Efficient Meta Learning via Minibatch Proximal Update

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Dec 11, 2019

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Meta-MinibatchProx learns a good **prior model initialization** w from observed tasks such that

w **is close to the optimal models of new similar tasks, promoting new task learning**

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- **Training model:** given a task distribution $\mathcal{T}$, we minimize a **bi-level** meta learning model

$$\min_{\boldsymbol{w}} \ \frac{1}{n}\sum_{i=1}^{n} \min_{\boldsymbol{w}_{T_i}} \mathcal{L}_{D_{T_i}}(\boldsymbol{w}_{T_i}) + \frac{\lambda}{2}\|\boldsymbol{w} - \boldsymbol{w}_{T_i}\|_2^2,$$

where each task $T_i \sim \mathcal{T}$ has K training samples $D_{T_i} = \{(\boldsymbol{x}_s, \boldsymbol{y}_s)\}_{s=1}^{K}$

$\mathcal{L}_{D_{T_i}} = \frac{1}{K}\sum_{(\boldsymbol{x},\boldsymbol{y})\in D_{T_i}} \ell(f(\boldsymbol{w},\boldsymbol{x}),\boldsymbol{y})$ is empirical loss with predictor f and loss ℓ.

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update task-specific solution

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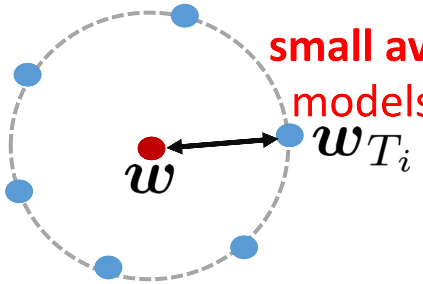

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$$\min_{\boldsymbol{w}_T} \ \mathcal{L}_{D_T}(\boldsymbol{w}_T) + \frac{\lambda}{2}\|\boldsymbol{w}^* - \boldsymbol{w}_T\|_2^2,$$

where $\boldsymbol{w}^*$ denotes the learnt prior initialization.

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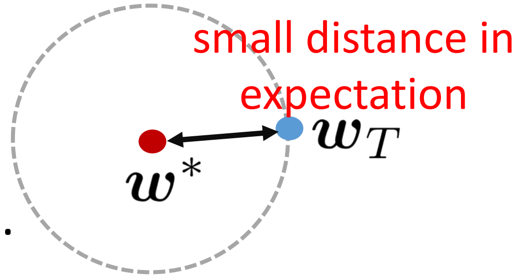
- **Benefit:** a few data is sufficient for adaptation

the learnt prior initialization $\boldsymbol{w}^*$ is close to optimum $\boldsymbol{w}_T$

when training and test tasks are sampled from the same distribution.



Optimization Algorithm

We use SGD based algorithm to solve bi-level training model :

$$\min_{\boldsymbol{w}} \ \left\{F(\boldsymbol{w}) := \frac{1}{n}\sum_{i=1}^{n} \min_{\boldsymbol{w}_{T_i}} \mathcal{L}_{D_{T_i}}(\boldsymbol{w}_{T_i}) + \frac{\lambda}{2}\|\boldsymbol{w} - \boldsymbol{w}_{T_i}\|_2^2\right\}$$

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- Step1. select a mini-batch of task $\{T_i\}$ of size b_s .
- Step2. for T_i, compute an approximate minimizer:

$$\boldsymbol{w}_{T_i} \approx \mathrm{argmin}_{\boldsymbol{w}_{T_i}} \{g(\boldsymbol{w}_{T_i}) := \mathcal{L}_{D_{T_i}}(\boldsymbol{w}_{T_i}) + \frac{\lambda}{2}\|\boldsymbol{w} - \boldsymbol{w}_{T_i}\|_2^2\}, \ \text{namely} \ \|\nabla g(\boldsymbol{w}_{T_i})\|_2^2 \leq \epsilon_s$$

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- Step3. update the prior initialization model:

$$\boldsymbol{w} = \boldsymbol{w} - \eta_s \lambda(\boldsymbol{w} - \frac{1}{b_s}\sum_{i=1}^{b_s} \boldsymbol{w}_{T_i})$$

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Theorem 1 (convergence guarantees, informal).

(1) Convex setting, i.e. convex $\phi_{D_{T_i}}(\boldsymbol{w})$. We prove $\mathbb{E}[\|\boldsymbol{w}^S - \boldsymbol{w}^*\|_2^2] \leq \mathcal{O}(\frac{1}{S})$.

(2) Nonconvex setting, i.e. smooth $\phi_{D_{T_i}}(\boldsymbol{w})$. We prove $\mathbb{E}_s[\|\nabla F(\boldsymbol{w}^s)\|_2^2] \leq \mathcal{O}(\frac{1}{\sqrt{S}})$.

Generalization Performance Guarantee

- Ideally, for a given task $T \sim \mathcal{T}$, one should train the model on the population risk

$$\text{Population solution: } \boldsymbol{w}_{T,P}^* = \text{argmin}_{\boldsymbol{w}_T} \{\mathcal{L}(\boldsymbol{w}_T) := \mathbb{E}_{(\boldsymbol{x},\boldsymbol{y})\sim T} \ell(f(\boldsymbol{w}_T, \boldsymbol{x}), \boldsymbol{y})\}.$$

- In practice, we has only K samples and adapt the learnt prior model $\boldsymbol{w}^*$ to the new task:

$$\text{Empirical solution: } \boldsymbol{w}_T^* = \text{argmin}_{\boldsymbol{w}_T} \mathcal{L}_{D_T}(\boldsymbol{w}_T) + \frac{\lambda}{2}\|\boldsymbol{w}^* - \boldsymbol{w}_T\|_2^2.$$

- **Since $\boldsymbol{w}_{T,P}^* \neq \boldsymbol{w}_T^*$, why $\boldsymbol{w}_T^*$ is good for generalization in few-shot learning problem?**

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- **Since $w_{T,P}^{*} \neq \boldsymbol{w}_T^*$, why w_T^* is good for generalization in few-shot learning problem?**

Theorem 2 (generalization performance guarantee, informal).

Suppose each loss $\phi_{D_{T_i}}(\boldsymbol{w})$ is convex and is smooth. Let $D_T = \{(\boldsymbol{x}_i, \boldsymbol{y}_i)\}_{i=1}^{K} \sim T$. Then we have

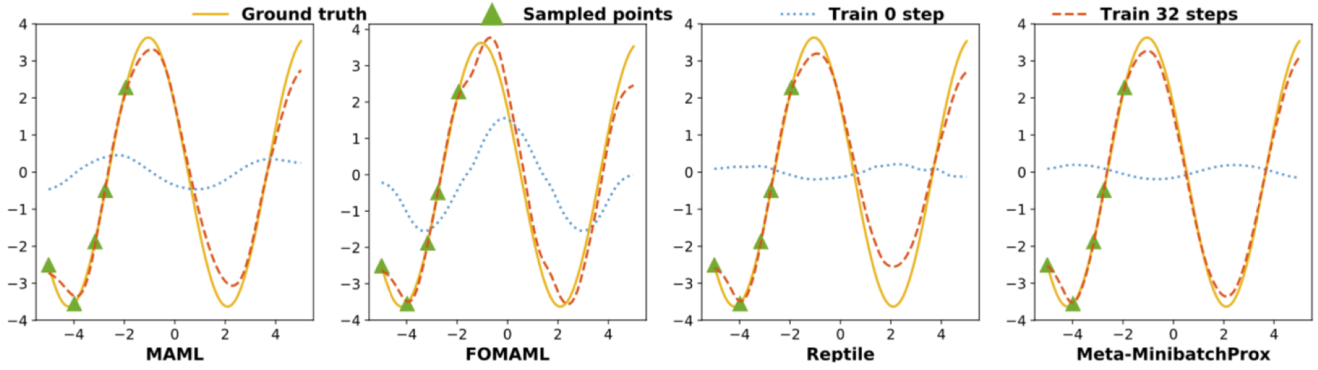
$$\mathbb{E}_{T\sim\mathcal{T}}\mathbb{E}_{D_T\sim T}(\mathcal{L}(\boldsymbol{w}_T^*) - \mathcal{L}(\boldsymbol{w}_{T,P}^*)) \leq \frac{c}{\sqrt{K}}\mathbb{E}[\|\boldsymbol{w}^* - \boldsymbol{w}_{T,P}^*\|_2^2]\quad \text{with a constant } c. \quad (1)$$

Remark: strong generalization performance, as our training model guarantees

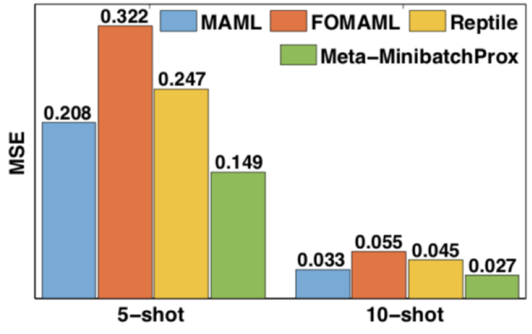
the learnt prior w^* is close to the optimum model $w_{T,P}^*$.

Experimental results

Few-shot regression : smaller mean square error (MSE) between prediction and ground truth

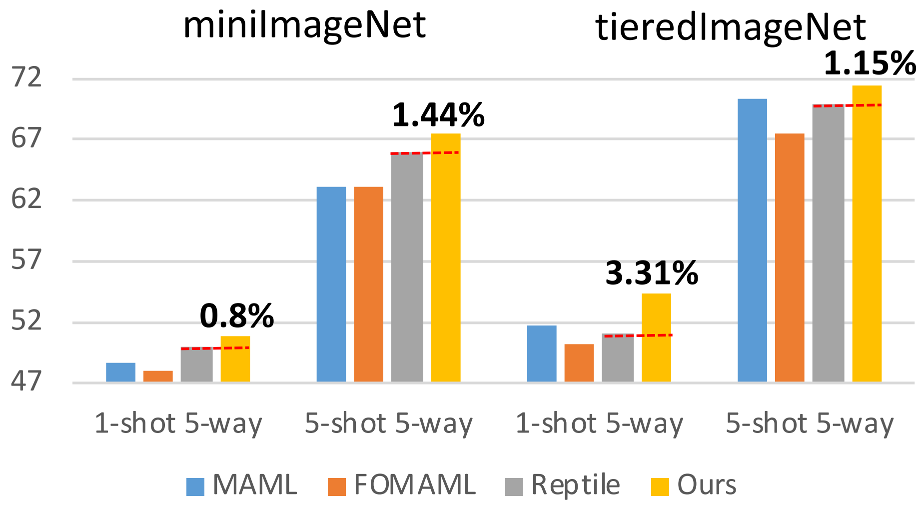


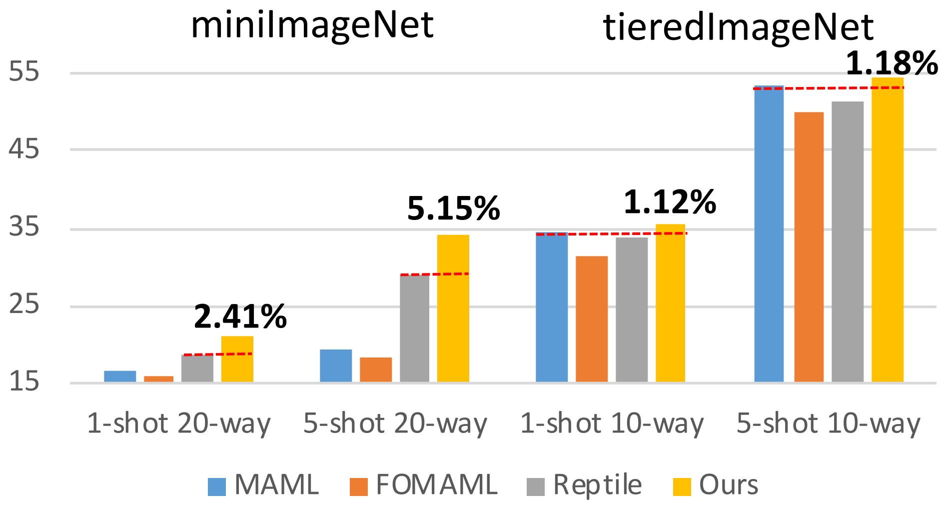
(a) Visual illustration



(b) MSE

Few-shot classification: higher classification accuracy





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05:00 -- 07:00 PM @ East Exhibition Hall B + C

Thanks!