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UNIVERSITY OF JYVÄSKYLÄ

Graphical model inference: Sequential Monte Carlo meets deterministic approximations

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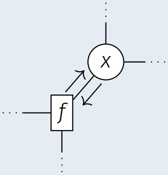
Jouni Helske (Linköping University)
Matti Vihola (University of Jyväskylä)

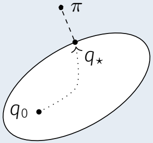
Approximate Bayesian inference

Deterministic methods

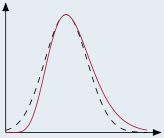
Message passing



Variational inference

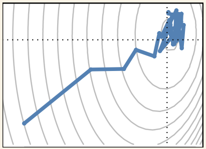


Laplace's method

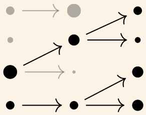


Monte Carlo methods

Markov chain Monte Carlo



Sequential Monte Carlo



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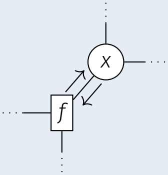

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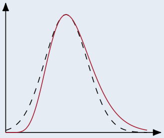

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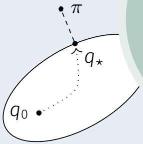

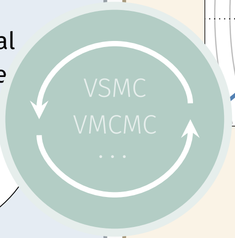

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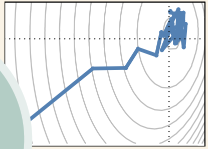
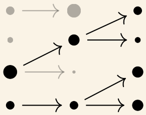


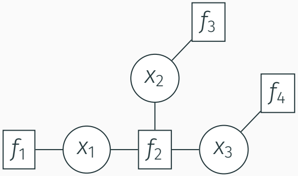
Sequential Monte Carlo



Probabilistic graphical models

We consider inference in *factor graphs* with joint distribution

$$\pi(x_{1:T}) = \frac{1}{Z} \prod_{j \in \mathcal{F}} f_j(x_{\mathcal{I}_j}).$$



Task:

- Compute expectations w.r.t. $\pi(x_{1:T})$.
- Compute the normalizing constant Z.

Sequential Monte Carlo

Sequential Monte Carlo (SMC) can be used for probabilistic graphical model inference via *sequential graph decompositions:*

Christian A. Naesseth, Fredrik Lindsten and Thomas B. Schön. **Sequential Monte Carlo methods for graphical models.** *Advances in Neural Information Processing Systems 27*, December, 2014.

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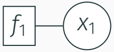
Iteration $t = 1$



$\gamma_1(x_1)$

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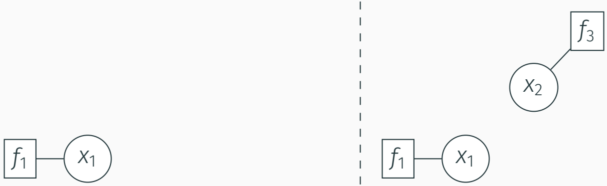
Iteration $t = 1$ | **Iteration** $t = 2$



$\gamma_1(x_1)$ | $\gamma_2(x_{1:2})$

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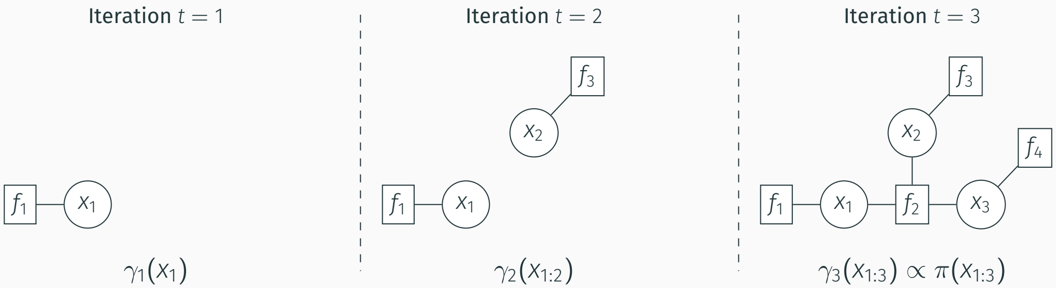

Twisted SMC

Dependencies on “future variables” are not taken into account!

Twisted intermediate targets:

$$\gamma_t^{\psi}(x_{1:t}) := \psi_t(x_{1:t})\gamma_t(x_{1:t}) = \psi_t(x_{1:t}) \prod_{j \in \mathcal{F}_t} f_j(x_{\mathcal{I}_j}).$$

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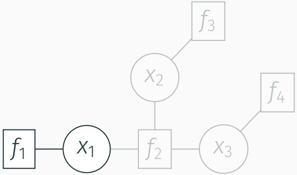

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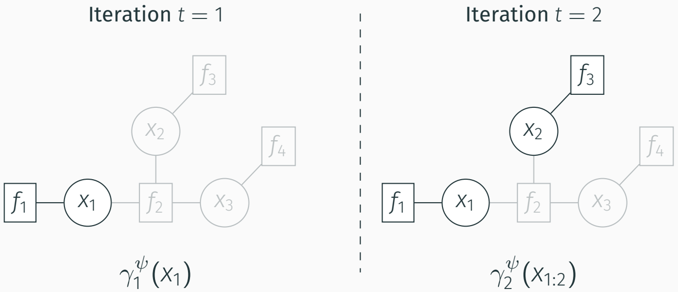

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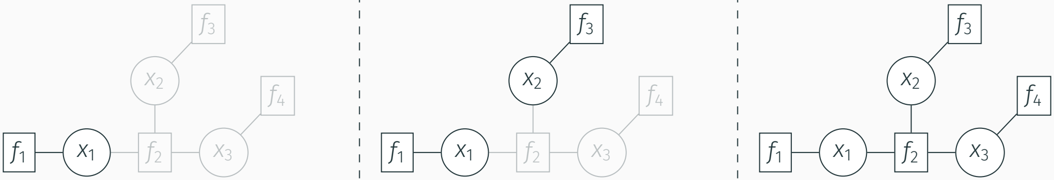
Iteration $t = 1$ | **Iteration** $t = 2$ | **Iteration** $t = 3$



$\gamma_1^{\psi}(x_1)$ | $\gamma_2^{\psi}(x_{1:2})$ | $\gamma_3^{\psi}(x_{1:3}) \propto \pi(x_{1:3})$

How do we choose the twisting functions?

Proposition (Optimal twisting). With

$$\psi_t^*(x_{1:t}) = \int \prod_{j \in \mathcal{F} \setminus \mathcal{F}_t} f_j(x_{\mathcal{I}_j}) dx_{t+1:T},$$

the SMC algorithm outputs i.i.d. draws from π and the normalizing constant estimate is exact; $\widehat{Z} = Z$ w.p.1.

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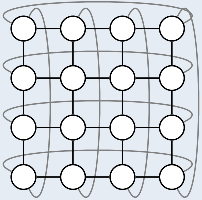
Optimal twisting functions are intractable, but:

- $\psi_t \approx \psi_t^*$ can be computed by various *deterministic inference methods*
- Sub-optimality only affects efficiency, not consistency or unbiasedness
- Can be seen as a bias post-correction

Twisting functions via deterministic approximations

Loopy Belief Propagation

ex) Square lattice Ising model



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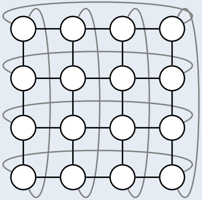
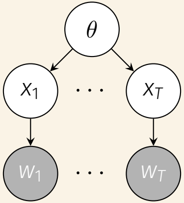


Expectation Propagation

ex) Topic model likelihood evaluation



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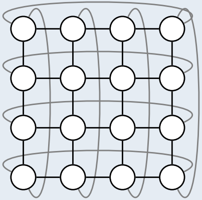

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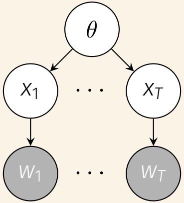
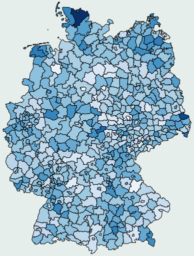


Laplace Approximation

ex) Gaussian Markov random field



Thank you for listening!

Come see the poster: #51

Code available at:

- `github.com/freli005/smc-pgm-twist`
- `github.com/helske/particlefield`