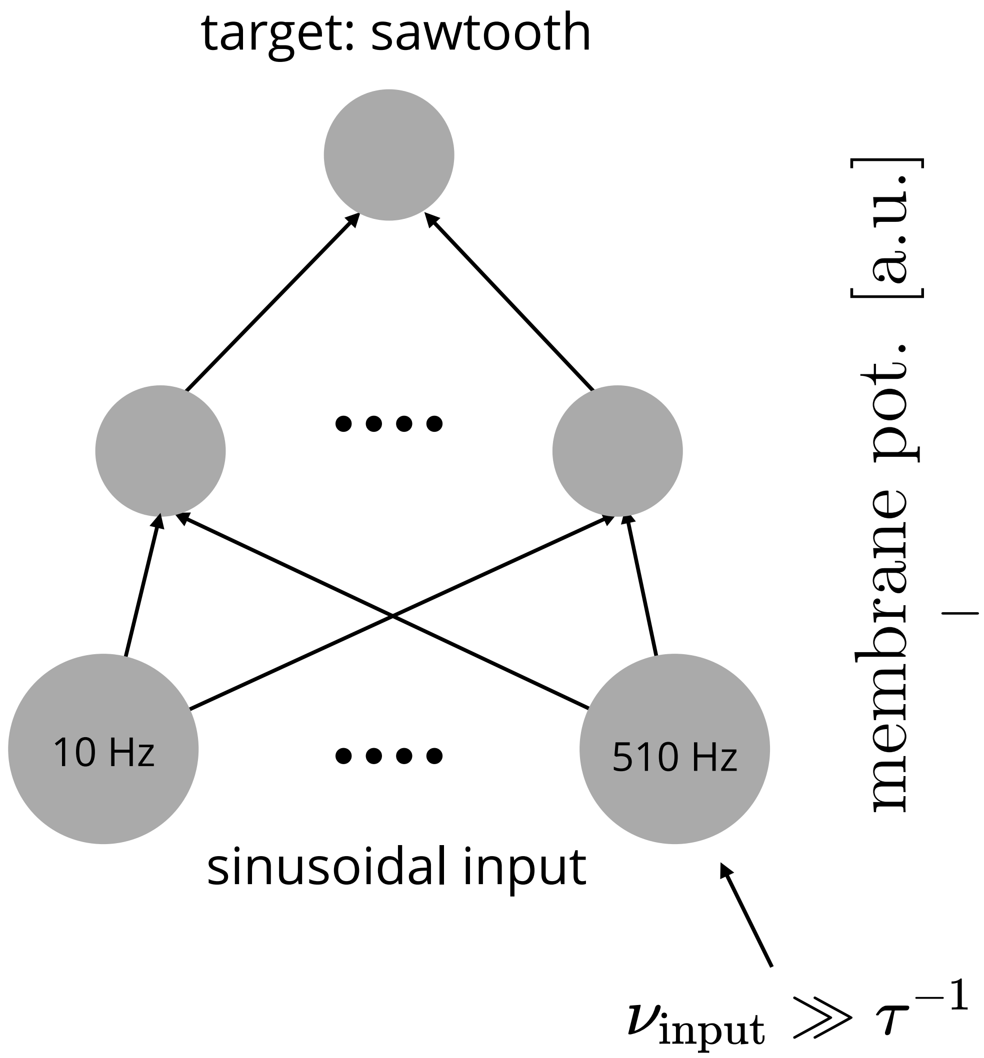
4. Synaptic plasticity

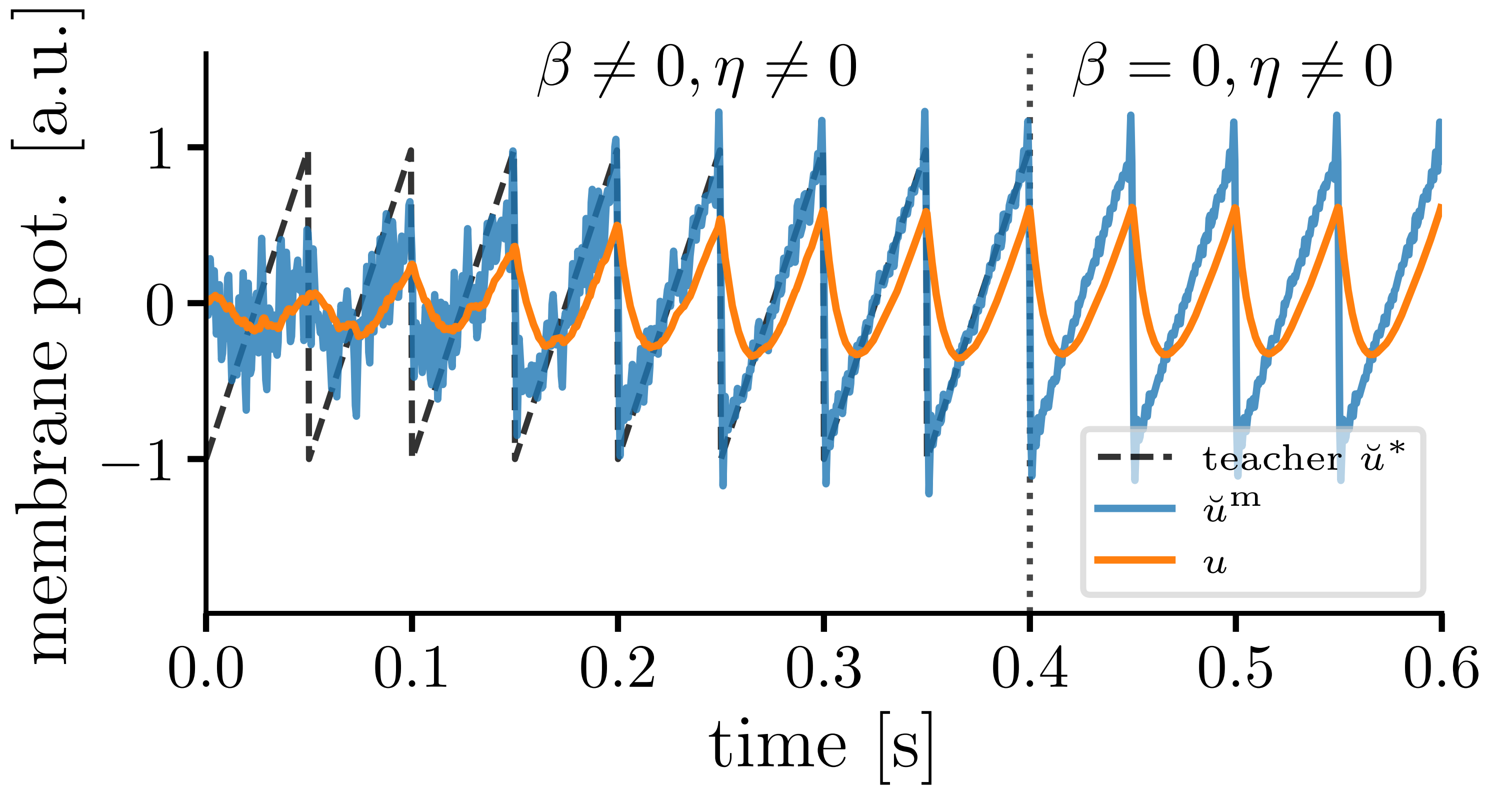
$$\dot{W} \propto -\nabla_W E \quad \Longrightarrow \quad \underbrace{\dot{W}_{ij} = \eta_W \left(\breve{u}_i - (\boldsymbol{W}\boldsymbol{r})_i\right) r_j}_{\text{learning rule by}}$$

learning rule by Urbanczik & Senn (2014)

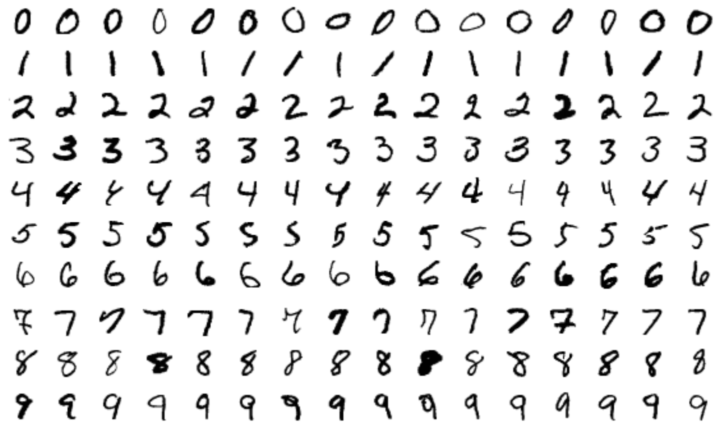


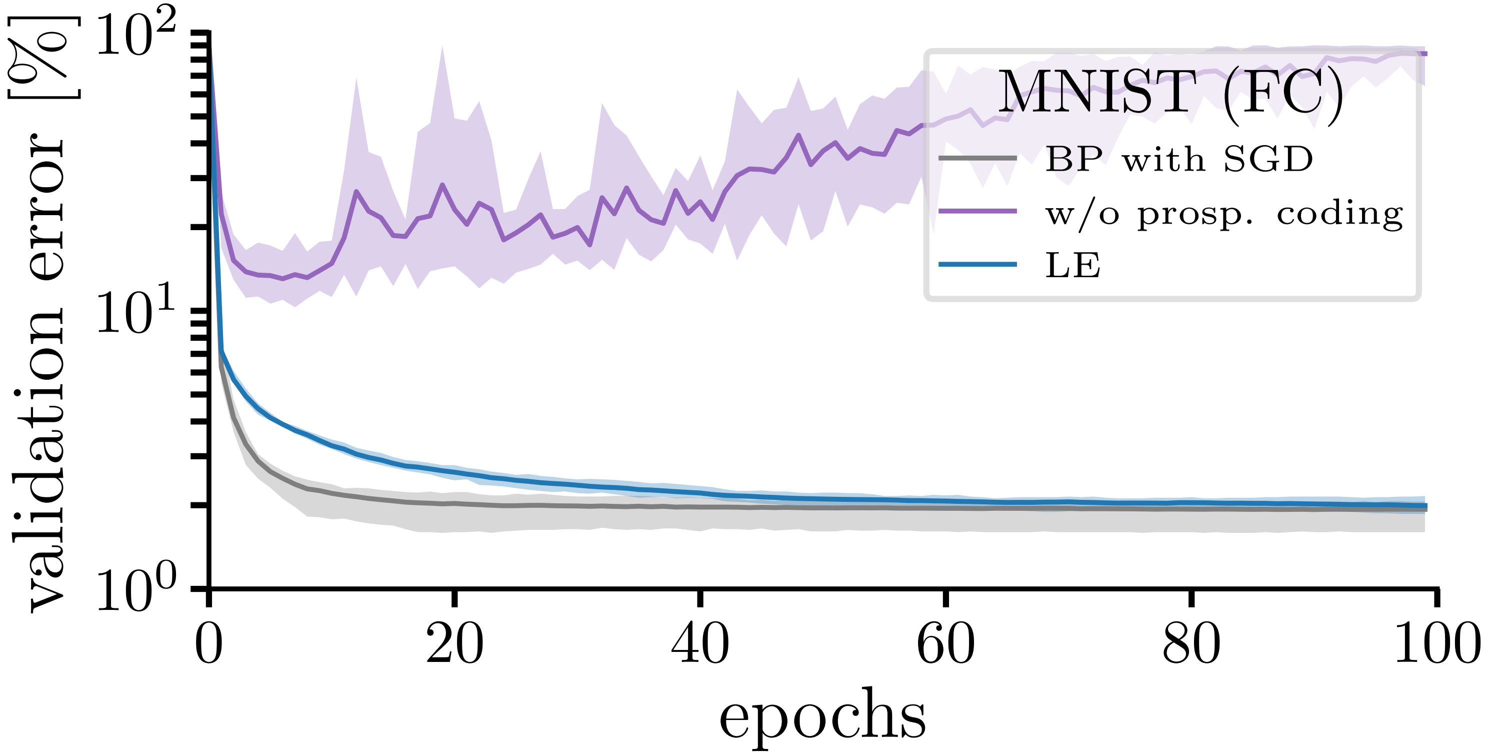
First results with LE: Fourier synthesis





Results on standard ML benchmarks: MNIST





architecture: fully-connected

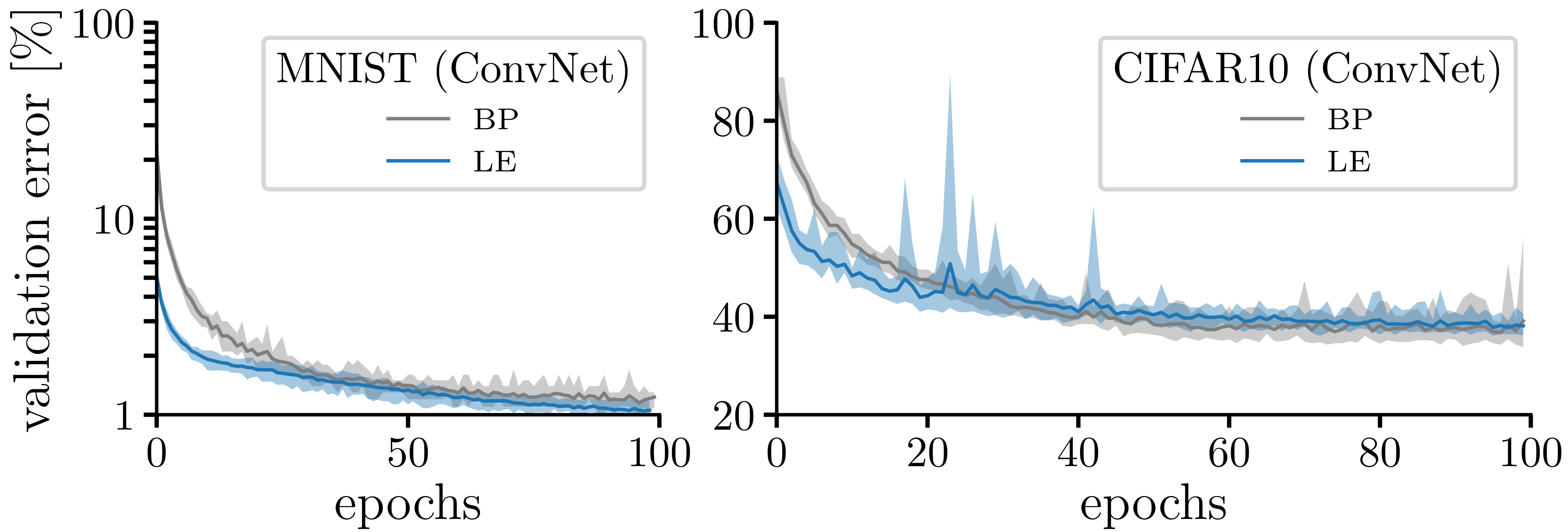
$784 - 300 - 100 - 10$

time-continuous dynamics with membrane time constant $\tau = 20\,\mathrm{ms}$

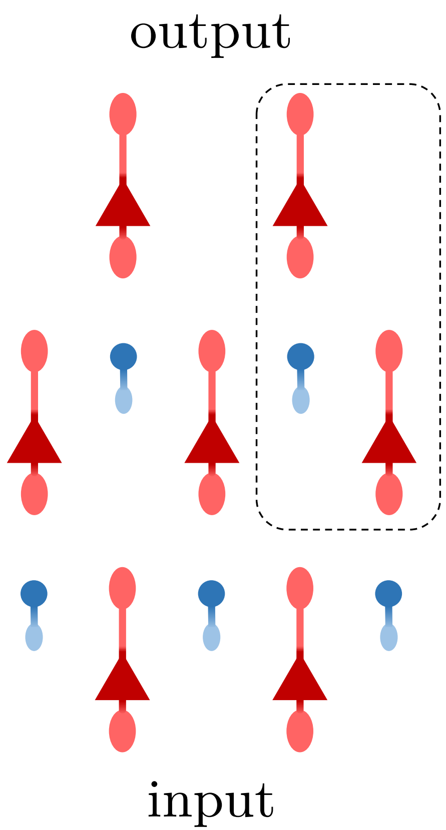
and presentation time $T_{\mathrm{pres}} = 1\,\mathrm{ms} = 0.05\,\tau$

LE with convolutional architectures

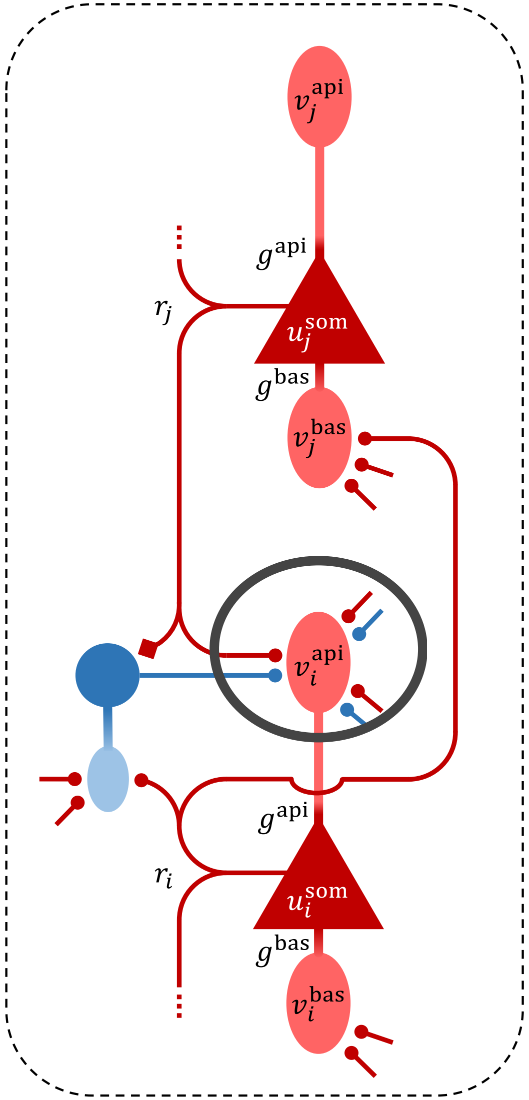
architecture: LeNet-5 (LeCun et al., 1989)



Microcircuit implementation



microcircuit based on
Sacramento et al. (2018)



apical dendrites encode errors:

$$e \approx v^{\mathrm{api}} = \varphi'(\breve{u}) W^{\mathrm{T}} (\breve{u} - W r)$$

for hierarchical networks:

$$e_\ell \approx v_\ell^{\mathrm{api}} = \varphi'(\breve{u}_\ell) W_{\ell+1}^{\mathrm{T}} e_{\ell+1}$$

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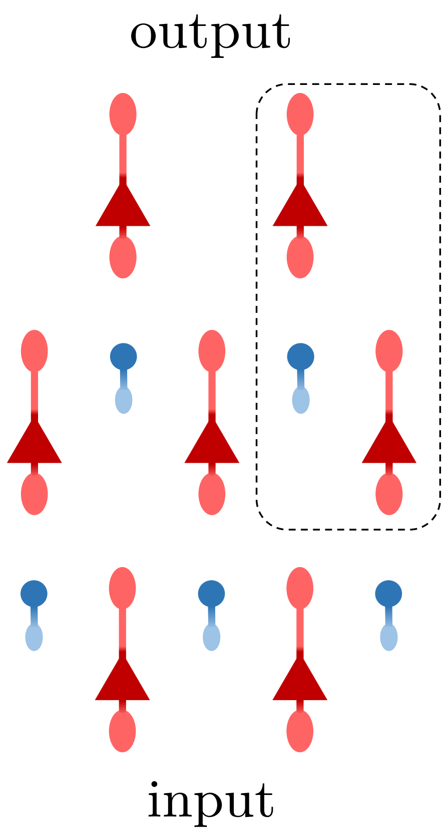
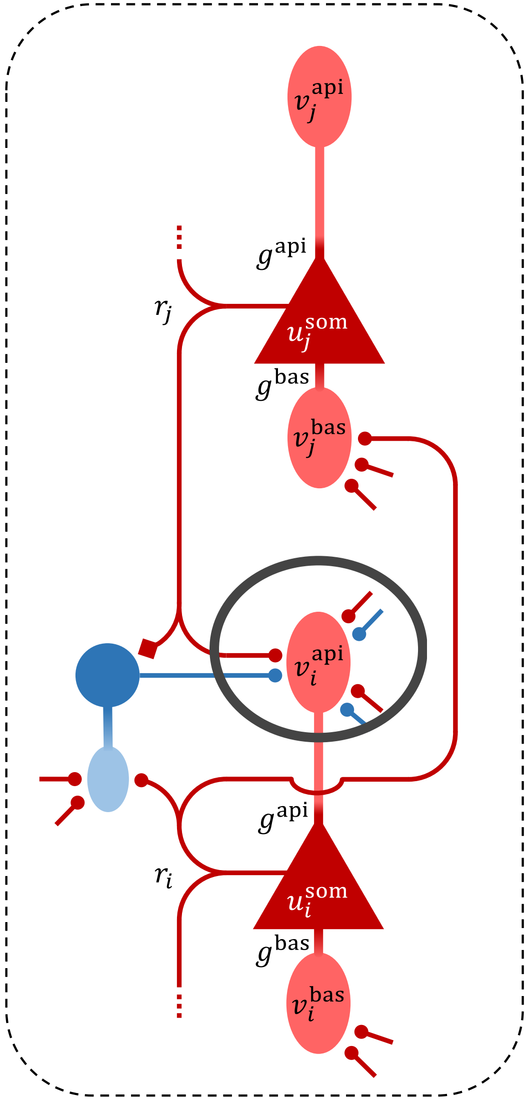
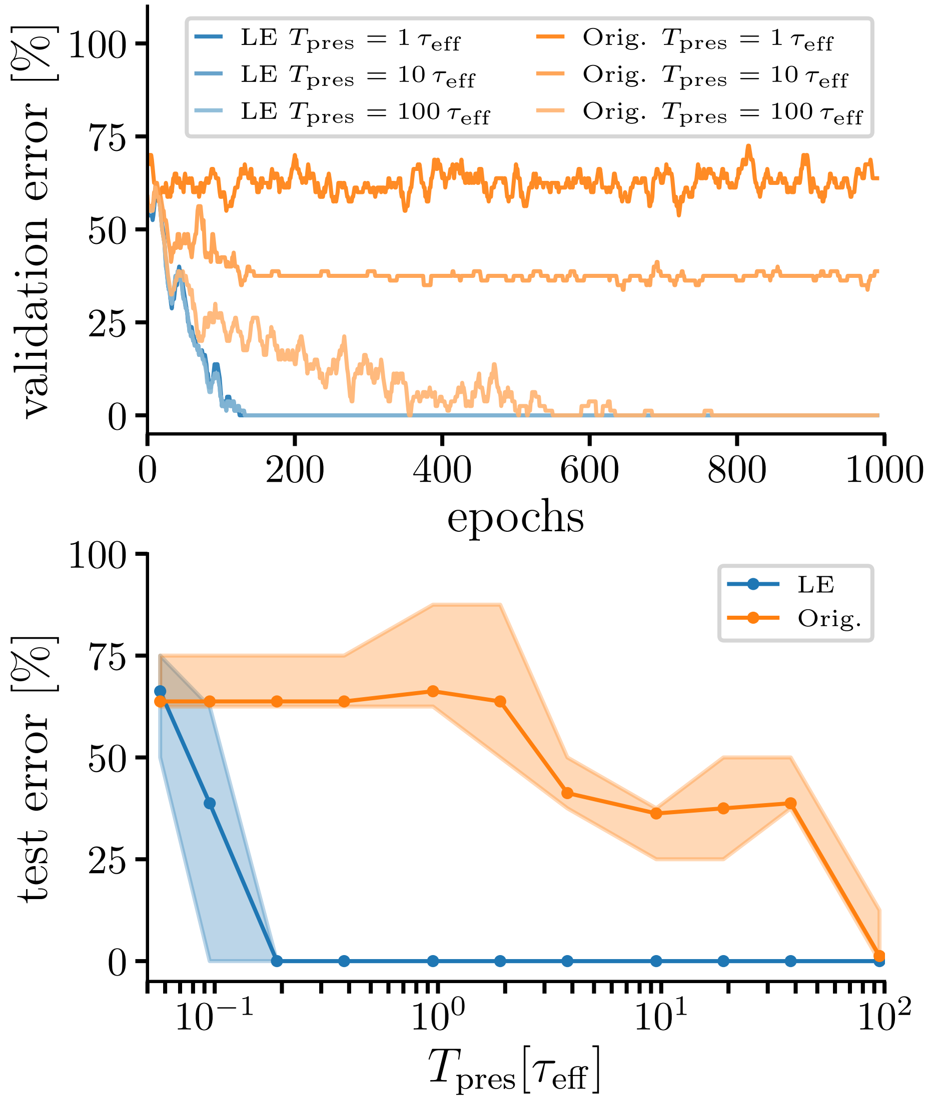
bio-plausible variant of BP with real-time dynamics and phase-free, continual local learning

Microcircuit implementation



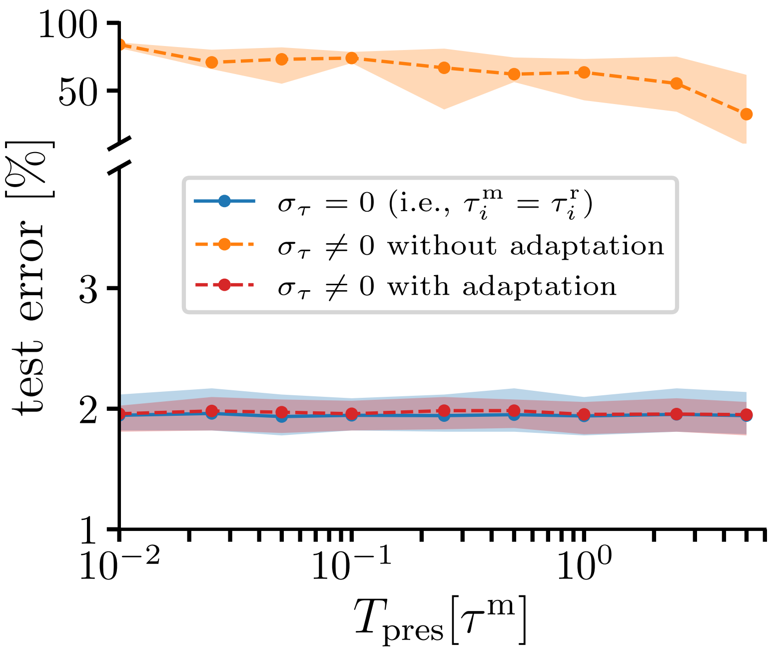




microcircuit based on
Sacramento et al. (2018)

Robustness to spatio-temporal noise in physical systems

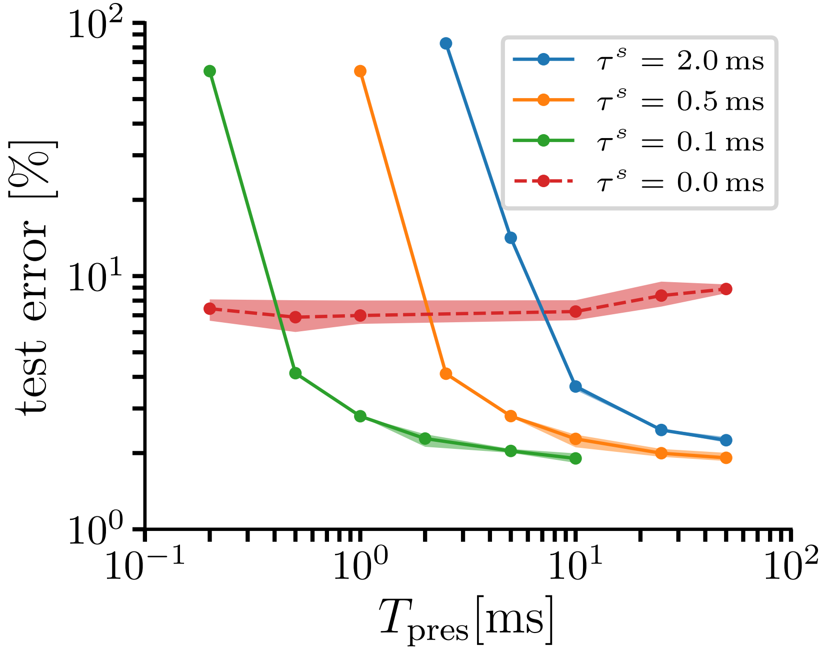
Spatial noise:



adaptation during development:

$$\dot{\tau}_i^{\mathrm{m}} = \eta_\tau \left(\breve{u}_i^{\mathrm{r}} - (W r)_i \right) \dot{u}_i$$

Temporal noise:



synaptic filtering:

$$r(t) \rightarrow \bar{r}^{\mathrm{s}}(t) = \frac{1}{\tau^{\mathrm{s}}} \int_{-\infty}^{t} r(t') \exp\left(\frac{t-t'}{\tau^{\mathrm{s}}}\right) \mathrm{d}t'$$

(dataset used: MNIST)

Conclusion

- problem: slow information processing **and disrupted learning** in slow physical systems outside of equilibrium

- solution: Latent Equilibrium
  - neuronal morphology, network structure, neuronal dynamics, synaptic plasticity from single **energy function**
  - continuous, but effectively **decoupled neuronal and synaptic dynamics**
- **bio-plausible** implementation in (recurrent) cortical microcircuits
- **robustness** against spatio-temporal distortions in physical substrates

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