A Spectral View of Adversarially Robust Features

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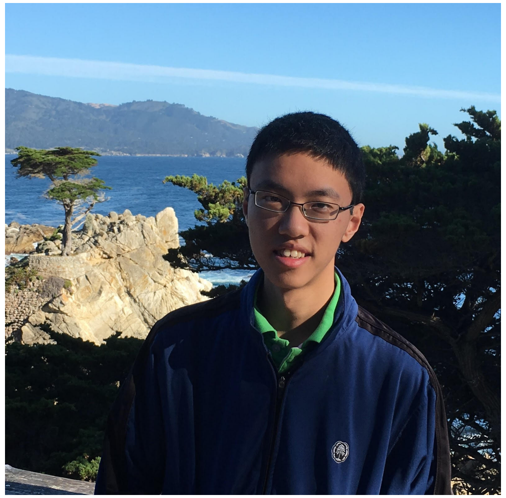

Brian Zhang*

Gregory Valiant

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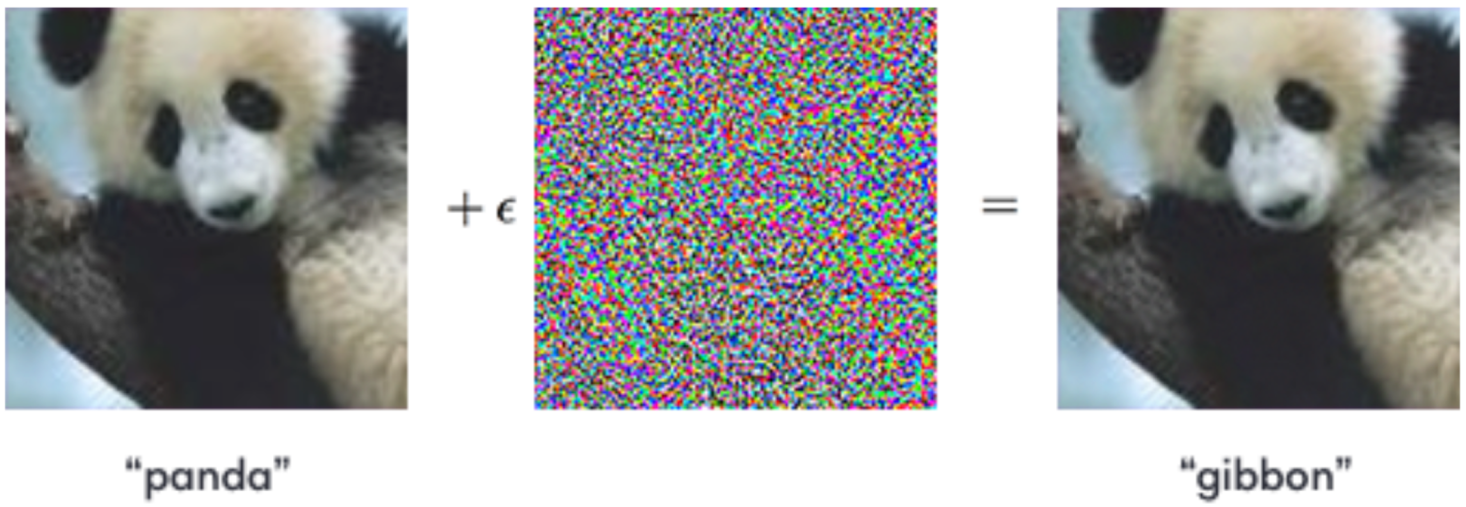
What are adversarial examples?



Adding small amount of well-crafted noise to the test data fools the classifier

More Questions than Answers

Intense ongoing research efforts, but we still don’t have a good understanding of many basic questions:

- What are the tradeoffs between the amount of data available, accuracy of the trained model, and vulnerability to adversarial examples?

- What properties of the geometry of a dataset make models trained on it vulnerable to adversarial attacks?

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- Disentangles the challenges of robustness and classification performance

- Train a classifier on top of robust features

Connections to Spectral Graph Theory

- Second eigenvector v of the Laplacian of a graph is the solution to:

$$\min_{v} \sum_{(i,j)\in E} (v_i - v_j)^2 \quad \text{s.t.} \quad \sum_i v_i = 0; \quad \sum_i v_i^2 = 1$$

- Assigns values to vertices that change smoothly across neighbors

- Constraints ensure sufficient variance among these values

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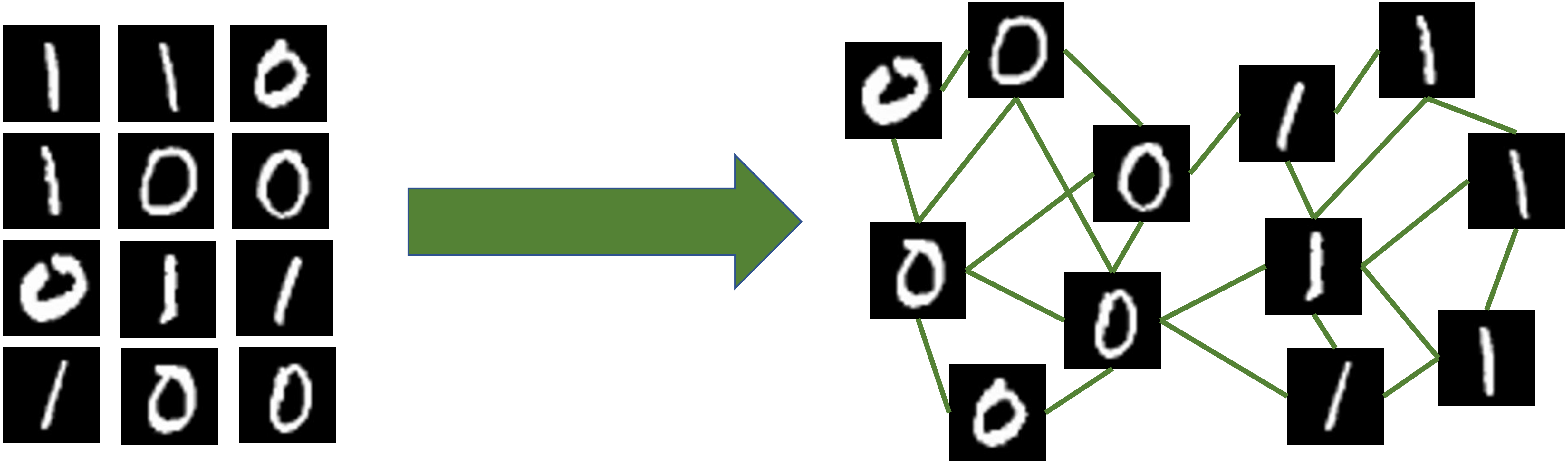
- **Upper bound**: Characterizes the robustness of features in terms of eigen values and spectral gap of the Laplacian

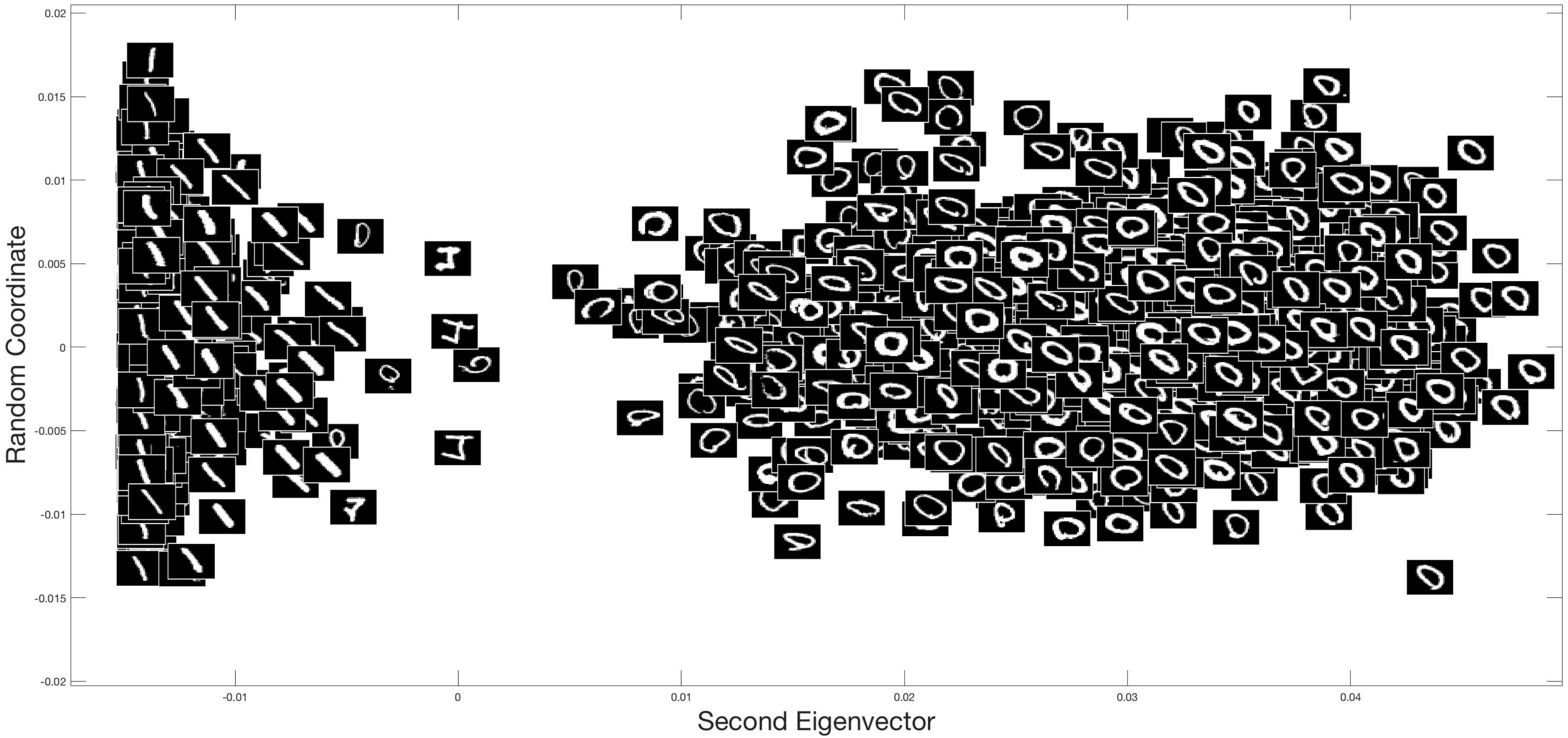
- **Lower bound**: Roughly says that if there exists a robust feature, the spectral approach would find it under certain conditions on the properties of Laplacian.

Illustration: Create a Graph



Create similarity graph according to a given distance metric
[the same metric that we hope to be robust wrt]

Illustration: Extract Feature from 2nd eigenvector



$$f(x_i) = v_2\,(x_i)$$

Takeaways

- Disentangling the two goals of robustness and classification performance may help us understand the extent to which a given dataset is vulnerable to adversarial attacks, and ultimately might help us develop better robust classifiers
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Thank you!