Adaptive Density Estimation for Generative Models



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Generative modelling

Goal

Given samples from target distribution p^* , train a model p_{θ} to match p^*

1

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• Maximum likelihood: Eval. training points under the model

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Given samples from target distribution p^* , train a model p_{θ} to match p^*

- Maximum likelihood: Eval. training points under the model
- \bullet Adversarial training 1 : Eval. samples under (approximation of) p^{\ast}

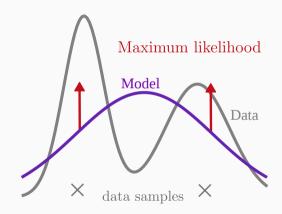
1

 $^{^{1}\}mbox{Ian}$ Goodfellow et al. (2014). "Generative adversarial nets". In: NIPS.

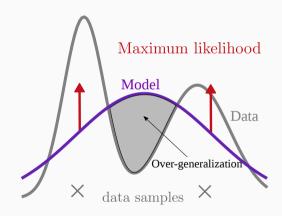
Schematic illustration



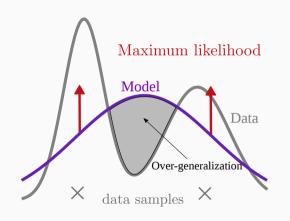
Maximum likelihood



Maximum likelihood



Maximum likelihood

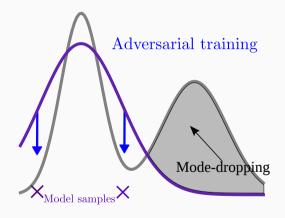


Consequences

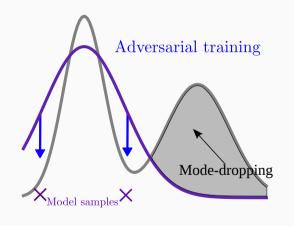
• MLE covers full support of distribution

• Produces unrealistic samples

Adversarial training



Adversarial training



Consequences

• Production of high quality samples

Parts of the support are dropped

Goal

• Explicitly optimize both dataset coverage and sample quality

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- Limiting parametric assumptions required for tractable MLE,
 e.g. Gaussianity, conditional independence

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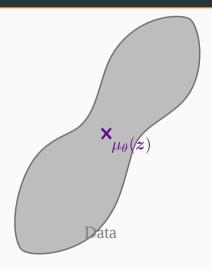
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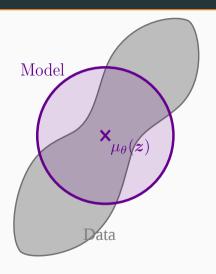
- Tradeoff between the two objectives: need more flexibility
- Limiting parametric assumptions required for tractable MLE,
 e.g. Gaussianity, conditional independence
- Often no likelihood in pixel space²

 $^{^2}$ A. Larsen et al. (2016). "Autoencoding beyond pixels using a learned similarity metric". In: ICML.

Conditional independence

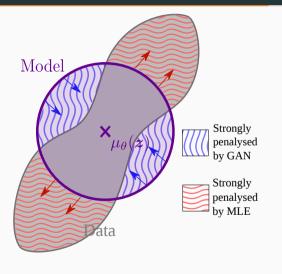


Conditional independence



$$p(\boldsymbol{x}|\boldsymbol{z}) = \prod_{i}^{N} \mathcal{N}(x_{i}|\mu_{\theta}(\boldsymbol{z}), \sigma I_{n})$$

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Going beyond conditional independence

Avoiding strong parametric assumptions

• Lift reconstruction losses into a feature space

Going beyond conditional independence

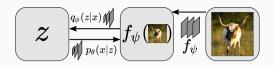
Avoiding strong parametric assumptions

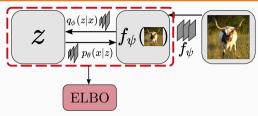
- Lift reconstruction losses into a feature space
- Deep invertible models: valid density in image space

Going beyond conditional independence

Avoiding strong parametric assumptions

- Lift reconstruction losses into a feature space
- Deep invertible models: valid density in image space
- Retain fast sampling for adversarial training

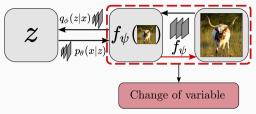




Amortized Variational inference in feature space:

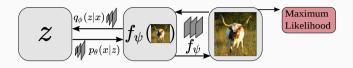
$$\mathcal{L}_{\theta,\phi,\psi}(\boldsymbol{x}) = \underbrace{-\mathbb{E}_{q_{\phi}(\boldsymbol{z}|\boldsymbol{x})}\left[\ln(p_{\theta}(f_{\psi}(\boldsymbol{x})|\boldsymbol{z}))\right] + D_{KL}(q_{\phi}(\boldsymbol{z}|\boldsymbol{x})||p_{\theta}(\boldsymbol{z}))}_{-\ln\left|\det\frac{\partial f_{\psi}}{\partial \boldsymbol{x}}\right|}$$

Evidence lower bound in feature space



Amortized Variational inference in feature space:

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Adversarial training with Adaptive Density Estimation:

$$\mathcal{L}_{\mathsf{adv}}(p_{\theta,\psi}) = \underbrace{-\mathbb{E}_{p_{\theta}(z)} \left[\ln \frac{D(f_{\psi}^{-1}(\mu_{\theta}(z)))}{1 - D(f_{\psi}^{-1}(\mu_{\theta}(z)))} \right]}$$

Adv. update using log ratio loss

Experiments on CIFAR10



Samples



Real images

q

Experiments on CIFAR10



Samples



Real images

Model	BPD ↓	IS ↑	FID ↓
1710461	GAN		112 4
WGAN-GP		7.9	
SNGAN		7.4	29.3
$SNGAN_{(R,H)}$		8.2	21.7
	MLE		
VAE-IAF	3.1	3.8^{\dagger}	73.5^{\dagger}
NVP	3.5	4.5^{\dagger}	56.8^{\dagger}
	Hybrid		
Ours (v1)	3.8	8.2	17.2
Ours (v2)	3.5	6.9	28.9
FlowGan	4.2	3.9	

Samples and real images (LSUN churches, 64×64)



Samples @ $4.3\ BPD$



Real images

Thank you for listening. Come see us at poster 71:)