

Learning Reward Machines for Partially Observable Reinforcement Learning

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ELEMENT^{AI}

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What is a Reward Machine (RM)?



*Using Reward Machines for High-Level Task Specification and Decomposition in Reinforcement Learning

by Toro Icarte et al. (ICML, 2018)

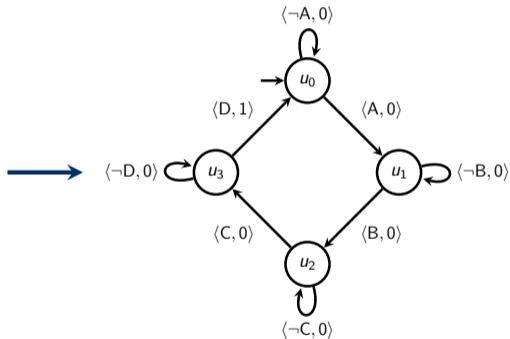
Reward Machines (RMs)

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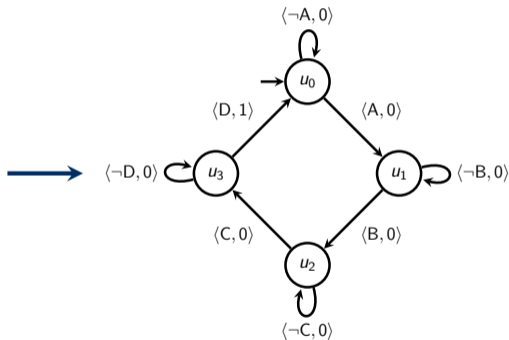
```
1 m = 0 # global variable
2 def get_reward(s):
3     if m == 0 and s.at("A"):
4         m = 1
5     if m == 1 and s.at("B"):
6         m = 2
7     if m == 2 and s.at("C"):
8         m = 3
9     if m == 3 and s.at("D"):
10        m = 0
11        return 1
12    return 0
```



Reward Machines (RMs)

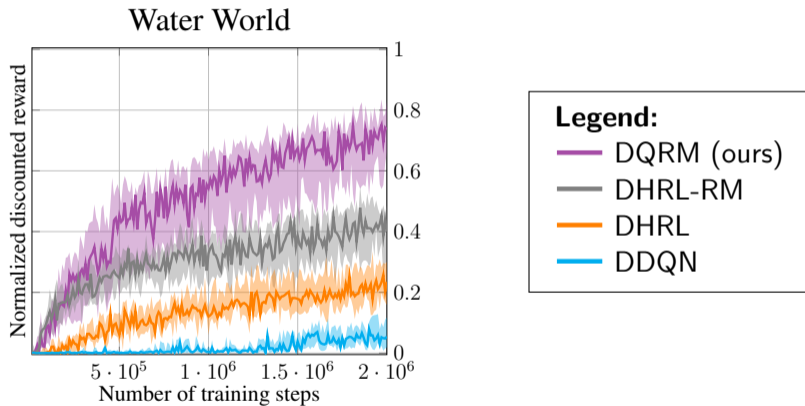
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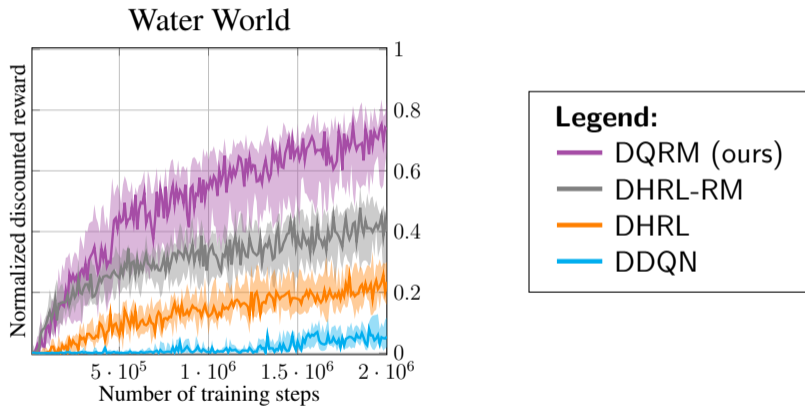


... that allow for **learning policies faster**.

Reward Machines (RMs)



Reward Machines (RMs)



... but the RMs were **handcrafted**.

This work:

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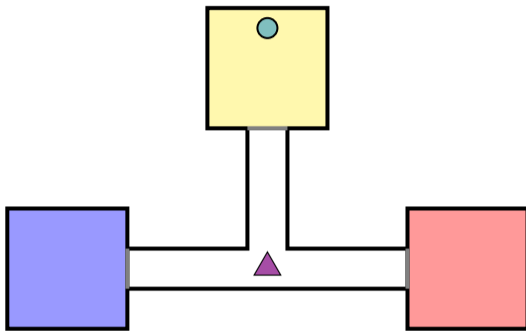
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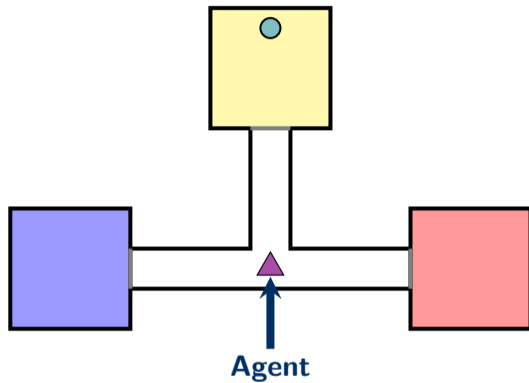
- 1 Shows how to learn RMs from experiences (LRM).
- 2 Uses RMs as memory for partially observable RL.
- 3 Extends QRM to work under partial observability.
- 4 Provides a theoretical and empirical analysis of LRM.

The Cookie Domain

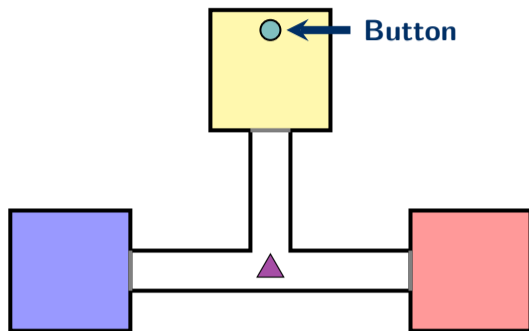
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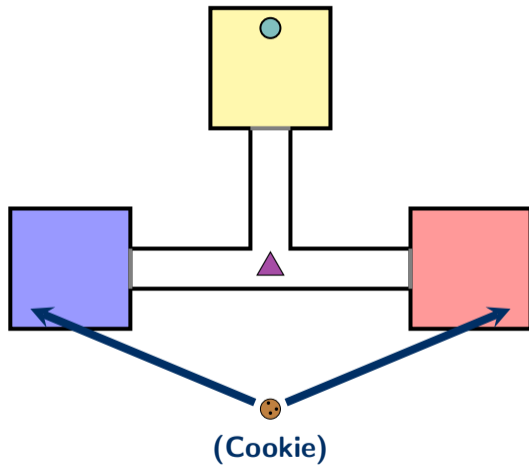
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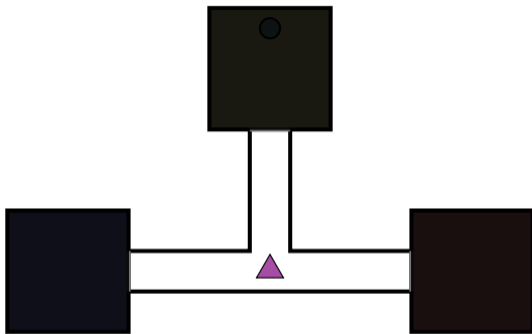
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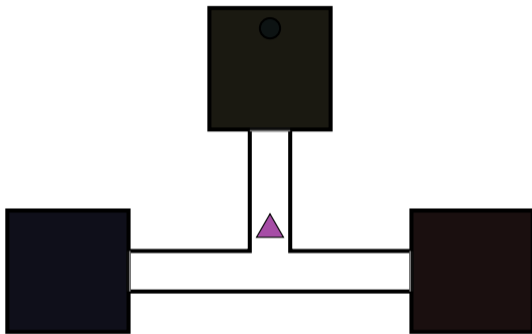
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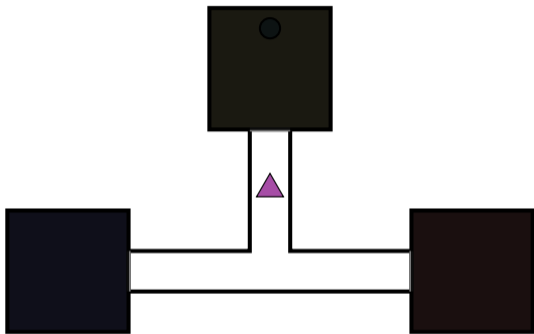
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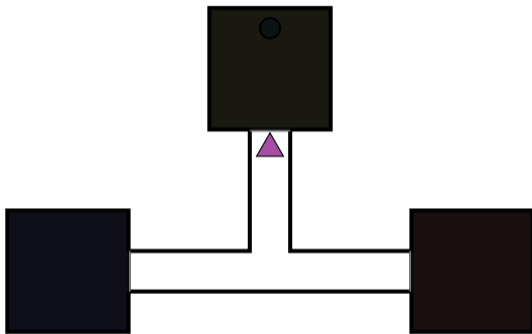
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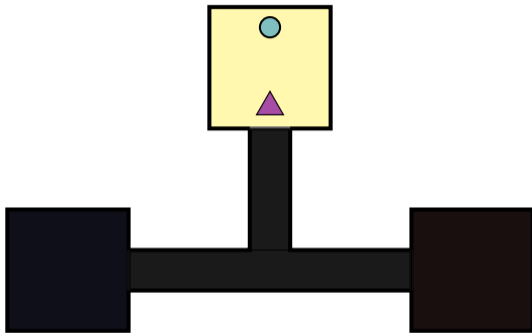
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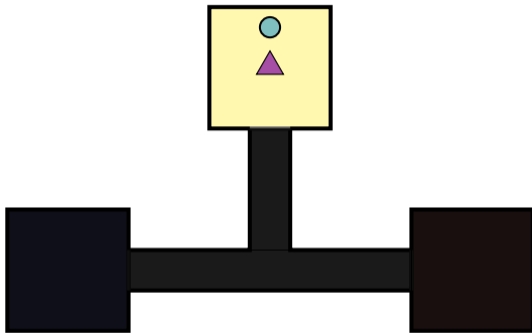
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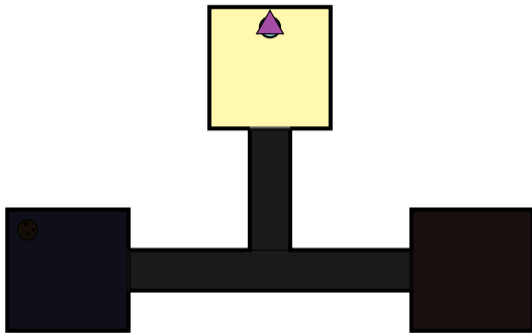
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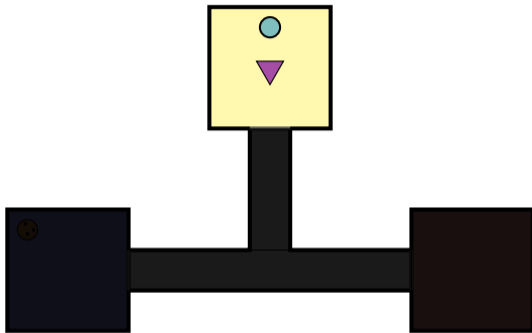
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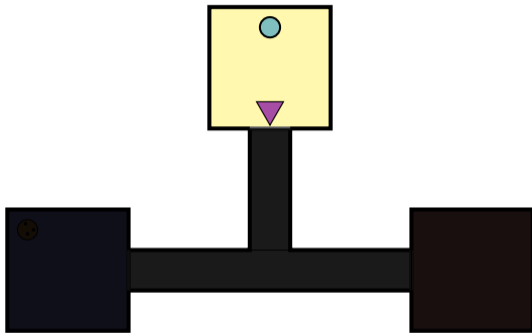
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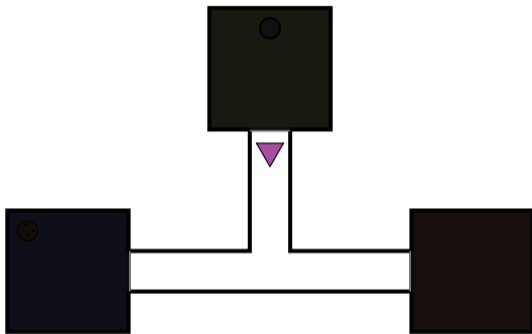
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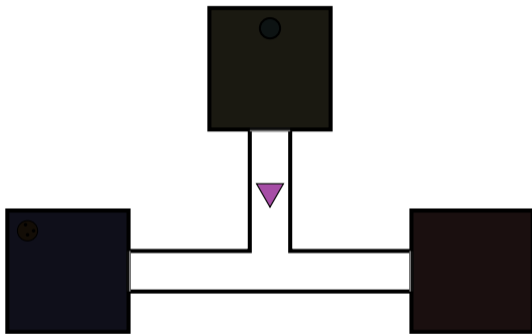
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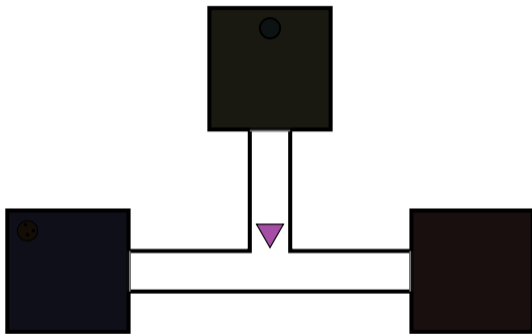
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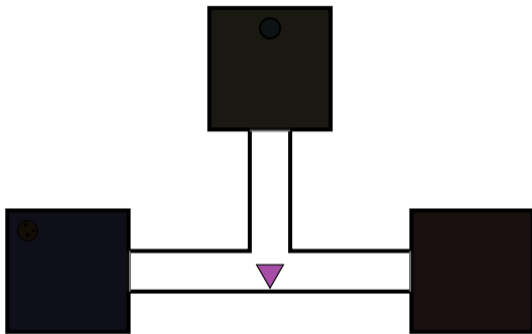
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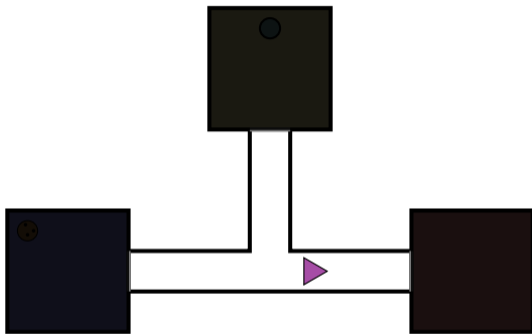
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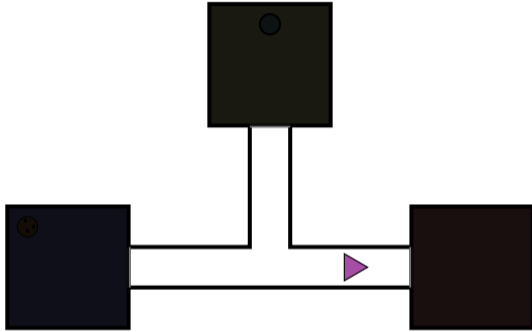
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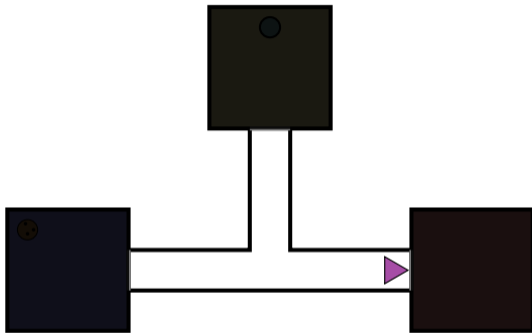
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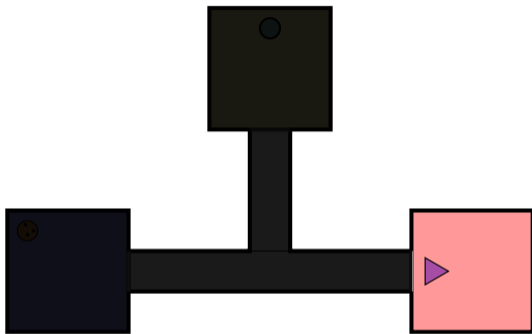
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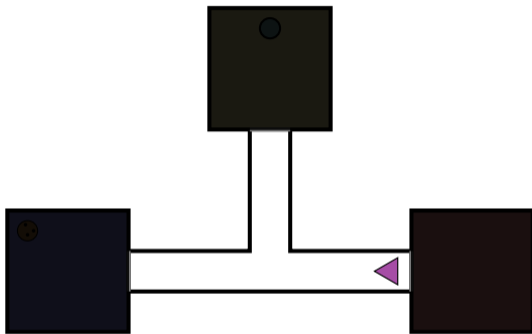
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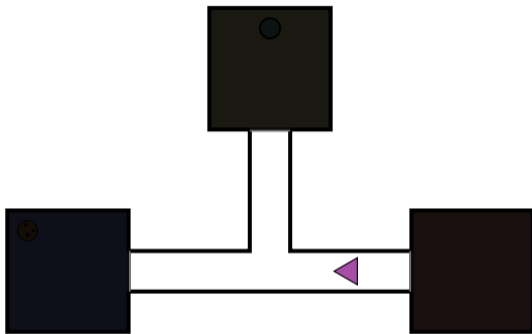
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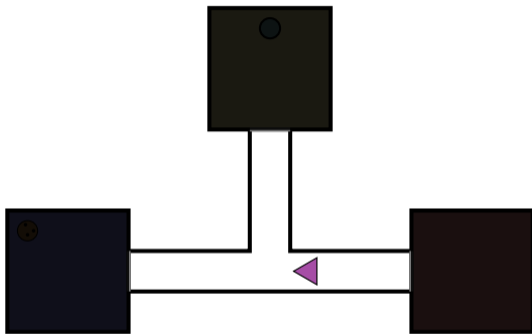
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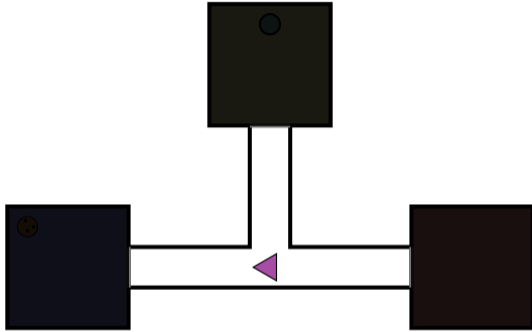
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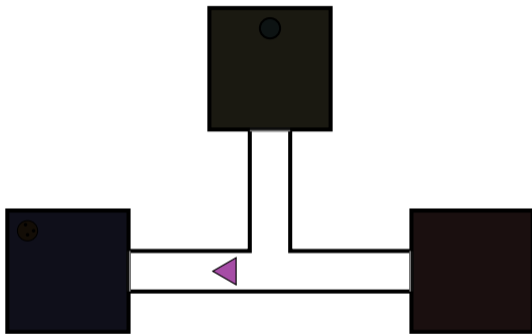
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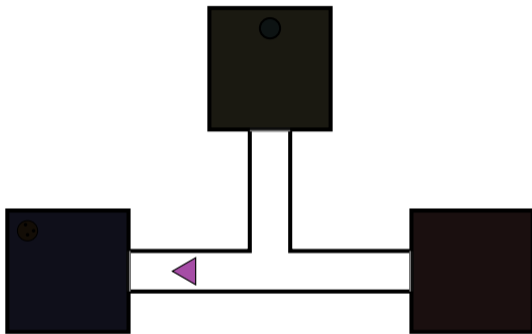
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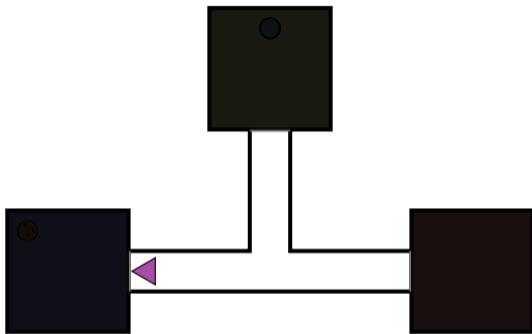
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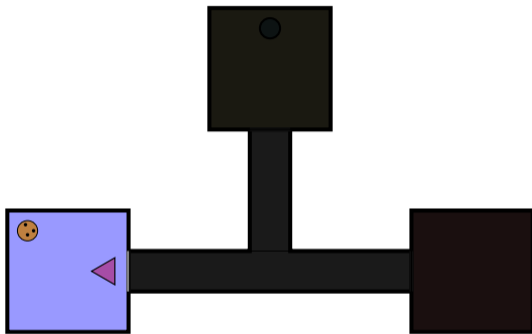
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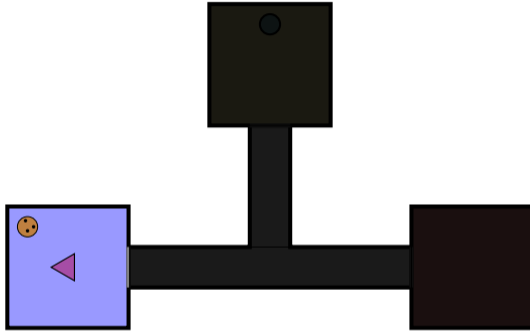
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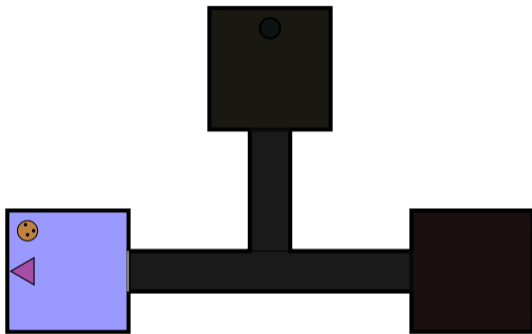
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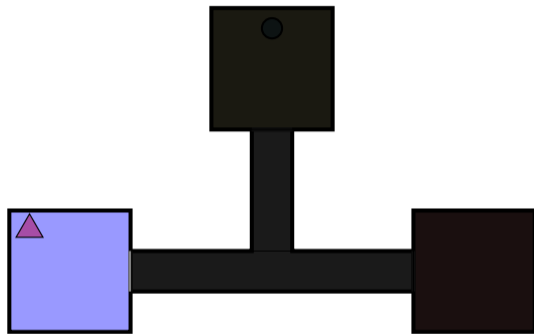
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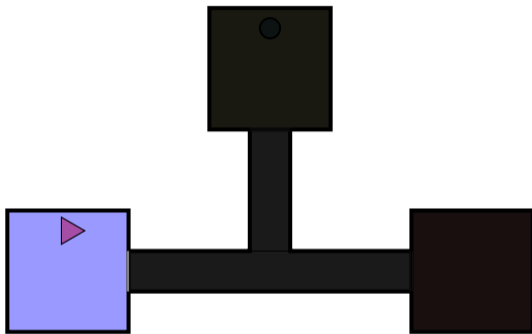


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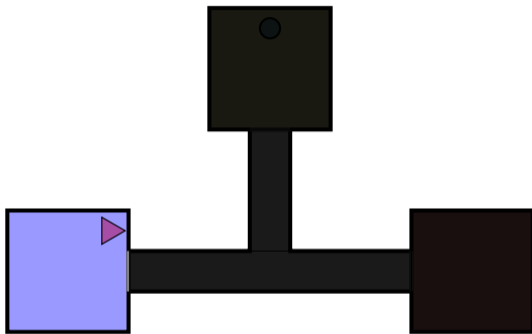


(+1 Reward)

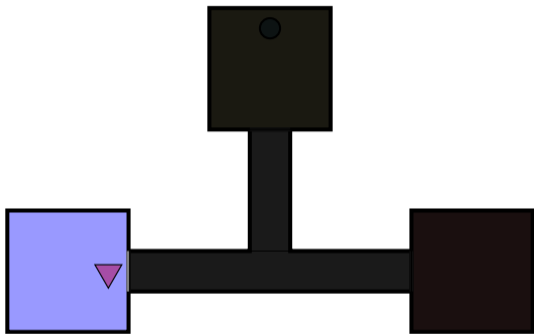
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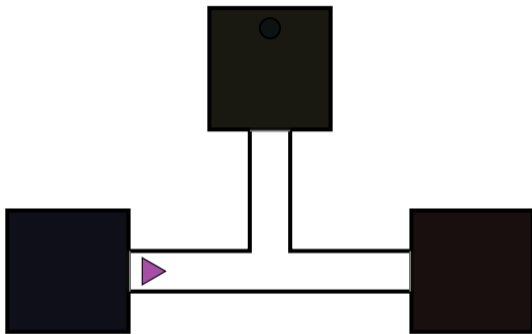
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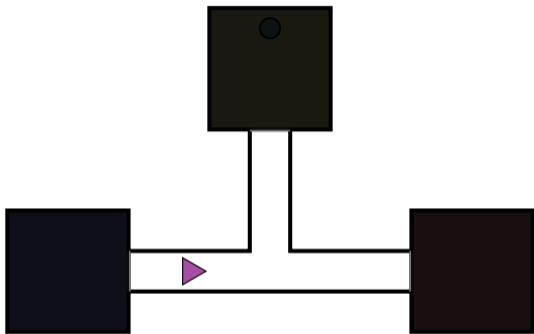
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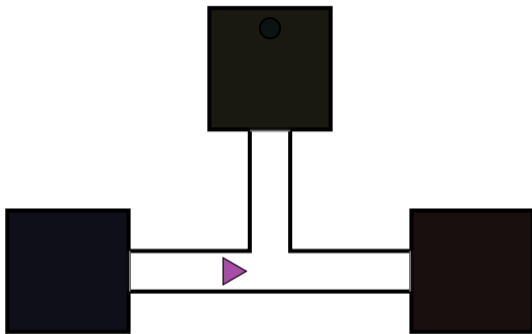
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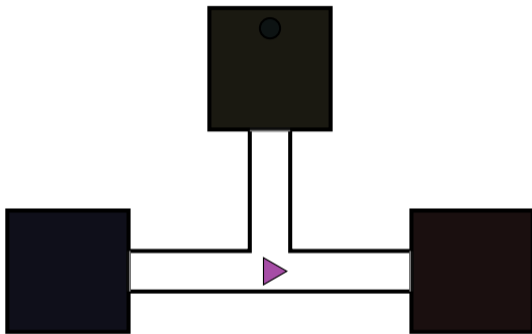
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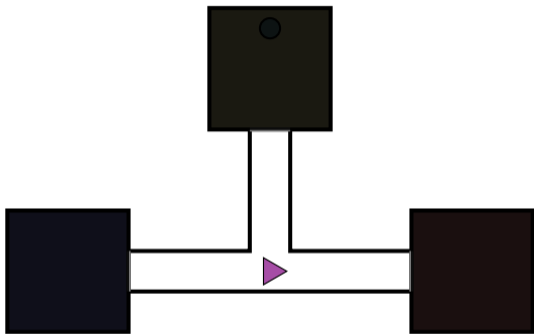
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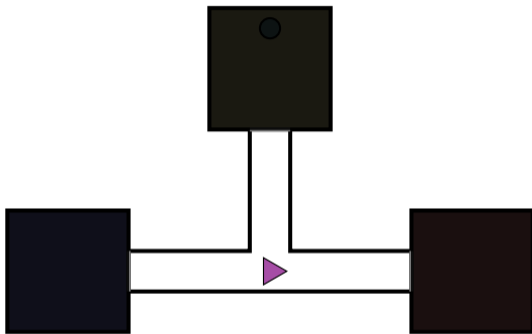


The cookie domain



Solving the cookie domain requires **memory**!

The cookie domain



Solving the cookie domain requires **memory!**

$$\pi^*(a|o_t) \neq \pi^*(a|o_0, \dots, o_t)$$

Partially Observable RL

The most popular approach:

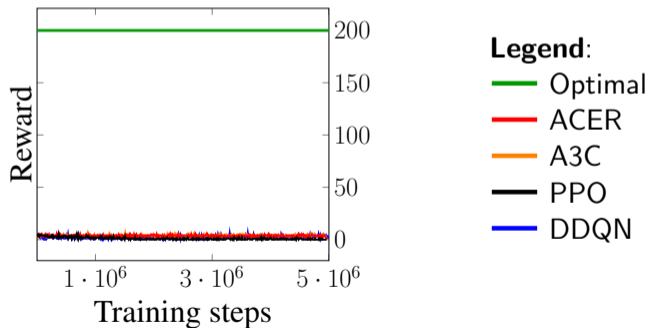
Training LSTMs policies using a policy gradient method.

Partially Observable RL

The most popular approach:

Training LSTMs policies using a policy gradient method.

... **starves** in the cookie domain.



RM as memory

Reward Machines as memory

If the agent can detect the color of the rooms (■, □, ■, ■),

Reward Machines as memory

If the agent can detect the color of the rooms (■, □, ■, ■), and when it presses the button (●),

Reward Machines as memory

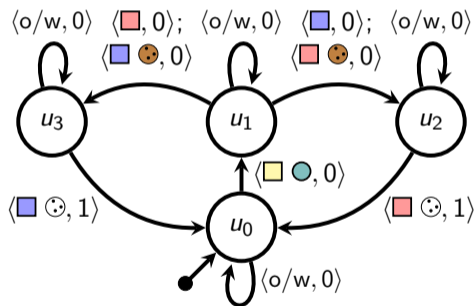
If the agent can detect the color of the rooms (■, □, ■, ■), and when it presses the button (●), eats a cookie (☺),

Reward Machines as memory

If the agent can detect the color of the rooms (🟥, 🟩, 🟦, 🟨), and when it presses the button (🟢), eats a cookie (😊), and sees a cookie (🍪),

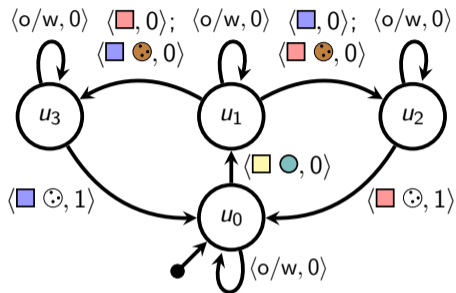
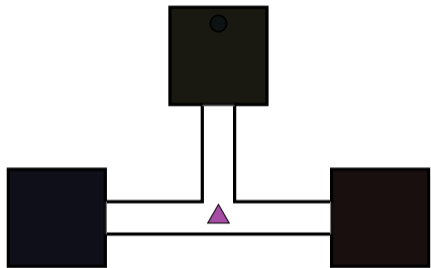
Reward Machines as memory

If the agent can detect the color of the rooms (■, □, ■, ■), and when it presses the button (●), eats a cookie (☺), and sees a cookie (☹), then:

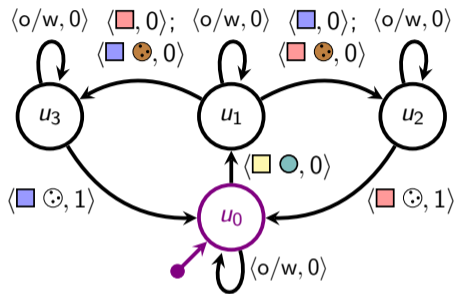
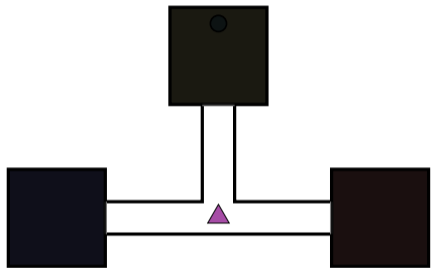


... becomes a **“perfect” memory** for the cookie domain.

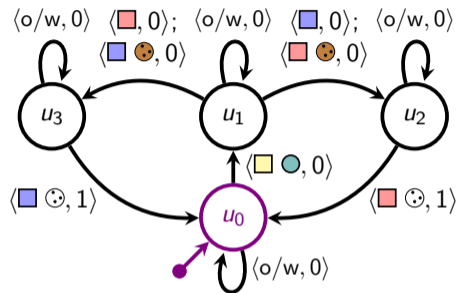
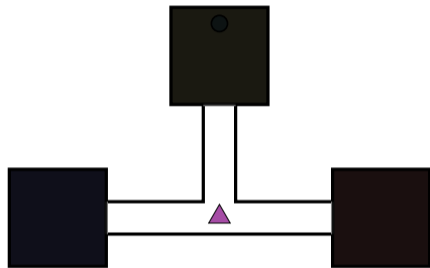
Reward Machines as memory



Reward Machines as memory



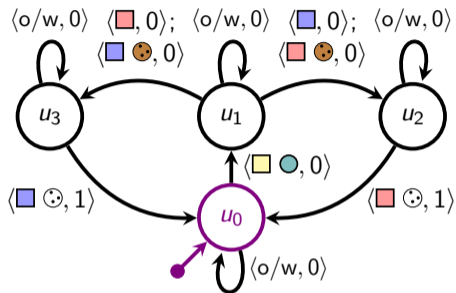
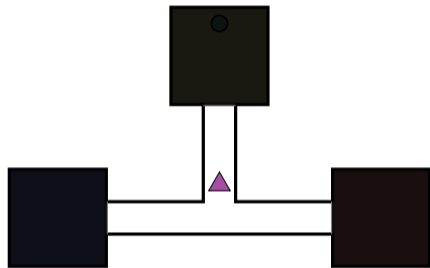
Reward Machines as memory



conditions at state u_0

if (yellow square teal circle)	→	goto u_1
else	→	goto u_0

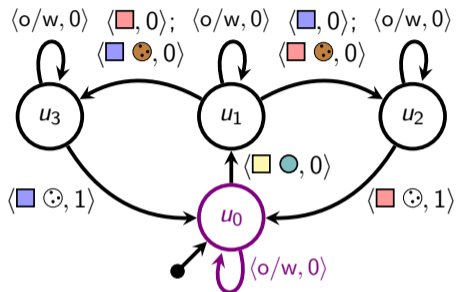
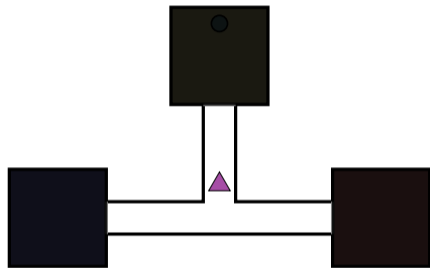
Reward Machines as memory



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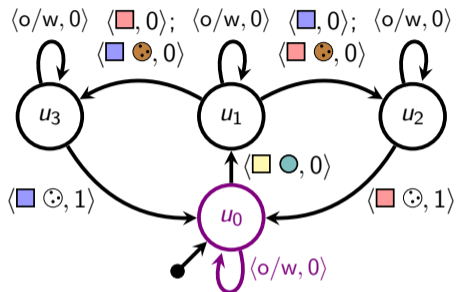
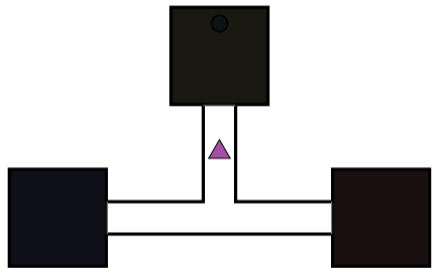
Reward Machines as memory



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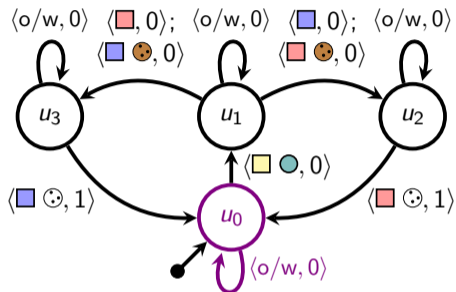
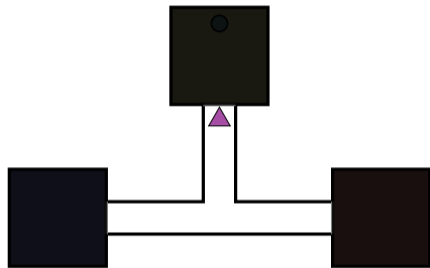
Reward Machines as memory



conditions at state u_0

if $\langle \square \bullet \rangle$	\rightarrow	goto u_1
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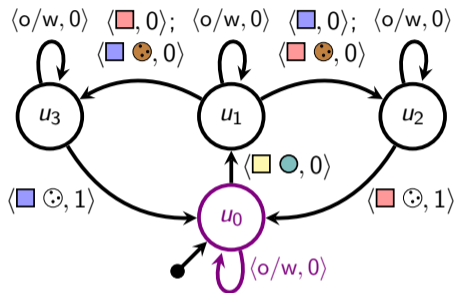
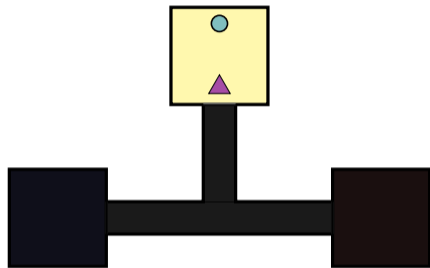
Reward Machines as memory



conditions at state u_0

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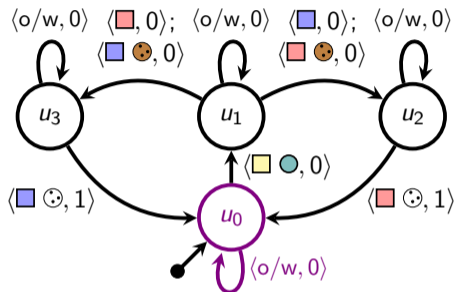
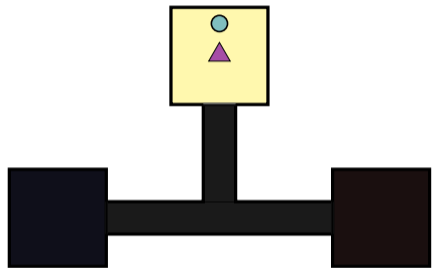
Reward Machines as memory



conditions at state u_0

if (yellow square blue circle)	\rightarrow	goto u_1
else	\rightarrow	goto u_0

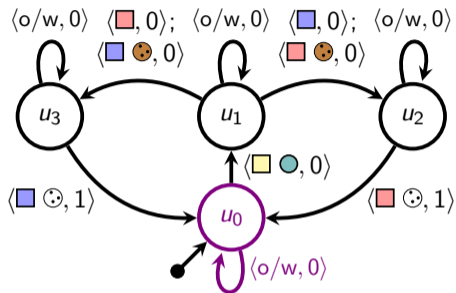
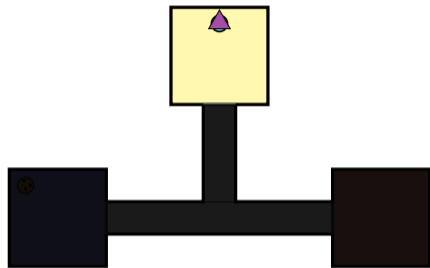
Reward Machines as memory



conditions at state u_0

if (yellow square cyan circle)	→	goto u_1
else	→	goto u_0

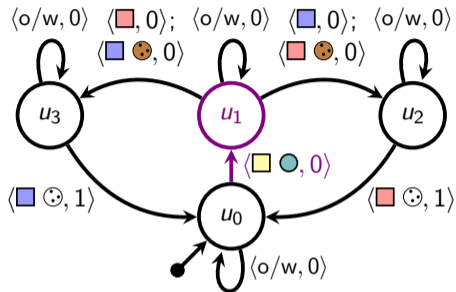
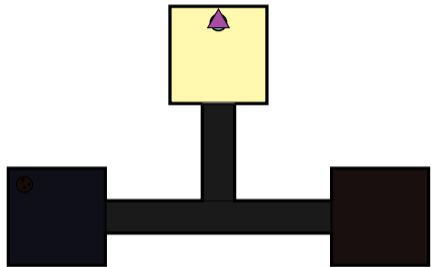
Reward Machines as memory



conditions at state u_0

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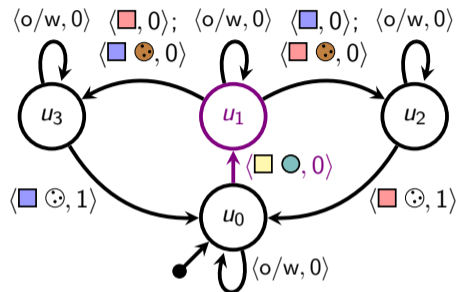
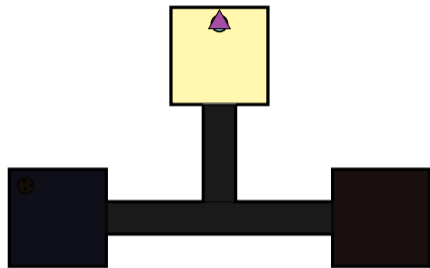
Reward Machines as memory



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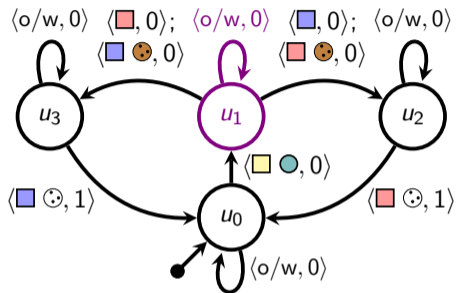
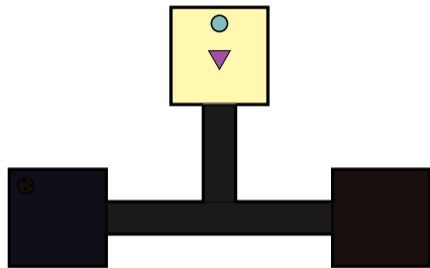
Reward Machines as memory



conditions at state u_1

if (blue or red smiley)	\rightarrow	goto u_2
if (red or blue smiley)	\rightarrow	goto u_3
else	\rightarrow	goto u_1

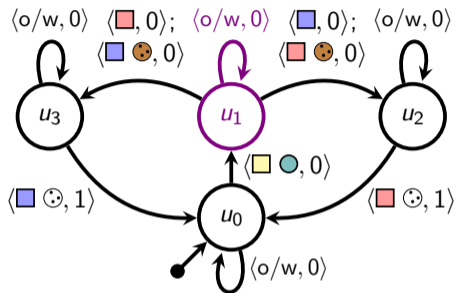
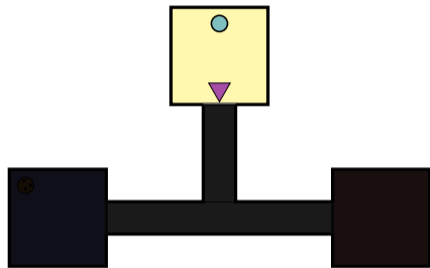
Reward Machines as memory



conditions at state u_1

if (blue square or $\text{red square, brown circle}$)	\rightarrow	goto u_2
if (red square or $\text{blue square, brown circle}$)	\rightarrow	goto u_3
else	\rightarrow	goto u_1

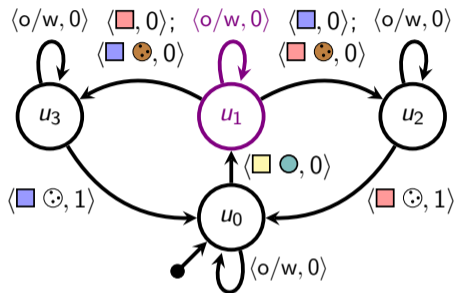
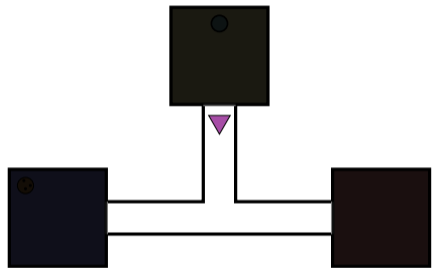
Reward Machines as memory



conditions at state u_1

if (blue square or $\text{red square, brown circle}$)	\rightarrow	goto u_2
if (red square or $\text{blue square, brown circle}$)	\rightarrow	goto u_3
else	\rightarrow	goto u_1

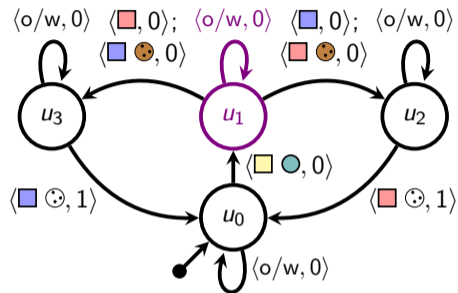
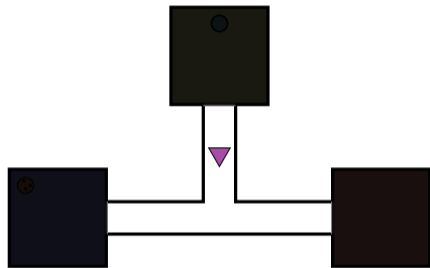
Reward Machines as memory



conditions at state u_1

if (blue square or $\text{red square smiley face}$)	\rightarrow	goto u_2
if (red square or $\text{blue square smiley face}$)	\rightarrow	goto u_3
else	\rightarrow	goto u_1

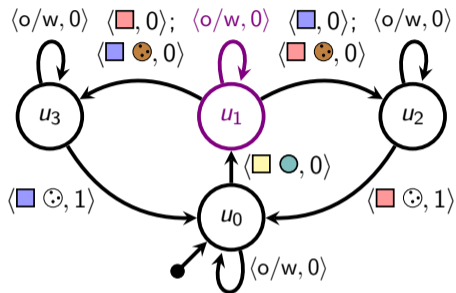
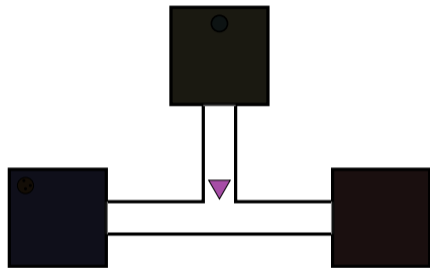
Reward Machines as memory



conditions at state u_1

if (blue square or $\text{red square smiley face}$)	\rightarrow	goto u_2
if (red square or $\text{blue square smiley face}$)	\rightarrow	goto u_3
else	\rightarrow	goto u_1

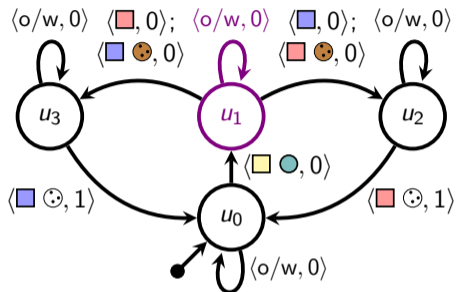
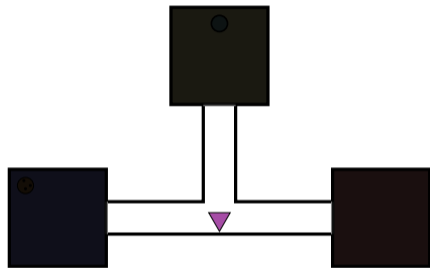
Reward Machines as memory



conditions at state u_1

if (blue square or $\text{red square, brown circle}$)	\rightarrow	goto u_2
if (red square or $\text{blue square, brown circle}$)	\rightarrow	goto u_3
else	\rightarrow	goto u_1

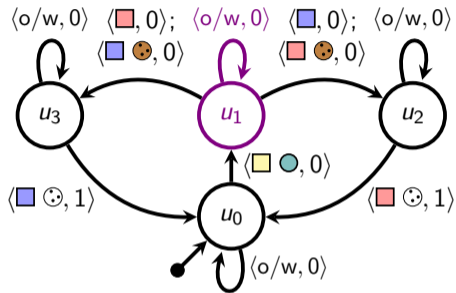
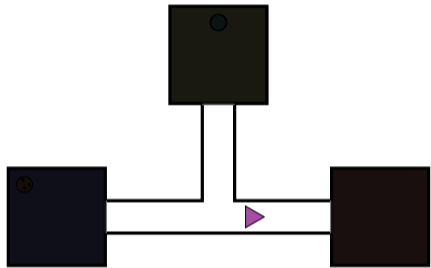
Reward Machines as memory



conditions at state u_1

if (blue square or $\text{red square, brown circle}$)	\rightarrow	goto u_2
if (red square or $\text{blue square, brown circle}$)	\rightarrow	goto u_3
else	\rightarrow	goto u_1

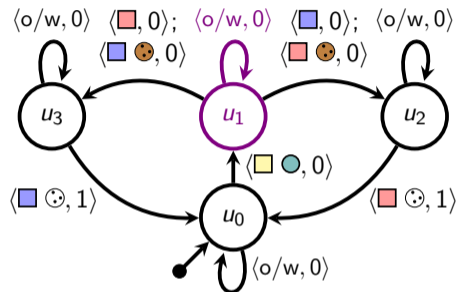
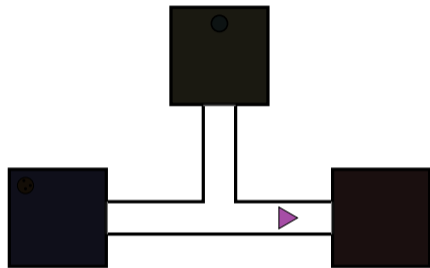
Reward Machines as memory



conditions at state u_1

if (blue square or $\text{red square smiley face}$)	\rightarrow	goto u_2
if (red square or $\text{blue square smiley face}$)	\rightarrow	goto u_3
else	\rightarrow	goto u_1

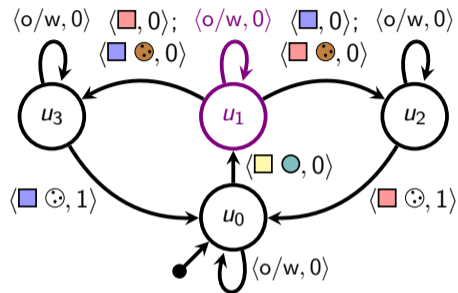
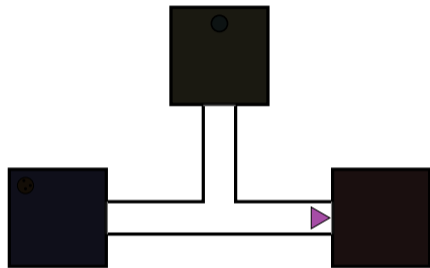
Reward Machines as memory



conditions at state u_1

if (blue square or $\text{red square, brown circle}$)	\rightarrow	goto u_2
if (red square or $\text{blue square, brown circle}$)	\rightarrow	goto u_3
else	\rightarrow	goto u_1

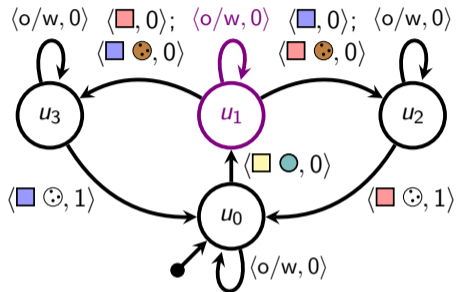
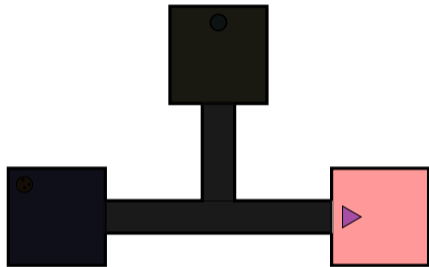
Reward Machines as memory



conditions at state u_1

if (blue square or $\text{red square smiley face}$)	\rightarrow	goto u_2
if (red square or $\text{blue square smiley face}$)	\rightarrow	goto u_3
else	\rightarrow	goto u_1

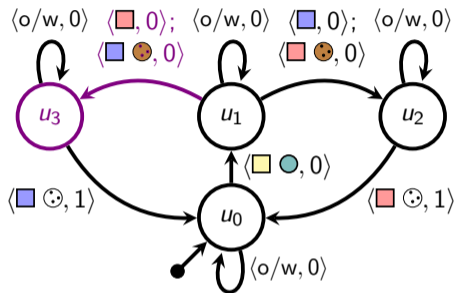
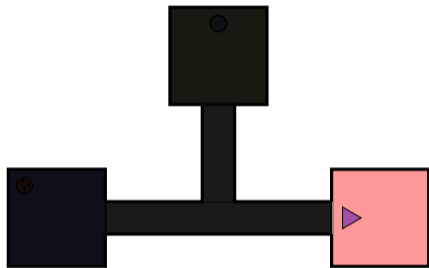
Reward Machines as memory



conditions at state u_1

if (blue square or red square smiley)	\rightarrow	goto u_2
if (red square or blue square smiley)	\rightarrow	goto u_3
else	\rightarrow	goto u_1

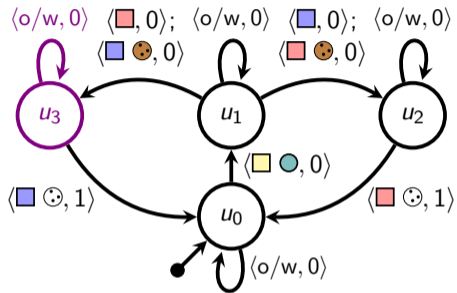
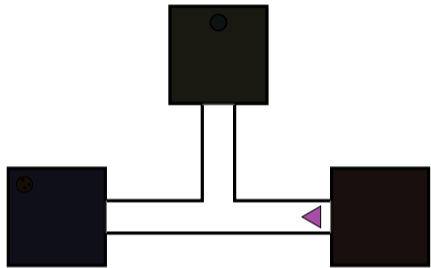
Reward Machines as memory



conditions at state u_3

if ($\langle \text{blue square, smiley} \rangle$)	\rightarrow	goto u_0
else	\rightarrow	goto u_3

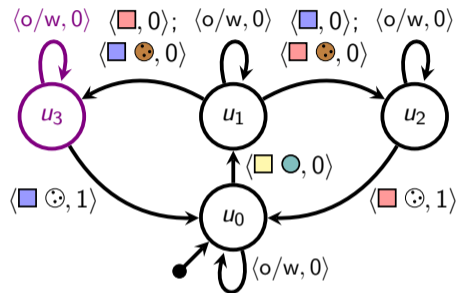
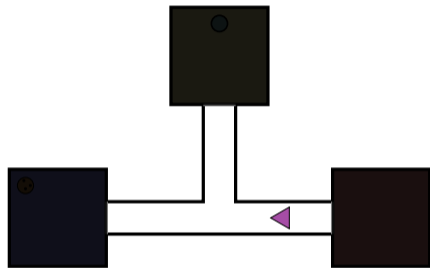
Reward Machines as memory



conditions at state u_3

if (blue square smiley face)	→	goto u_0
else	→	goto u_3

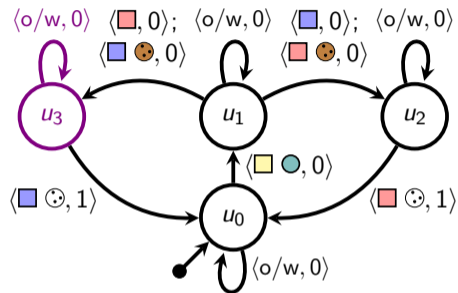
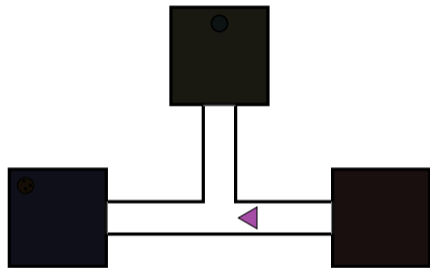
Reward Machines as memory



conditions at state u_3

if ($\langle \text{blue square}, \text{smiley face} \rangle$)	\rightarrow	goto u_0
else	\rightarrow	goto u_3

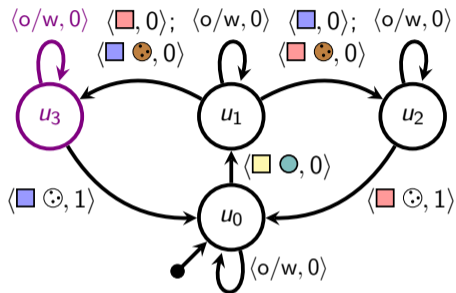
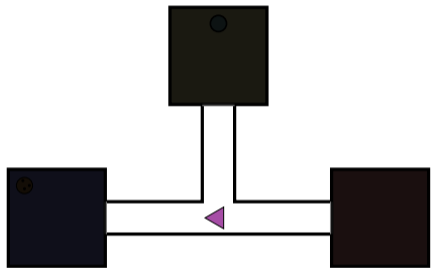
Reward Machines as memory



conditions at state u_3

if ($\langle \text{blue square, smiley face} \rangle$) \rightarrow goto u_0
else \rightarrow goto u_3

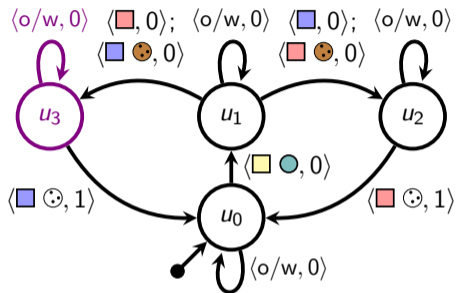
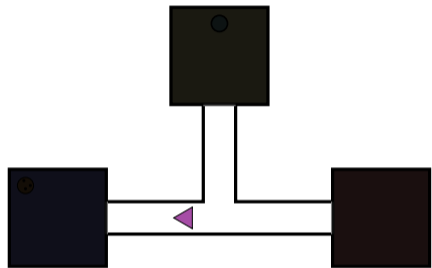
Reward Machines as memory



conditions at state u_3

if ($\text{blue circle smiley face}$) \rightarrow goto u_0
else \rightarrow goto u_3

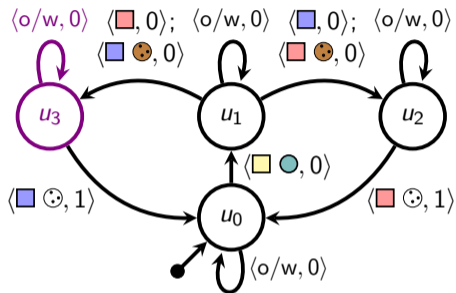
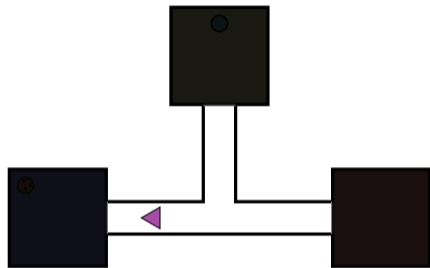
Reward Machines as memory



conditions at state u_3

if (blue square smiley face)	→	goto u_0
else	→	goto u_3

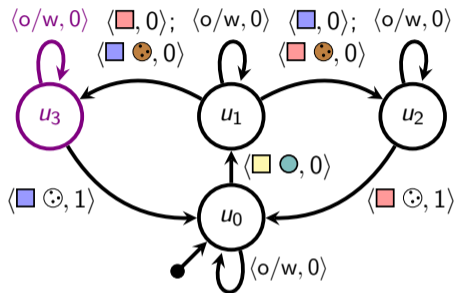
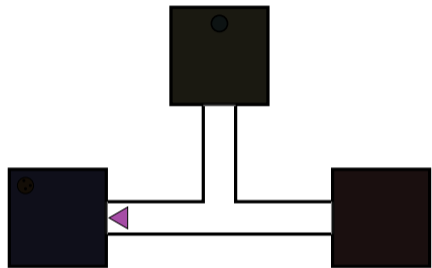
Reward Machines as memory



conditions at state u_3

if (blue square smiley)	→	goto u_0
else	→	goto u_3

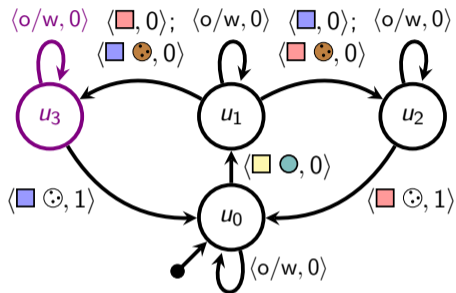
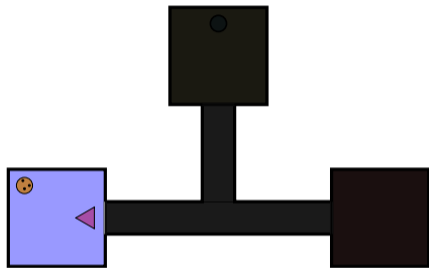
Reward Machines as memory



conditions at state u_3

if ($\langle \text{blue square, smiley face} \rangle$) \rightarrow goto u_0
else \rightarrow goto u_3

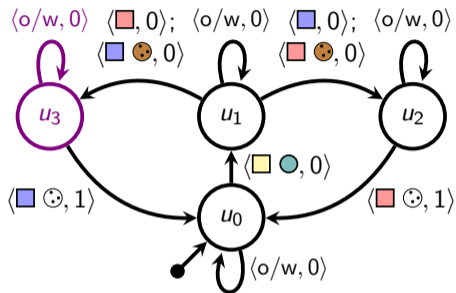
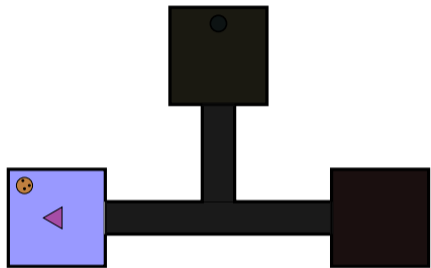
Reward Machines as memory



conditions at state u_3

if (blue square smiley) \rightarrow goto u_0
else \rightarrow goto u_3

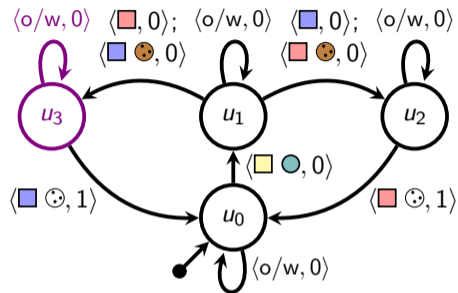
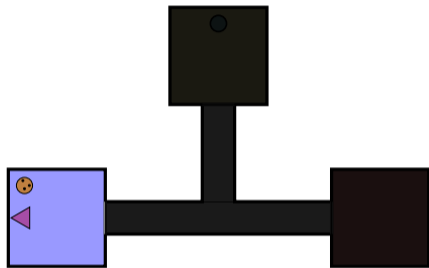
Reward Machines as memory



conditions at state u_3

if (blue square smiley face)	→	goto u_0
else	→	goto u_3

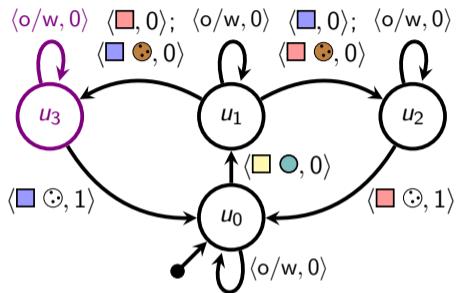
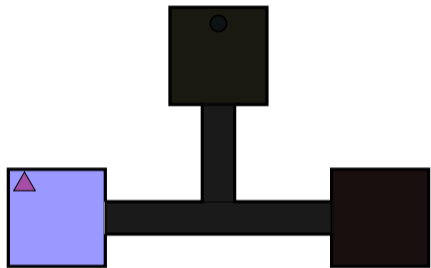
Reward Machines as memory



conditions at state u_3

if ($\langle \text{blue square, smiley} \rangle$)	\rightarrow	goto u_0
else	\rightarrow	goto u_3

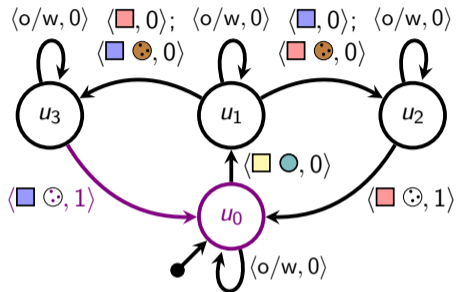
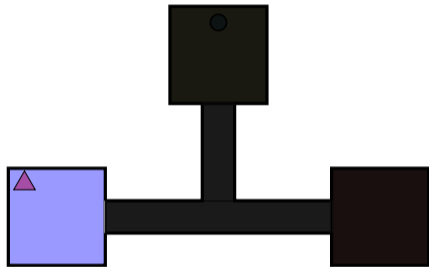
Reward Machines as memory



conditions at state u_3

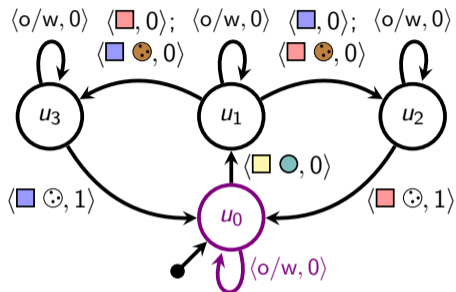
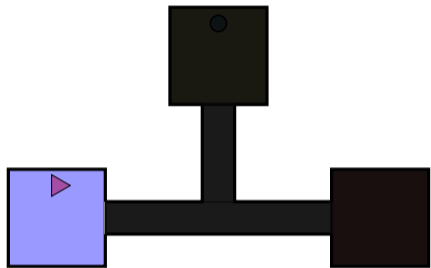
if ($\langle \text{blue square}, \text{smiley face} \rangle$) \rightarrow goto u_0
else \rightarrow goto u_3

Reward Machines as memory



conditions at state u_0	
if (yellow square teal circle)	→ goto u_1
else	→ goto u_0

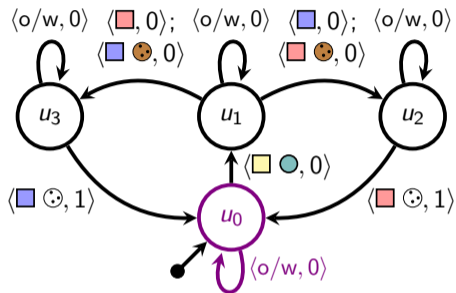
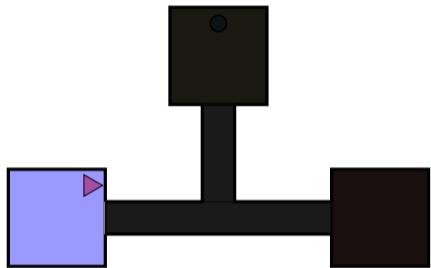
Reward Machines as memory



conditions at state u_0

if (yellow square teal circle)	\rightarrow	goto u_1
else	\rightarrow	goto u_0

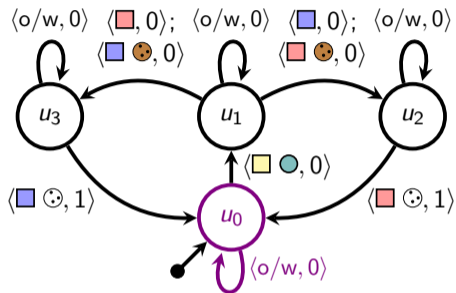
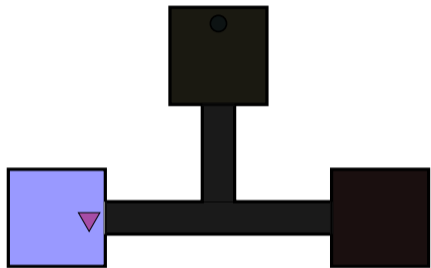
Reward Machines as memory



conditions at state u_0

if (yellow square teal circle)	→	goto u_1
else	→	goto u_0

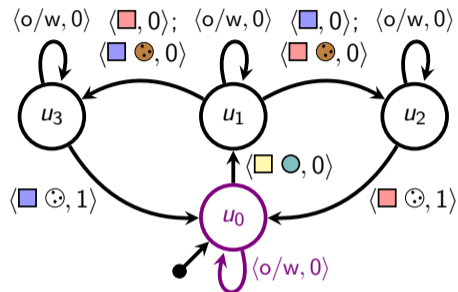
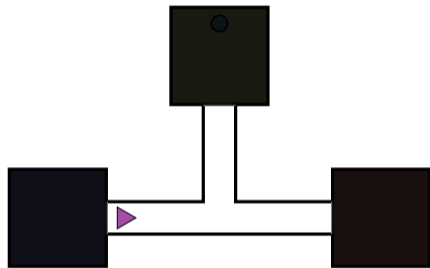
Reward Machines as memory



conditions at state u_0

if (yellow square teal circle)	→	goto u_1
else	→	goto u_0

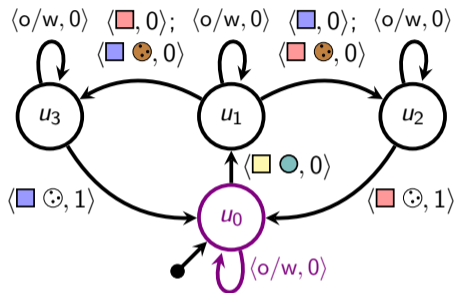
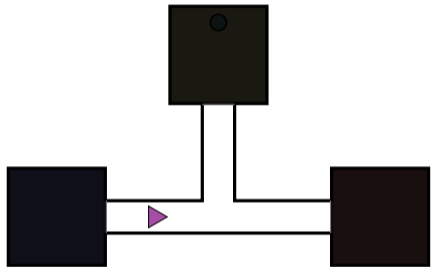
Reward Machines as memory



conditions at state u_0

if (yellow square teal circle)	→	goto u_1
else	→	goto u_0

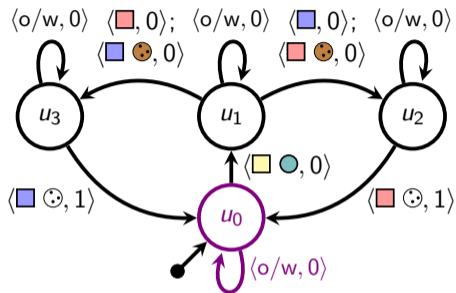
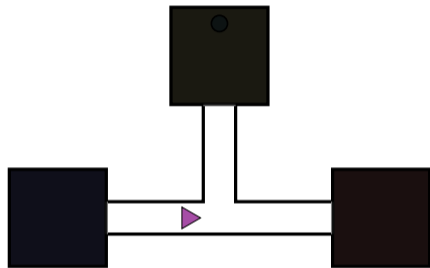
Reward Machines as memory



conditions at state u_0

if (yellow square teal circle)	\rightarrow	goto u_1
else	\rightarrow	goto u_0

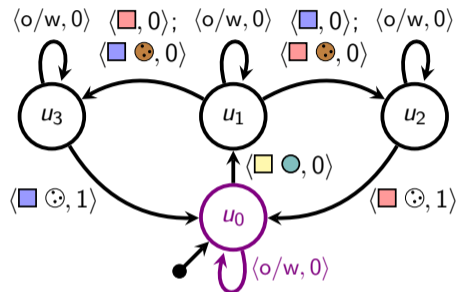
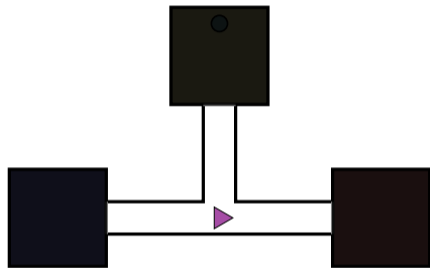
Reward Machines as memory



conditions at state u_0

if (yellow square teal circle)	→	goto u_1
else	→	goto u_0

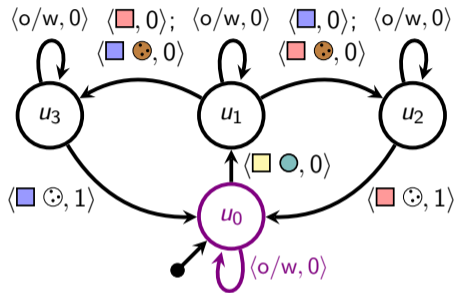
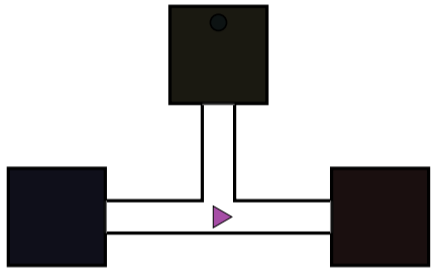
Reward Machines as memory



conditions at state u_0

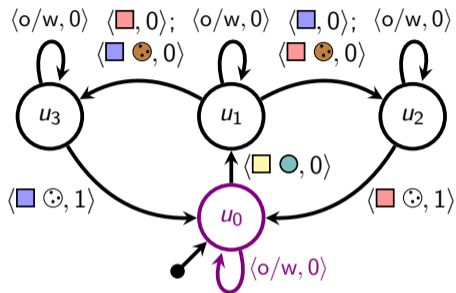
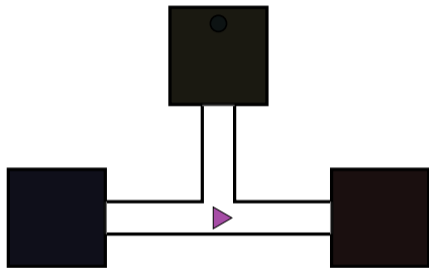
if (yellow square teal circle)	→	goto u_1
else	→	goto u_0

Reward Machines as memory



Why is this a perfect memory?

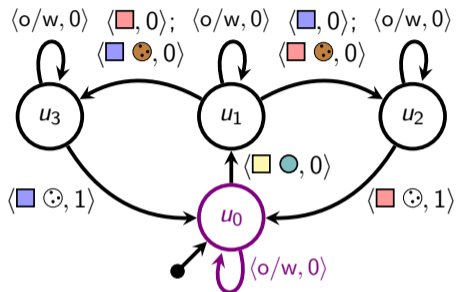
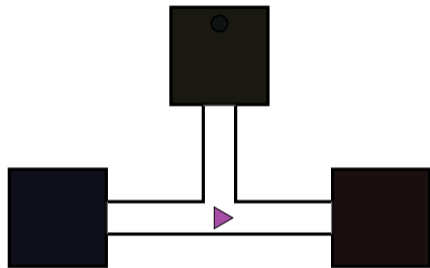
Reward Machines as memory



Why is this a perfect memory?

$$\pi^*(a|o_0, \dots, o_t) = \pi^*(a|o_t, u_t)$$

Reward Machines as memory




Why is this a perfect memory?

$$\pi^*(a|o_0, \dots, o_t) = \pi^*(a|o_t, u_t)$$

Hard problem $\xrightarrow{\text{RM}}$ Easy problem

How to learn such RMs?

Learning Reward Machines

Given a set of detectors (e.g., {, , , , , , }) and traces \mathcal{T} ,

Learning Reward Machines

Given a set of detectors (e.g., {, , , , , , , , , , , , , , , , , , , , , , , , }) and traces \mathcal{T} , learning RMs is a **discrete optimization** problem:

Learning Reward Machines

Given a set of detectors (e.g., $\{\text{red square, white square, blue square, yellow square, green circle, brown circle, smiley face}\}$) and traces \mathcal{T} , learning RMs is a **discrete optimization** problem:

$$\underset{\langle U, u_0, \delta_u, \delta_r \rangle}{\text{minimize}} \sum_{i \in I} \sum_{t \in T_i} \log(|N_{x_{i,t}, L(e_{i,t})}|) \quad (\text{LRM})$$

$$\text{s.t. } \langle U, u_0, \delta_u, \delta_r \rangle \in \mathcal{R}_{\mathcal{P}} \quad (1)$$

$$|U| \leq u_{\max} \quad (2)$$

$$x_{i,t} \in U \quad \forall i \in I, t \in T_i \cup \{t_i\} \quad (3)$$

$$x_{i,0} = u_0 \quad \forall i \in I \quad (4)$$

$$x_{i,t+1} = \delta_u(x_{i,t}, L(e_{i,t+1})) \quad \forall i \in I, t \in T_i \quad (5)$$

$$N_{u,l} \subseteq 2^{2^{\mathcal{P}}} \quad \forall u \in U, l \in 2^{\mathcal{P}} \quad (6)$$

$$L(e_{i,t+1}) \in N_{x_{i,t}, L(e_{i,t})} \quad \forall i \in I, t \in T_i \quad (7)$$

Learning Reward Machines

Given a set of detectors (e.g., $\{\color{red}\square, \color{orange}\square, \color{blue}\square, \color{yellow}\square, \color{green}\circ, \color{brown}\odot, \color{grey}\odot\}$) and traces \mathcal{T} , learning RMs is a **discrete optimization** problem:

$$\underset{\langle U, u_0, \delta_u, \delta_r \rangle}{\text{minimize}} \sum_{i \in I} \sum_{t \in T_i} \log(|N_{x_{i,t}, L(e_{i,t})}|) \quad (\text{LRM})$$

$$\text{s.t. } \langle U, u_0, \delta_u, \delta_r \rangle \in \mathcal{R}_{\mathcal{P}} \quad (1)$$

$$|U| \leq u_{\max} \quad (2)$$

$$x_{i,t} \in U \quad \forall i \in I, t \in T_i \cup \{t_i\} \quad (3)$$

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$$N_{u,l} \subseteq 2^{\mathcal{P}} \quad \forall u \in U, l \in 2^{\mathcal{P}} \quad (6)$$

$$L(e_{i,t+1}) \in N_{x_{i,t}, L(e_{i,t})} \quad \forall i \in I, t \in T_i \quad (7)$$

Learn a
"simple"
causal model

Learning Reward Machines

Given a set of detectors (e.g., $\{\text{red square, white square, blue square, yellow square, green circle, brown circle, sad face}\}$) and traces \mathcal{T} , learning RMs is a **discrete optimization** problem:

$$\underset{\langle U, u_0, \delta_u, \delta_r \rangle}{\text{minimize}} \sum_{i \in I} \sum_{t \in T_i} \log(|N_{x_{i,t}, L(e_{i,t})}|) \quad (\text{LRM})$$

$$\text{s.t. } \langle U, u_0, \delta_u, \delta_r \rangle \in \mathcal{R}_{\mathcal{P}} \quad (1)$$

$$|U| \leq u_{\max} \quad (2)$$

$$x_{i,t} \in U \quad \forall i \in I, t \in T_i \cup \{t_i\} \quad (3)$$

$$x_{i,0} = u_0 \quad \forall i \in I \quad (4)$$

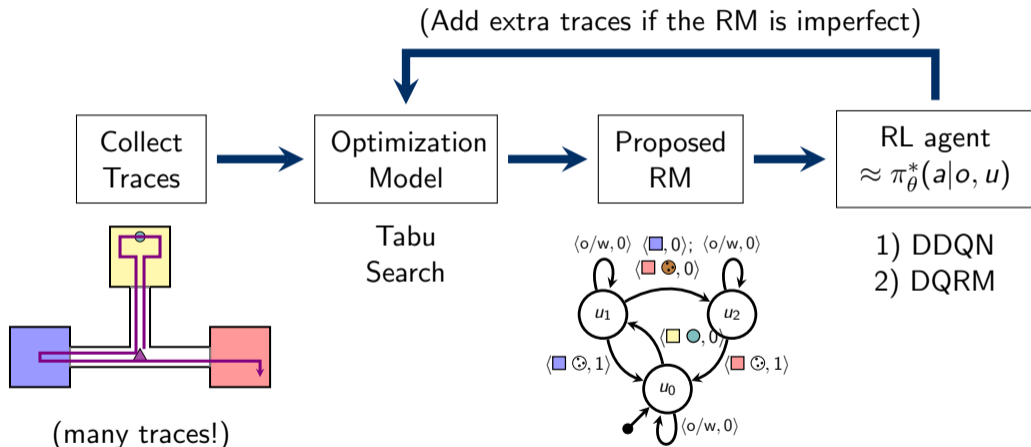
$$x_{i,t+1} = \delta_u(x_{i,t}, L(e_{i,t+1})) \quad \forall i \in I, t \in T_i \quad (5)$$

$$N_{u,l} \subseteq 2^{\mathcal{P}} \quad \forall u \in U, l \in 2^{\mathcal{P}} \quad (6)$$

$$L(e_{i,t+1}) \in N_{x_{i,t}, L(e_{i,t})} \quad \forall i \in I, t \in T_i \quad (7)$$

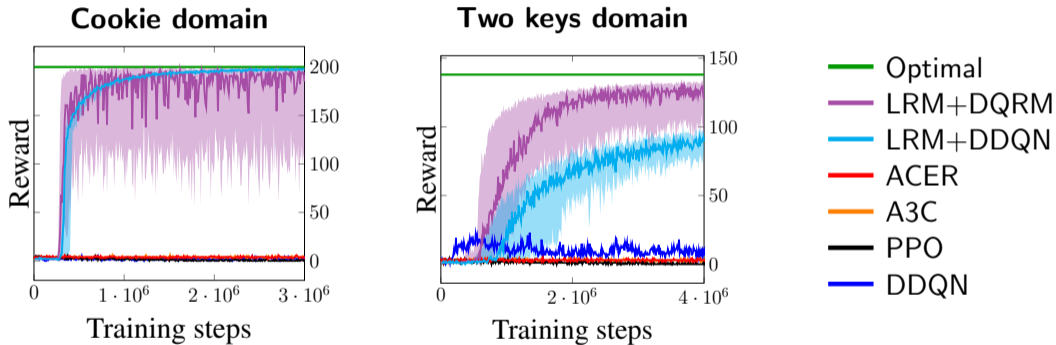
... that we solved using **Tabu Search**.

Overall approach



Results

Results



***Note:** The detectors were also given to the baselines.

Discussion at poster #210

<https://bitbucket.org/RToroIcarte/lrm>

Thanks! :)



Rodrigo



Ethan



Toryn



Rick



Margarita



Sheila