Online Learning via the Differential Privacy Lens

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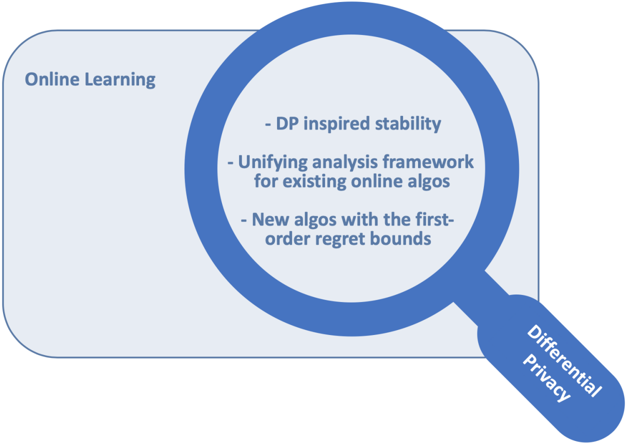
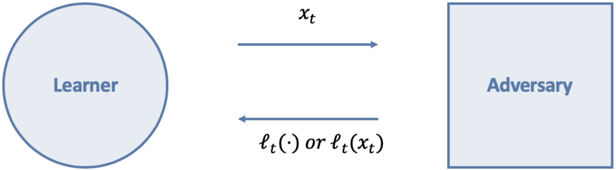


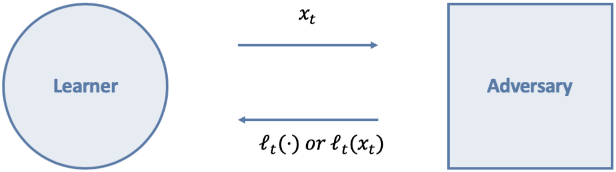
DP inspired stability is well-suited to analyzing OL algorithms

Adversarial Online Learning Problems



- A sequential game between *Learner* and *Adversary*
- Learner chooses its action $x_t \in \mathcal{X}$, which can be *random*
- Adversary chooses a loss function $\ell_t \in \mathcal{Y}$ (NOT random)
- *Full Info.*: the entire function ℓ_t is revealed to the learner
- *Partial Info.*: only the function value $\ell_t(y_t)$ is revealed

Adversarial Online Learning Problems



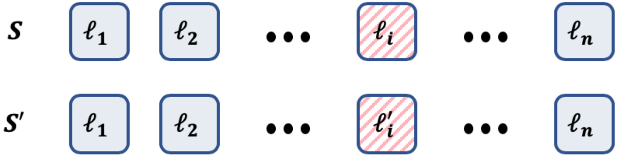
- The learner's goal is to minimize the *expected regret*:

$$\mathbb{E}[\text{Regret}_T] = \mathbb{E}[\sum_{t=1}^{T} \ell_t(x_t)] - L_T^\star, \text{ where } L_T^\star = \min_{x \in \mathcal{X}} \sum_{t=1}^{T} \ell_t(x).$$

- *Zero-order* bound proves $\mathbb{E}[\text{Regret}_T] = o(T)$
- *First-order* bound proves $\mathbb{E}[\text{Regret}_T] = o(L_T^\star)$
  - The first-order bound is more desirable if $L_T^\star = o(T)$
- OCO, OLO, expert problems, MABs, bandits with experts

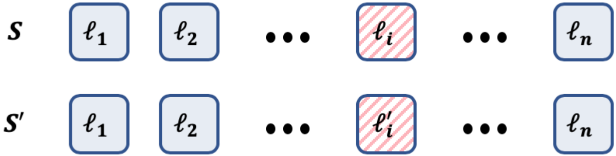
Differential Privacy

Let $\mathcal{A}$ be a randomized algorithm that maps a data set S to a decision rule in $\mathcal{X}$



- $\mathcal{A}(S)$ will be available to users but NOT S itself
- We do *NOT* want the users to infer our data set S from $\mathcal{A}(S)$
- Suppose S and S' differ only by a single entry
  $\Rightarrow$ We want $\mathcal{A}(S)$ and $\mathcal{A}(S')$ to be similar

Differential Privacy



- The δ-*approximate max-divergence* between two distributions P and Q is (sup takes over all measurable sets)

$$D_\infty^\delta(P, Q) = \sup_{P(B) > \delta} \log \frac{P(B) - \delta}{Q(B)}$$

- We say $\mathcal{A}$ is (ϵ, δ)-DP if $D_\infty^\delta(\mathcal{A}(S), \mathcal{A}(S')) < \epsilon$

New Stability Notions

Main Observation

In online learning, Follow-The-Leader algorithm performs badly while F-T-Purturbed-L or F-T-Regularized-L do well.

Definition 1 (One-step differential stability)

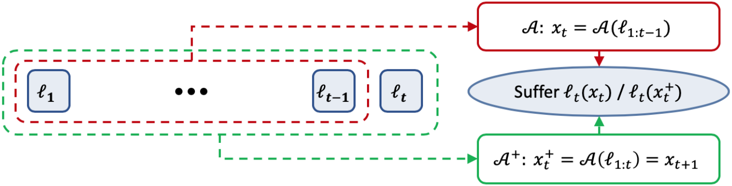
For a divergence D, $\mathcal{A}$ is called DiffStable(D) at level ϵ iff for any t and any $\ell_{1:t} \in \mathcal{Y}^t$, we have $D(\mathcal{A}(\ell_{1:t-1}), \mathcal{A}(\ell_{1:t})) \leq \epsilon$

Definition 2 (DiffStable, when losses are vectors)

For a norm $||\cdot||$, $\mathcal{A}$ is called DiffStable(D,$||\cdot||$) at level ϵ iff for any t and any $\ell_{1:t} \in \mathcal{Y}^t$, we have $D(\mathcal{A}(\ell_{1:t-1}), \mathcal{A}(\ell_{1:t})) \leq \epsilon ||\ell_t||$

Remark. $\ell_{1:t-1}$ and $\ell_{1:t}$ only differ by one item!

Key Lemma



Suppose loss functions always belong to $[0, B]$ for some B and $\mathcal{A}$ is DiffStable(D_∞^δ) at level $\epsilon \leq 1$. Then the regret of $\mathcal{A}$ satisfies

$$\mathbb{E}[\text{Regret}(\mathcal{A})_T] \leq 2\epsilon L_T^* + 3\mathbb{E}[\text{Regret}(\mathcal{A}^+)_T] + \delta BT.$$

- We can adopt DiffStable algorithms from DP community
- $\mathbb{E}[\text{Regret}(\mathcal{A}^+)_T]$ is usually small (independent of T)
- δ can be set to be as small as $1/BT$

Online Convex Optimization

Algorithm 1 Online convex optimization using Obj-Pert

1: **Given** Obj-Pert solves the convex optimization while preserving DP
2: **for** $t = 1, \cdots, T$ **do**
3: Play $x_t = \text{Obj-Pert}(\ell_{1:t-1}; \epsilon, \delta, \beta, \gamma)$
4: **end for**

- Algorithm 1 is automatically DiffStable due to Obj-Pert (object perturbation) algorithm from DP literature
- When applying the Key Lemma, $\mathbb{E}[\text{Regret}(\mathcal{A}^+)_T]$ scales as $\frac{1}{\epsilon}$

$$\mathbb{E}[\text{Regret}(\mathcal{A})_T] \leq 2\epsilon L_T^* + 3\mathbb{E}[\text{Regret}(\mathcal{A}^+)_T] + \delta BT$$

- Tuning ϵ and setting $\delta = 1/BT$, we get the first-order regret bound of $O(\sqrt{L_T^\star})$

Other Applications

- OLO/OCO, Expert Learning, MABs, Bandits with Experts
- Zero-order and First-order regret bounds
- Provide a unifying framework to analyze OL algorithms
- Come to Poster #53 @ East Exhibition Hall B + C (that starts NOW!) for more details

Thanks!